Optimal Portfolio Choice with Contagion Risk and Restricted Information

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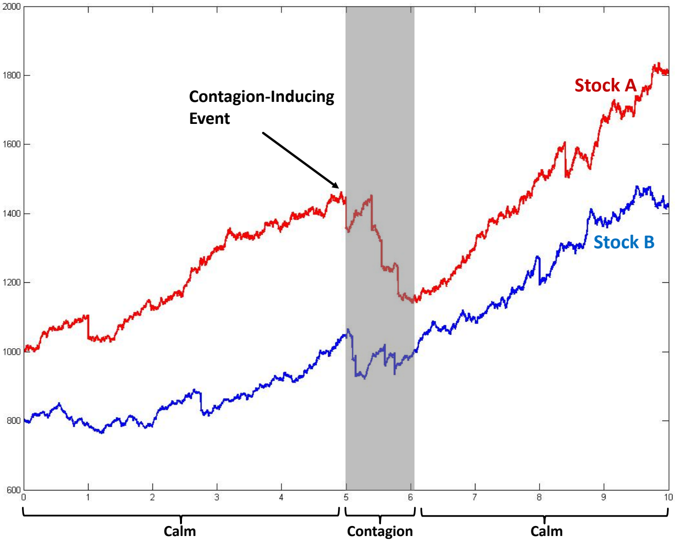
Contagion Risk



How to deal with contagion risk in an asset allocation model?

- Starting point: **asset allocation in a jump-diffusion setup**
  - → Merton (1969, 1971), Liu/Pan (2003), Liu/Longstaff/Pan (2003), Branger/Schlag/Schneider (2008),...
- First extension: **joint Poisson jumps**
  - → Das/Uppal (2004), Kraft/Steffensen (2008), Ait-Sahalia/Cacho-Diaz/Hurd (2009), ...
  - → disregard the time dimension of contagion
- Second extension: **regime-switching models**
  - → Ang/Bekaert (2002) Guidolin/Timmermann (2005, 2007, 2008), Kole/Koedijk/Verbeek (2006), ...
  - → state variable and asset prices are not linked directly
  - → up to now, mainly diffusion models

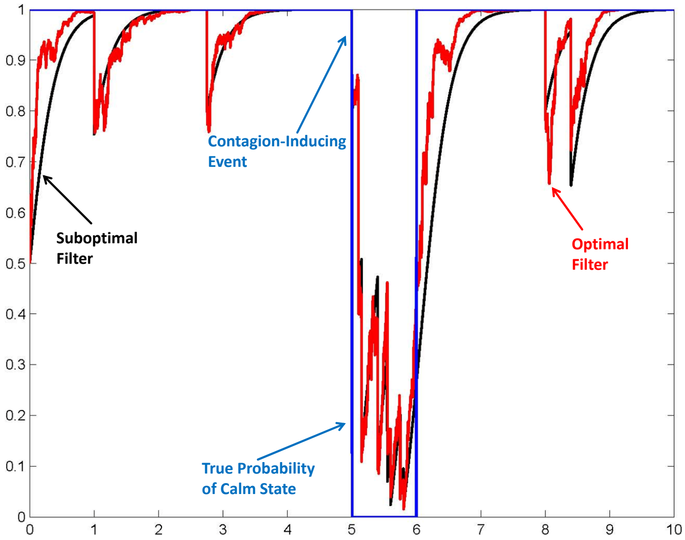
- **Our approach**
  - Two economic regimes ('calm', 'contagion')
  - Regime switches and asset prices are linked directly: some (not all) asset price jumps trigger contagion
  - **Explicitly takes time dimension of contagion into account**
  - See Branger, Kraft, Meinerding (2009) (focus on model risk)

