The Bachelier Legacy:

Why do asset prices move?

Impact and Second Generation models

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Why ask?

- Crucial question in theoretical Economics and Finance: what is the information reflected by prices & what are markets good for?

- Crucial question for investment strategies: is there any way to predict how prices will move?

- Crucial question for risk control/regulation: understanding why and how prices move allows one to devise efficient risk models and useful regulation (?)

The Sacred Lore of Theoretical Economics

- Efficient market theory: Agents are rational and Markets are in equilibrium

  – still deemed by many to be a good starting point

- Prices reflect faithfully the Fundamental Value of assets and only move because of exogeneous unpredictable news.

- Platonian markets which merely reveal fundamental values without influencing them – or is it a mere tautology??

  Black's definition of efficiency: price right to within a factor 2! ✓

- Crashes can only be exogenous, not induced by markets dynamics itself – oh really??

By the way...

- Agents (us humans) do make errors and have regrets, (cognitive or sensorial biases, imperfect or superabundant information, urgency, negligence, etc.)

- Problems can be algorithmically so complex that we have to make suboptimal decisions based on heuristic rules

- Agents are deeply influenced by the behaviour of others – who might have more information (??)

- $\longrightarrow$ Even silly trades do impact prices and may create positive feedback loops

First generation models of markets

- Rooted in the idea that dynamics is exogenous and markets are efficient, Financial Engineering:

- (1) postulate any process that

  - is tractable

  - looks vaguely similar to real data – or not even

- (2) brute force calibrate, on "liquid" markets (supposed to be efficient) and price options or more exotic derivatives

- Examples: Brownian motion (Black-Scholes), GARCH, Heston, Local vol., Lévy, Multifractal, etc. , etc., etc.

BUT

- NONE of these models are justified by "first principles", or agent based models, such that parameters can be (at least in principle) *computed*

- Inspiration from physics: macroscopic (or hydrodynamic) laws from microscopic elements

  – Navier-Stokes from molecular collisions

  – Magnetic properties from individual spins

  – Phase diagram of bodies from individual atoms, etc. etc.

BUT

- Uncontrolled brute force calibration are often

  *) based on models absurdly remote from reality (e.g. local volatility models, Archimedean copulas, etc.)

  *) can be extremely dangerous ("liquid" markets are in fact not liquid and not efficient, errors and biases are amplified in a non-linear way, etc.) – cf. the BS feedback loop in 1987...

- To calibrate does not mean to understand. A perfect fit is not a theory – often a red-herring

- Let's try to undestand what's going on at the micro level

Some empirical facts

- Financial markets offer Terabytes of information (weekly) to try to investigate why and how prices move

- A) Are news really the main determinant of volatility?

  Exogenous vs. endogenous dynamics

- B) Are price really such that supply instantaneously equals demands? How fast information is included in prices?

A) Exogenous or endogenous dynamics?

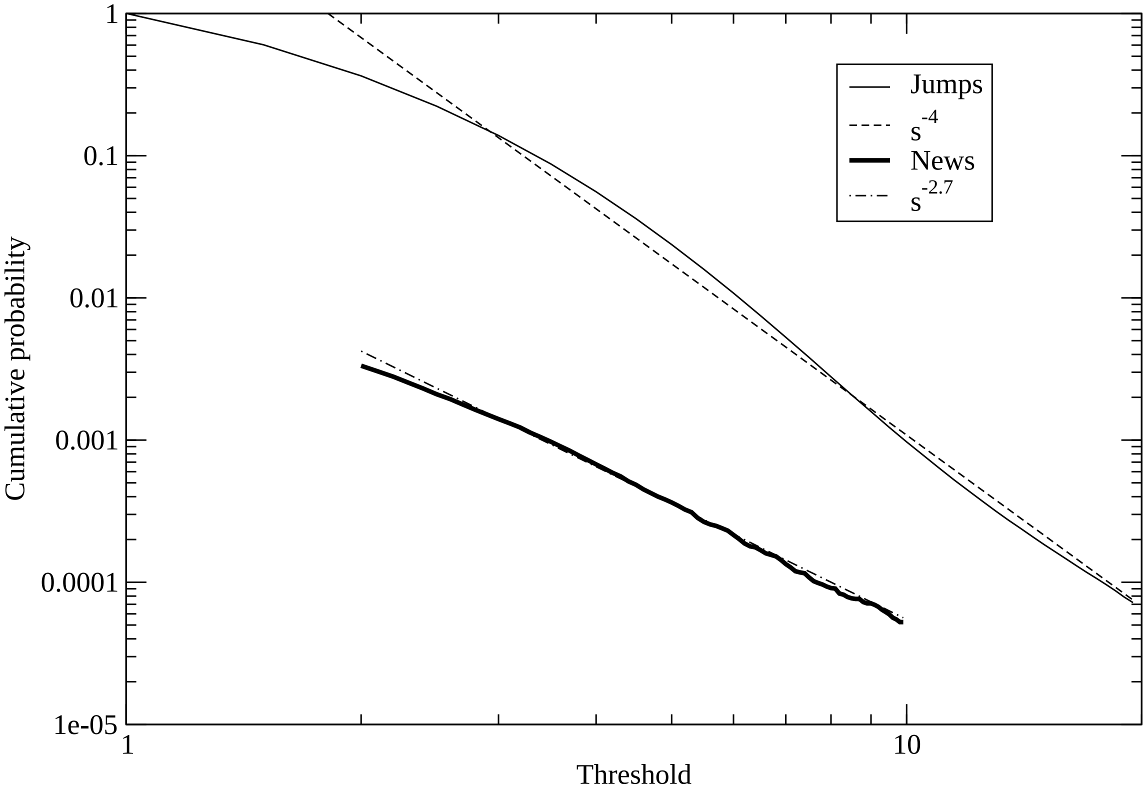
- Yes, some news make prices jump, sometimes a lot, but jump freq. is much larger than news freq.

