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A CDO OPTION MARKET MODEL FOR STANDARDIZED CDS INDEX TRANCHES

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DORN J. INDEX TRANCHE MARKET MODEL

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ECONOMIC MOTIVATION

MARKET CONTEXT

- CDO is a OTC Product \Rightarrow High Transaction Costs
- "Liquidity Gap" costs precious Basis Points

 \Rightarrow Initialization of a standardized synthetic CDO Market (CDX/iTraXX)

MODELING CONSTRAINTS

- Credit Derivatives : "Static" Models \Rightarrow The investor does not pay for the Véga !
- Pricing of CDO tranches with option alike pay-offs (Deal Spread, Cumulative Loss as underlying)
- "Maturity Trap"
- \rightarrow need for Spread Dynamics !

Preliminaries	Motivation
The Model	Recall : CDO Structure
Conclusion	Related Literature
Implementation	CDO Spread Determinants

STANDARDIZATION ASSUMPTIONS

- Underlying CDS Portfolio restricted to components of CDX / iTraXX index series
- Pre-Set Attachment / Detachment Points
- \rightarrow Success Story \Rightarrow option trading possible

CDS INDEX TRANCHES

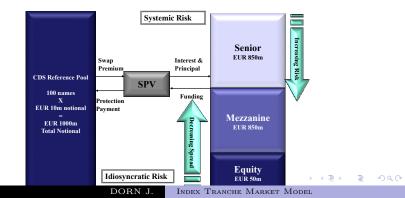
- CDS Index Tranches securitize CDS Index Series.
- Attachment / Detachment Points are standardized [0%, 3%, 6%, 9%, 12%, 22%, 100%]
 ⇒ improves liquidity, reduces ramp-up costs for structurers

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Synthetic CDO Structure

- Only synthetic CDOs (CDOs on a CDS portfolio) allow for product standardization and hence for liquidity
- $\bullet\,$ CDOs securitize credit spreads and issue tranches $\rightarrow\,$ Leverage



Related Literature

- BGM model : Arbitrage-Free model for other than instantaneous, continuously compounded forward rates
 - The idea is to chose a different numeraire other than the risk-free account
 - \blacktriangleright Leads to Black's formula \rightarrow we refer to as "market models"
 - First attempt to model a market-implied term structure of forward rates
- CDS option MM : Brigo & Mercurio transferred the idea of a market model into the credit derivatives environment
 - One-Period Spread modeling approach applied to the CDS market, with approximation constraints
- D. Filipovic, L. Overbeck and T. Schmidt. "Dynamic CDO Term Structure Modelling" (2008)

Scope

- provide a market model for tranche spread dynamics
- practitioner-oriented approximation method which avoids modeling the 125 underlying CDS
- incorporates CDS correl assumptions market-implied (wrong or false)
- $\bullet\,$ underlier are traded tranche by tranche $\Rightarrow\,$ no need for cross tranche model
- $\bullet\,$ every tranche has its own correl assumption $\Rightarrow\,$ individual asset
- Dynamics become even more relevant with upcoming CDS clearing chamber

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 PRELIMINARIES
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 THE MODEL
 RECALL : CDO STRUCTURE

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 RELATED LITERATURE

 IMPLEMENTATION
 CDO SPREAD DETERMINANTS

Scope

We aim to provide a framework that justifies B&S market practice application.

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Scope

- We consider a CDO tranche with AP *D*% and DP *E*% and tenor [*T_a*; *T_b*].
- The aim consists in finding a recursive formula for market-implied Spread Dynamics !
- \Rightarrow need for liquid market data.

LEMMA

Let $\Pi_{CallCDO_{a,b}^{D,E}}(t, K)$ describe the t-time pay-off of a forward start call option written on standardized CDO tranche with boundaries [D%; E%]. The tenor is $[T_a; T_b]$. Within the Black & Scholes framework the Call option takes the value

$$\Pi_{CallCDO_{a,b}^{D,E}}(t,K) = \hat{C}_{a,b}^{D,E}(t) \times \left[S_{a,b}^{D,E}(t)N(d_1) - K \times N(d_2)\right]$$
(1)

with

$$\hat{C}_{a,b}^{D,E}(T_a) =: \sum_{i=a+1}^b \delta_i B(T_a, T_i) E_Q^t T_i [X(T_i)]$$

and

$$d_{1,2} = \frac{ln\left(\frac{S_{a,b}^{D,E}(t)}{K}\right) \pm (T_a - t)\frac{1}{2}\int_t^{T_a}\sigma_{a,b}^2(s)ds}{\sigma_{a,b}(T_a - t)\sqrt{T_a - t}}$$

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RECALL CDO SPREAD DETERMINANTS

DEFINITION "CDO PREMIUM LEG" :

• Sum of discounted Cash-Flows perceived by the Trancheholder

DEFINITION "CDO PROTECTION LEG" :

• Sum of the discounted reductions of a tranche's notional inherent to credit events which lead to a decrease in the Trancheholder's "spread revenue".