- **Restricted information**
  - **Investor cannot identify the state directly** (... but has to learn from historical asset prices)
  - (Subjective) probability of being in the calm state:

$$\widehat{p}_t \in [0, 1]$$

  - Investor optimizes conditional upon the **state variable** $\widehat{p}_t$

Restricted Information and Filtering



1. **Contagion and learning have a substantial impact**
   - underreaction to contagion-triggering jumps
   - overreaction to noncontagious jumps
     (and subsequent re-adjustment of portfolio)
2. **Complete and incomplete market differ structurally**
   - complete market: largest reaction to first jump
     ('risk of contagion')
   - incomplete market: largest reaction to subsequent jumps
     ('confirmation of contagion')
   - larger trading volume in complete market
3. **Significant hedging demand**
   - up to 50% of speculative demand
   - may be nonmonotonic function of state variable $\widehat{p}_t$

- **Two risky assets** (A and B) with dynamics

$$\frac{dS_i(t)}{S_i(t)} = \mu_i^{Z(t)}dt + \sigma_i^{Z(t)}dW_i(t) - \sum_{K \neq Z(t-)} L_i^{Z(t-),K} dN^K(t)$$

 under the 'large' filtration $\{\mathcal{F}_t\}_{t\in[0,T]}$

 $Z(t)$: current state of the economy (calm/contagion)
- **Riskless asset** (constant interest rate r)
- **Derivatives** (only if needed for market completeness)
- **Economy switches between 2 states** ('calm', 'contagion')
  - **two types of jumps**
    1. **jump induces loss in one asset**
    2. **jump induces loss in one asset and triggers contagion**
  - overall jump intensity larger in contagion state (reflecting turbulence in the market)
  - constant loss size for each sort of jump
  - N^K counts number of jumps into state K

- **Investor**
  - can perfectly distinguish jumps and diffusion
  - **... but cannot distinguish the different types of jumps**
  - filters a subjective probability of the calm state $\widehat{p}_t$ out of historical asset prices
  - decides on his optimal portfolio using the 'small' filtration $\{\mathcal{G}_t\}_{t\in[0,T]} \subset \{\mathcal{F}_t\}_{t\in[0,T]}$
  - CRRA utility (with RRA γ=3 in the benchmark case)
  - maximizes utility from terminal wealth only
  - investment horizon: 5 years (in the benchmark case)
- **Complete Market**
  - investor chooses exposures against the four risk factors (which he can distinguish with restricted information)
  - investor uses derivatives to disentangle the risk factors
- **Incomplete Market**
  - investor chooses portfolio weights for the two risky assets
  - investor has to accept the whole package of risk factors

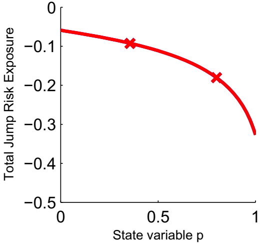
Parametrization

- Main parameters taken from the literature (EJP 2003, BCJ 2007): $r = 0.01$, $\sigma = 0.15$, $\rho = 0.5$, $L = 0.04$
- **Only jump parameters differ across both states**
- Jump intensities are calibrated via
  - ξ: jump intensity multiplicator calm-contagion
  - α: (conditional) probability of contagion-triggering jumps
- **Benchmark case** (**identical assets**)
  - $\xi_i = 5$, $\alpha_i = 0.2$
  - average (unconditional) jump intensity per year: 0.62
- **Second case** (**different assets**)
  - $\xi_A = 5$, $\alpha_A = 0.2$ (A is more severely hit by contagion)
  - $\xi_B = 2.5$, $\alpha_B = 0.5$ (B is more likely to trigger contagion)
- **Risk Premia**
  - diffusion risk: 0.0525
  - jump risk: 0.08 (calm state) and 0.016 (contagion state)

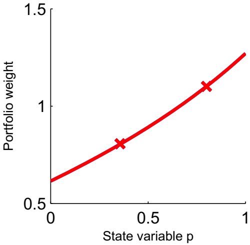
$\rightarrow$ Optimal and suboptimal filter equal