- On stocks, only $\sim 5\%$ of $4-\sigma$ jumps can be attributed to news, most jumps appear to be endogeneous

- Similar conclusions on daily data in seminal papers (Cutler, Poterba, Summers; Shiller)

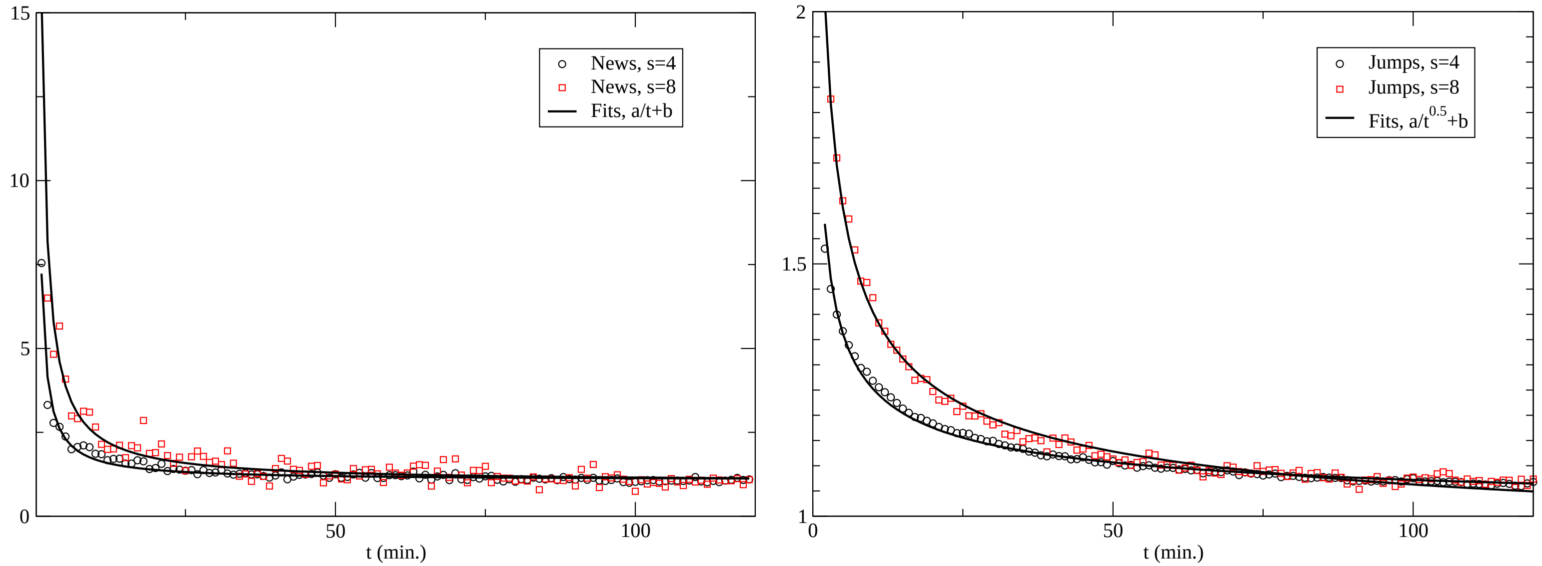
- Different statistics: return distributions and 'aftershocks' (volatility relaxation)

