Wavelet design via algorithmic real algebraic geometry

Bernd Bank & Lutz Lehmann

Humboldt Universität zu Berlin, Germany

Joint work with
Marc Giusti, Joos Heintz, Luis Miguel Pardo

Workshop on Complexity of Numerical Computation
Fields Institute, University Toronto, Canada
October 20–24, 2009

Abstract

As a result of the TERA–project a new type, highly efficient probabilistic algorithm for the solution of systems of polynomial equations was developed and implemented for the complex case. The geometry of polar varieties allows to extend this algorithm to a method that finds real solutions of systems of polynomial equations.
In order to test this method special emphasis was placed on the fact that example problems are of real-life and practical importance.
In the talk we report on the application of the method to real polynomial equation system solving basic for the design of fast wavelet transforms. The wavelet transforms we have in mind should reflect the practical important properties of symmetry and orthogonality. These requirements are expressible by a finite number of real parameters satisfying a finite system of polynomial equations.
If these equations have a real solution at all, the solution set can be finite or a variety of positive dimension. Examples with real solution sets of positive dimension have the advantage that one can search for optimal solutions in the sense that the wavelets have additionally desired analytic properties.
It turns out that our algorithm performes very well with this task and is able to solve larger systems than the best known commercial polynomial equation solvers.