Solution of the Portfolio Planning Problem with Identical Assets

Complete Market



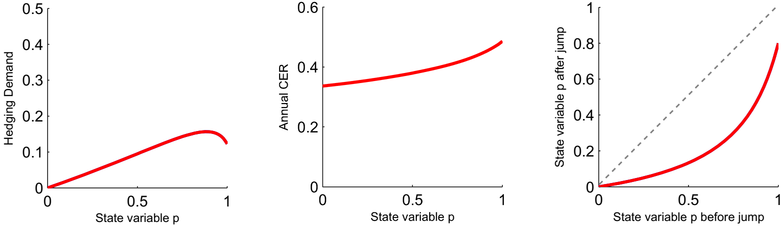
Incomplete Market



1. Impact of **restricted information**
   - Noncontagious jump: overreaction (and subsequent correction)
   - Contagious jump: underreaction

2. **Complete versus incomplete market**
   - Complete market: largest reaction to first jump ('risk of contagion')
   - Incomplete market: largest reaction to subsequent jumps ('confirmation of contagion')

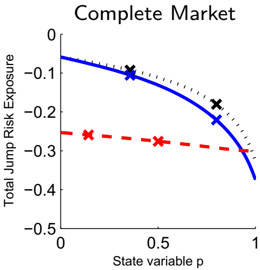
Solution with Identical Assets: Complete Market

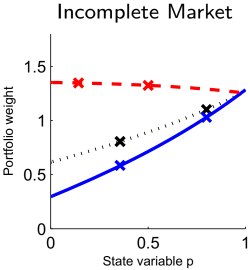


3 **Hedging Demand** for jump risk

- Worse investment opportunities in contagion state
  $\rightarrow$ positive hedging demand
- Largest probability update for $\widehat{p}_t \approx 0.8$
- Largest influence of $\widehat{p}_t$ on utility for $\widehat{p}_t = 1$
  $\rightarrow$ largest hedging demand for $\widehat{p}_t \approx 0.9$

Solution of the Portfolio Planning Problem with Different Assets





- **Asset A**
  - heavily affected by contagion ($\xi_A = 5$, $\alpha_A = 0.2$)
  - **largest trading volume**
- **Asset B**
  - more likely to trigger contagion ($\xi_B = 2.5$, $\alpha_B = 0.5$)
  - **induces largest portfolio adjustments**
- Jump risk 'spills over' from asset B to asset A

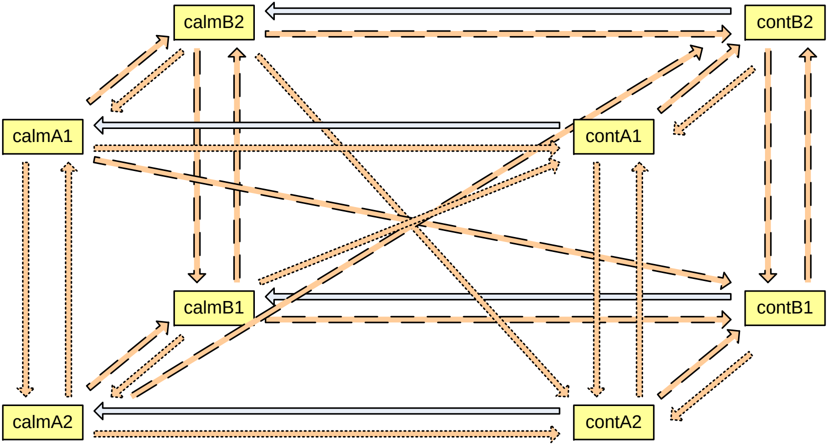
- Increasing **Diffusion Risk**
  - no impact on complete market
  - less impact of contagion in incomplete market
  - differences between complete and incomplete market increase
- **Loss size**
  - no qualitative changes
- **Investment horizon**
  - utility functions flatten out with larger horizons
- **Relative risk aversion**
  - no qualitative changes
- **Jump risk premia**
  - no qualitative changes
- **Average duration of the contagion regime**
  - has only marginal effects
  - main driver of our results:
    Contagion is a state (not a one-time event)

1. **Learning has a substantial impact**
   - underreaction to contagion-triggering jumps
   - overreaction to noncontagious jumps
   - stocks that are most hit by contagion
     $\rightarrow$ largest trading volume
   - stocks that most likely trigger contagion
     $\rightarrow$ induce largest portfolio adjustments
2. **Complete and incomplete market differ structurally**
   - complete market: largest reaction to 'risk of contagion'
   - incomplete market: largest reaction to 'confirmation'
3. **Significant hedging demand**
   - up to 50% of speculative demand
   - may be nonmonotonic function of state variable $\widehat{p}_t$