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DEFINITION "FAIR SPREAD" :

• The t-time Fair Spread is the Spread the investor should have contracted instead of Deal Spread + Euribor/Libor at issuing date in order to allow the tranche quote at par at time *t*.

$$\mathsf{Fair Spread} = \frac{\mathsf{Protection Leg}}{\mathsf{Premium Leg}}$$

Notional Erosion \rightarrow Spread has to be calculated on the outstanding Tranche Notional

DEFINITION FORWARD FAIR TRANCHE SPREAD

Fwd Fair Spread =
$$\frac{B(t, T_i) \text{Protection Leg}_i}{\sum_i B(t, T_i) \text{Premium Leg}_i}$$

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Overview Spread Dynamics

THE MODEL - CENTRAL IDEA

Describe a fwd-start option on a synth. CDS Index Tranche (B&S framework)

Define forward Fair Tranche Spread as a function of the numeraire Possibility to select any mtgle dynamics of the fwd spread rate under associated probability measure

The Expected outstanding Tranche Notional is a tfwd neutral martingale

Derive forward spread dynamics for different time horizons Calculate its volatility in function of the Spread Rate and the associated observable volatility

Overview Spread Dynamics

STEP 1- THE FWD SPREAD DYNAMICS

DEFINITION

FWD-NEUTRAL MEASURE

$$S_{a,b}^{D,E}(t) = \frac{Protleg(t)}{Premleg(t)}$$

$$\frac{dQ^{D,E}_{a,b}}{dQ} = \frac{\approx}{\frac{\mathsf{Premleg(t)}}{\mathsf{Premleg(t)}}}$$

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Overview Spread Dynamics

STEP 1- THE FWD SPREAD DYNAMICS

DEFINITION

FWD-NEUTRAL MEASURE

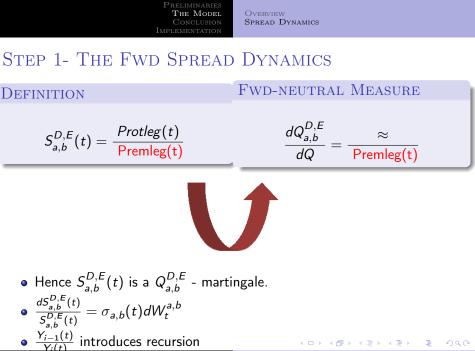
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INDEX TRANCHE MARKET MODEL

Overview Spread Dynamics

STEP 2 - SHORTFALL DYNAMICS

The expected outstanding tranche notional $Y_i(t)$ is a Q^{T_i} -martingale. Its dynamics under the forward-neutral probability Q^t follows :

$$\frac{dY_i(t)}{Y_i(t)} = \gamma_i(t) dZ_t$$

STEP 3 - DERIVING THE VOLATILITY

LEMMA

 $\forall k \in [a + 1; b]$ the volatility of the process Y_k related to tenor $[T_a, T_b]$ is given by

$$\gamma_k(t) = -\sum_{j=a+1}^k \left(\frac{\delta_j S_{j-1,j}^{D,E}(t)}{1 + \delta_j S_{j-1,j}^{D,E}(t)} \sigma_{j-1,j}(t) \right)$$

Overview Spread Dynamics

STEP 4 - THE FWD ONE-PERIOD SPREAD DYNAMICS

COROLLARY

Consider a deal with tenor $[T_a, T_b]$ and tranche [D, E]. The dynamics of the forward one-period Fair Tranche Spread on tenor $[T_{i-1}, T_i]$ is given by :

$$\frac{dS_{i-1,i}^{D,E}(t)}{S_{i-1,i}^{D,E}(t)} = \sigma_{i-1,i}(t)\rho \sum_{j=a+1}^{i} \left(\frac{\delta_{j}S_{j-1,j}^{D,E}(t)}{1+\delta_{j}S_{j-1,j}^{D,E}(t)} (\sigma_{j-1,j}(t))' \right) dt \\ + \sigma_{i-1,i}(t)dZ_{t}$$

More precisely, for a deal with tenor $[T_{i-1}, T_i]$, the forward one-period Fair Tranche Spread dynamics for the same tenor amounts to :

$$\frac{dS_{i-1,i}^{D,E}(t)}{S_{i-1,i}^{D,E}(t)} = \frac{\delta_i S_{i-1,i}^{D,E}(t)}{1 + \delta_i S_{i-1,i}^{D,E}(t)} |\sigma_{i-1,i}(t)|^2 dt + \sigma_{i-1,i}(t) dW_t$$