Jumps



Power-law distribution of news jumps and no-news jumps. With A. Joulin, D. Grunberg, A. Lefevre

Two jump types: Aftershocks



Volatility relaxation after news (t^{-1}, left) and endogenous jumps ($t^{-1/2}$, right). With A. Joulin, D. Grunberg, A. Lefevre

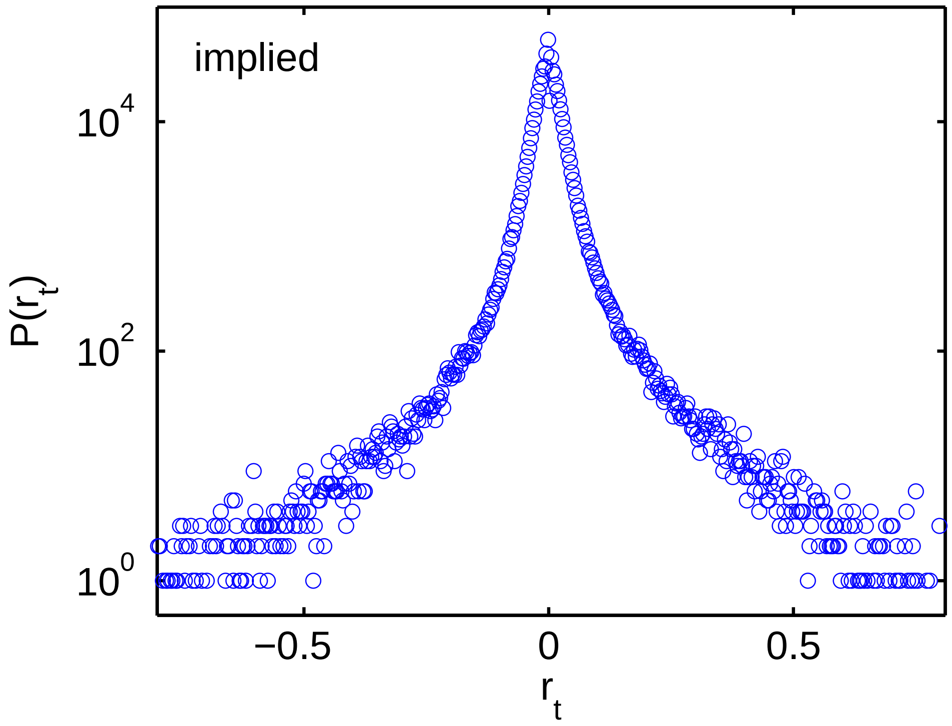
A) Exogenous or endogenous dynamics?

- Power-law distribution of price changes for anything that is traded

- Excess volatility, with long range memory – looks like endogeneous intermittent noise in complex systems (turbulence, Barkhausen noise, etc.)

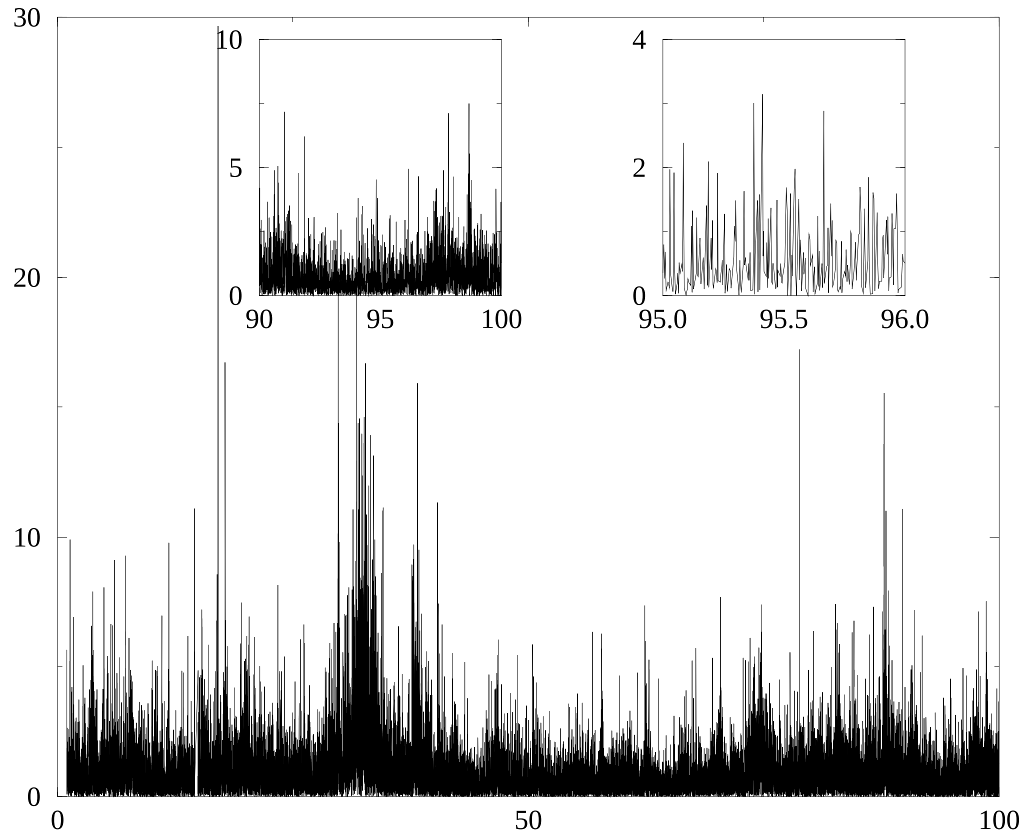
- Universal observations !!