Future research

- Analyze the difference between optimal and suboptimal filter
- General equilibrium ($\rightarrow$ **market price of contagion risk**)

The Markov Chain



The Suboptimal Filter

$$
\begin{aligned}
d\widehat{p}_t \quad = \quad & \left((1-\widehat{p}_t)\lambda^{cont,calm} - \widehat{p}_t(\lambda_A^{calm,cont} + \lambda_B^{calm,cont})\right) dt \\
& + \widehat{p}_t \left(\frac{\lambda_A^{calm,calm}}{\widehat{\lambda}_A(\widehat{p}_t)} - 1 \right) \left(d\widehat{N}_A(t) - \widehat{\lambda}_A(\widehat{p}_t) dt \right) \\
& + \widehat{p}_t \left(\frac{\lambda_B^{calm,calm}}{\widehat{\lambda}_B(\widehat{p}_t)} - 1 \right) \left(d\widehat{N}_B(t) - \widehat{\lambda}_B(\widehat{p}_t) dt \right)
\end{aligned}
$$

where the estimated subjective intensity of $\widehat{N}_i$ equals

$$
\widehat{\lambda}_i(\widehat{p}_t) = \widehat{p}_t \left(\lambda_i^{calm,calm} + \lambda_i^{calm,cont} \right) + (1-\widehat{p}_t)\lambda_i^{cont,cont}
$$

The Optimal Filter

$$
\begin{aligned}
dp_t \quad = \quad & p_t(1-p_t)\left[\lambda_A^{cont,cont} + \lambda_B^{cont,cont} - \lambda_A^{calm,calm} - \lambda_B^{calm,calm} - \lambda_A^{calm,cont} - \lambda_B^{calm,cont}\right] dt \\
& +(1-p_t)\lambda^{cont,calm} dt \\
& + p_t(1-p_t)\left[\frac{(\mu_A^{calm})^2 - (\mu_A^{cont})^2}{(1-\rho^2)\sigma_A^2} + \frac{(\mu_B^{calm})^2 - (\mu_B^{cont})^2}{(1-\rho^2)\sigma_B^2} - 2\rho\frac{\mu_A^{calm}\mu_B^{calm} - \mu_A^{cont}\mu_B^{cont}}{(1-\rho^2)\sigma_A\sigma_B}\right. \\
& \qquad + \frac{(1-p_t)(\mu_A^{cont})^2 - p_t(\mu_A^{calm})^2}{\sigma_A^2} + \frac{(1-p_t)(\mu_B^{cont})^2 - p_t(\mu_B^{calm})^2}{(1-\rho^2)\sigma_B^2}\left(1-\rho\frac{\sigma_B}{\sigma_A}\right)^2 \\
& \qquad \left. + \frac{(p_t-(1-p_t))\mu_A^{calm}\mu_A^{cont}}{\sigma_A^2} + \frac{(p_t-(1-p_t))\mu_B^{calm}\mu_B^{cont}}{(1-\rho^2)\sigma_B^2}\left(1-\rho\frac{\sigma_B}{\sigma_A}\right)^2\right] dt \\
& + p_t(1-p_t)\left[\frac{\mu_A^{calm} - \mu_A^{cont}}{\sigma_A} dW_t^A + \frac{\mu_B^{calm} - \mu_B^{cont}}{\sigma_B} dW_t^B\right] \\
& + \left(\frac{\lambda_A^{calm,calm} p_{t-}}{\lambda_A^{cont,cont}(1-p_{t-}) + (\lambda_A^{calm,calm} + \lambda_A^{calm,cont})p_{t-}} - p_{t-}\right) dN_t^{A,obs} \\
& + \left(\frac{\lambda_B^{calm,calm} p_{t-}}{\lambda_B^{cont,cont}(1-p_{t-}) + (\lambda_B^{calm,calm} + \lambda_B^{calm,cont})p_{t-}} - p_{t-}\right) dN_t^{B,obs}
\end{aligned}
$$