Overview Spread Dynamics

STEP 5 - THE MULTI-PERIOD EXTENSION

LEMMA

Again consider a deal with tenor $[T_a, T_b]$ and tranche [D, E]. The forward multi-period spread dynamics with the same tenor, note $S_{a,b}^{D,E}$, can be written as

$$\frac{dS_{a,b}^{D,E}(t)}{S_{a,b}^{D,E}(t)} = \left(\Lambda(t) + \varsigma(t)\right)\rho\left(\Lambda(t)\right)'dt - \left(\Lambda(t) + \varsigma(t)\right)dZ_t$$

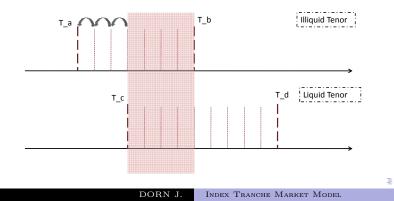
with

$$\Lambda(t) = \sum_{i=a+1}^{b} \frac{\delta_i A(t, T_i) Y_i(t)}{\hat{C}_{a,b}^{D,E}(t)} \gamma_i(t)$$

$$\varsigma(t) = \frac{A(t, T_b) Y_b(t)}{A(t, T_a) Y_a(t) - A(t, T_b) Y_b(t)} \gamma_b(t)$$

CONCLUSION

 Market Model allows for calibration of options with bespoke exercise periods to options with more liquid tenors thanks to multi-period fwd Tranche Spread Dynamics ⇒ More realistic prices.



CONCLUSION

- Possibility of pricing options on tranches with future ramp-up dates ⇒ Fwd spread is no longer a martingale ⇒Calculate expectations of the fwd spread dynamics !
- Fwd spread dynamics allow for modeling of deals with complicated pay-offs !
- The investor finally pays for the Véga !

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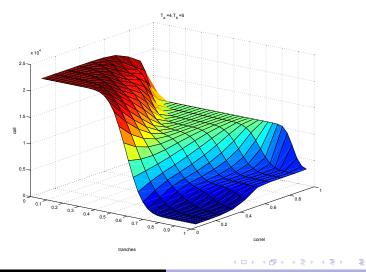
IMPLEMENTATION (1)

Interest rate assumptions	Deterministic
Recovery Rate	40% (market standard)
Implied volatilities of fwd-start options (in the money)	
for 3 maturities : 1 months, 3 months and 7 months	Semi-parametric approach (cf Gatarek [?])
Strike Spread K	0,02
Method used for implying the default intensity	student-t copula, degrees of freedom
	chosen in analogy to Hull [?]
Dataset used	iTraXX Series 8, daily data
	throughout may 2008
	Bloomberg ID : ITRXEB58
	(most recent and liquid series in May 2008)
Forward rates, yield curve	Provided by SGSS, Euro-VL

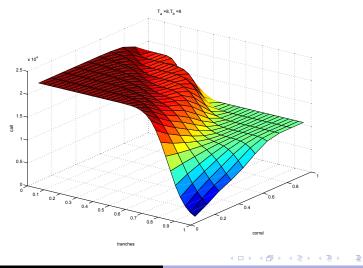
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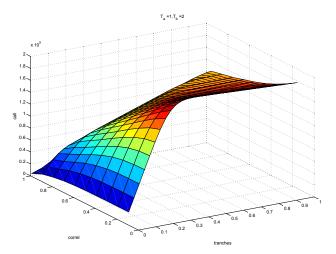
IMPLEMENTATION (1)



IMPLEMENTATION (2)



IMPLEMENTATION (3)

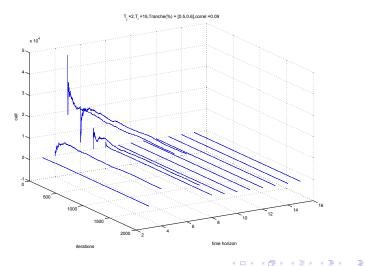


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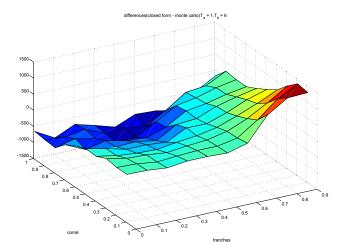
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IMPLEMENTATION (4)



IMPLEMENTATION (5)



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Outlook

- Approach might serve to model bespoke CDOs.
- The spread on a CDO tranche can be replicated by a Call Spread on the CDO's cumulative Loss Given Default (LGD) with strikes being the respective Attachment/Detachment Points.
- Hence by modeling the LGD dynamics there should be a way to price bespoke CDO tranches.

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THANK YOU FOR YOUR ATTENTION!

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