Optimization problem in a complete or incomplete market

$$G(t, X_t, \widehat{p}_t) = \max_{\Pi \in \mathcal{A}(t, \widehat{p}_t)} \left\{ E\left[u(X_T) | \widehat{p}_t \right] \right\}$$

$$\begin{aligned}
\text{s.t. } \frac{dX_t}{X_t} &= rdt \\
&+\theta_A^{diff}(t, \widehat{p}_t) \cdot (d\widehat{W}_A(t) + \widehat{\eta}_A^{diff} dt) \\
&+\theta_B^{diff}(t, \widehat{p}_t) \cdot (d\widehat{W}_B(t) + \widehat{\eta}_B^{diff} dt) \\
&+\theta_A^{jump}(t, \widehat{p}_t) \left[d\widehat{N}_A(t) - \widehat{\lambda}_A(\widehat{p}_t)dt - \widehat{\eta}_A^{jump}(\widehat{p}_t)\widehat{\lambda}_A(\widehat{p}_t)dt \right] \\
&+\theta_B^{jump}(t, \widehat{p}_t) \left[d\widehat{N}_B(t) - \widehat{\lambda}_B(\widehat{p}_t)dt - \widehat{\eta}_B^{jump}(\widehat{p}_t)\widehat{\lambda}_B(\widehat{p}_t)dt \right]
\end{aligned}$$

$$\begin{aligned}
\text{or } \frac{dX(t)}{X(t)} &= \pi_A(t, \widehat{p}_t)\frac{dS_A(t)}{S_A(t)} + \pi_B(t, \widehat{p}_t)\frac{dS_B(t)}{S_B(t)} \\
&+ \left[1 - \pi_A(t, \widehat{p}_t) - \pi_B(t, \widehat{p}_t)\right] rdt
\end{aligned}$$

Complete Market System of PDAEs

$$
\begin{aligned}
f_t(t,\widehat{p}_t) + f(t,\widehat{p}_t)\cdot(\mathcal{D}+\mathcal{E}) + f_p(t,\widehat{p}_t)\cdot\mathcal{B} \qquad & \\
+\left(1+\theta_A^{jump}\right)^{1-\gamma}\widehat{\lambda}_A f(t,\widehat{p}_A^{+}) + \left(1+\theta_B^{jump}\right)^{1-\gamma}\widehat{\lambda}_B f(t,\widehat{p}_B^{+}) &= 0 \\
-f(t,\widehat{p}_t)\cdot(1+\widehat{\eta}_A^{jump}) + f(t,\widehat{p}_A^{+})\cdot\left(1+\theta_A^{jump}\right)^{-\gamma} &= 0 \\
-f(t,\widehat{p}_t)\cdot(1+\widehat{\eta}_B^{jump}) + f(t,\widehat{p}_B^{+})\cdot\left(1+\theta_B^{jump}\right)^{-\gamma} &= 0
\end{aligned}
$$

- $\mathcal{B}$, $\mathcal{D}$ and $\mathcal{E}$ depend on the model parameters, $\widehat{p}_t$ and θ_i^{jump}
- $\widehat{p}_i^{+} = \frac{\lambda_i^{calm,calm}}{\widehat{\lambda}_i}\cdot\widehat{p}_t$ denotes the updated probability after a jump in stock i

Incomplete Market System of PDAEs

$$f_t(t,\widehat{p}_t) + f(t,\widehat{p}_t)\cdot\left[(1-\gamma)\cdot\mathcal{A}^* - 0.5\gamma(1-\gamma)\cdot\mathcal{C}^* - \widehat{\lambda}_A - \widehat{\lambda}_B\right]$$
$$+f_p(t,\widehat{p}_t)\cdot\mathcal{B} + \left[(1-\pi_A L_A)^{1-\gamma}\cdot f(t,\widehat{p}_A^+)\right]\widehat{\lambda}_A$$
$$+\left[(1-\pi_B L_B)^{1-\gamma}\cdot f(t,\widehat{p}_B^+)\right]\widehat{\lambda}_B = 0$$

$$f(t,\widehat{p}_t)\cdot(\widehat{\mu}_A - r) - \gamma\pi_B\rho\widehat{\sigma}_A\widehat{\sigma}_B\cdot f(t,\widehat{p}_t) - \gamma\widehat{\sigma}_A^2\pi_A\cdot f(t,\widehat{p}_t)$$
$$-L_A\cdot(1-\pi_A L_A)^{-\gamma}\cdot f(t,\widehat{p}_A^+)\cdot\widehat{\lambda}_A = 0$$
$$f(t,\widehat{p}_t)\cdot(\widehat{\mu}_B - r) - \gamma\pi_A\rho\widehat{\sigma}_A\widehat{\sigma}_B\cdot f(t,\widehat{p}_t) - \gamma\widehat{\sigma}_B^2\pi_B\cdot f(t,\widehat{p}_t)$$
$$-L_B\cdot(1-\pi_B L_B)^{-\gamma}\cdot f(t,\widehat{p}_B^+)\cdot\widehat{\lambda}_B = 0$$

- $\mathcal{A}^*$, $\mathcal{B}$ and $\mathcal{C}^*$ depend on the model parameters, $\widehat{p}_t$ and π_i
- $\widehat{p}_i^+ = \frac{\lambda_i^{calm,calm}}{\widehat{\lambda}_i}\cdot\widehat{p}_t$ denotes the updated probability after a jump in stock i

Benchmark Parametrization

		Benchmark (equal stocks)	Different stocks Stock A	Stock B
Data-generating process	σ_i^{calm}, σ_i^{cont}	0.15	0.15	0.15
	ρ^{calm}, ρ^{cont}	0.50	0.50	0.50
	$\lambda_i^{calm,calm}$	0.32	0.32	0.20
	$\lambda_i^{calm,cont}$	*0.08*	*0.08*	*0.20*
	$\lambda_i^{cont,cont}$	*2.00*	*2.00*	*1.00*
	$\lambda^{cont,calm}$	*1.00*	*0.75*	
	$L_i^{calm,calm}$	0.04	0.04	0.04
	$L_i^{calm,cont}$	0.04	0.04	0.04
	$L_i^{cont,cont}$	0.04	0.04	0.04
	$L_i^{cont,calm}$	0.00	0.00	0.00
	ξ_i	5.00	5.00	2.50
	α_i	0.20	0.20	0.50
	ψ	0.25	0.25	
Market prices of risk	η_i^{calm}, η_i^{cont}	0.35	0.35	0.35
	$\eta_i^{calm,calm}$	2.00	2.00	2.00
	$\eta_i^{calm,cont}$	*17.0*	*17.0*	*8.00*
	$\eta_i^{cont,cont}$	*0.20*	*0.20*	*1.40*
	$\eta^{cont,calm}$	0.00	0.00	0.00
Risk premia	diffusion risk calm/contagion	*0.0525*	*0.0525*	*0.0525*
	jump risk calm state	0.08	0.08	0.08
	jump risk contagion state	*0.016*	*0.016*	*0.056*

Risk Premia

- Investor knows the model and all parameters **except the state of the economy**
- Suboptimal filter: **from jump processes only**
  - Optimal if drift and diffusion terms equal across states
  - Resulting restrictions in the complete market

$$\widehat{\eta}_i^{diff} = \eta_i^{diff,calm} = \eta_i^{diff,cont} =: \eta_i^{diff}$$

$$\begin{aligned}\widehat{\lambda}_i \left(1+\widehat{\eta}_i^{jump}\right) &= \lambda_i^{calm,calm}\left(1+\eta_i^{calm,calm}\right) + \lambda_i^{calm,cont}\left(1+\eta_i^{calm,cont}\right) \\ &= \lambda_i^{cont,cont}\left(1+\eta_i^{cont,cont}\right)\end{aligned}$$

  - Similar restrictions hold in the incomplete market
- **Resulting jump risk premia**
  - 0.08 in the calm state
  - 0.016 in the contagion state
- Constant diffusion risk premium: 0.0525