Power-law tails



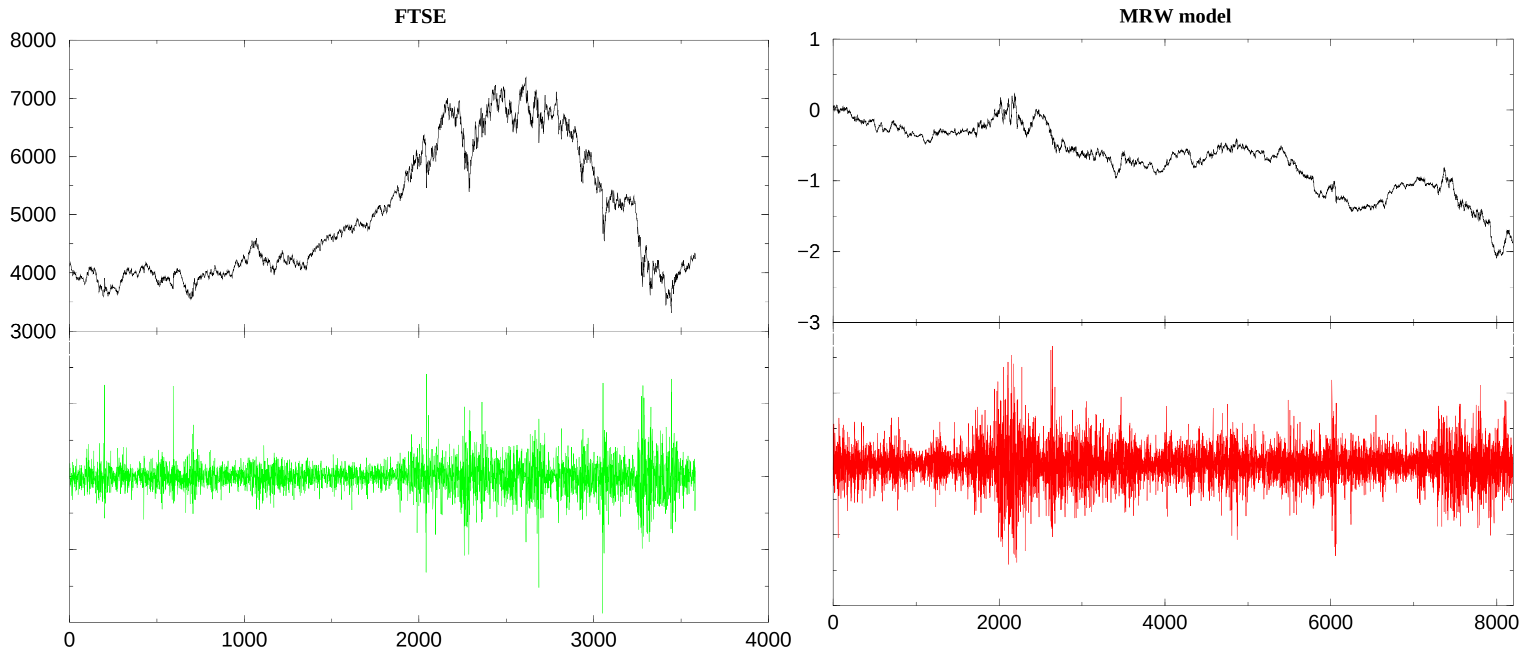
Distribution of daily volatility moves on option markets or *any other traded stuff*: inverse cubic law

Multiscale intermittency



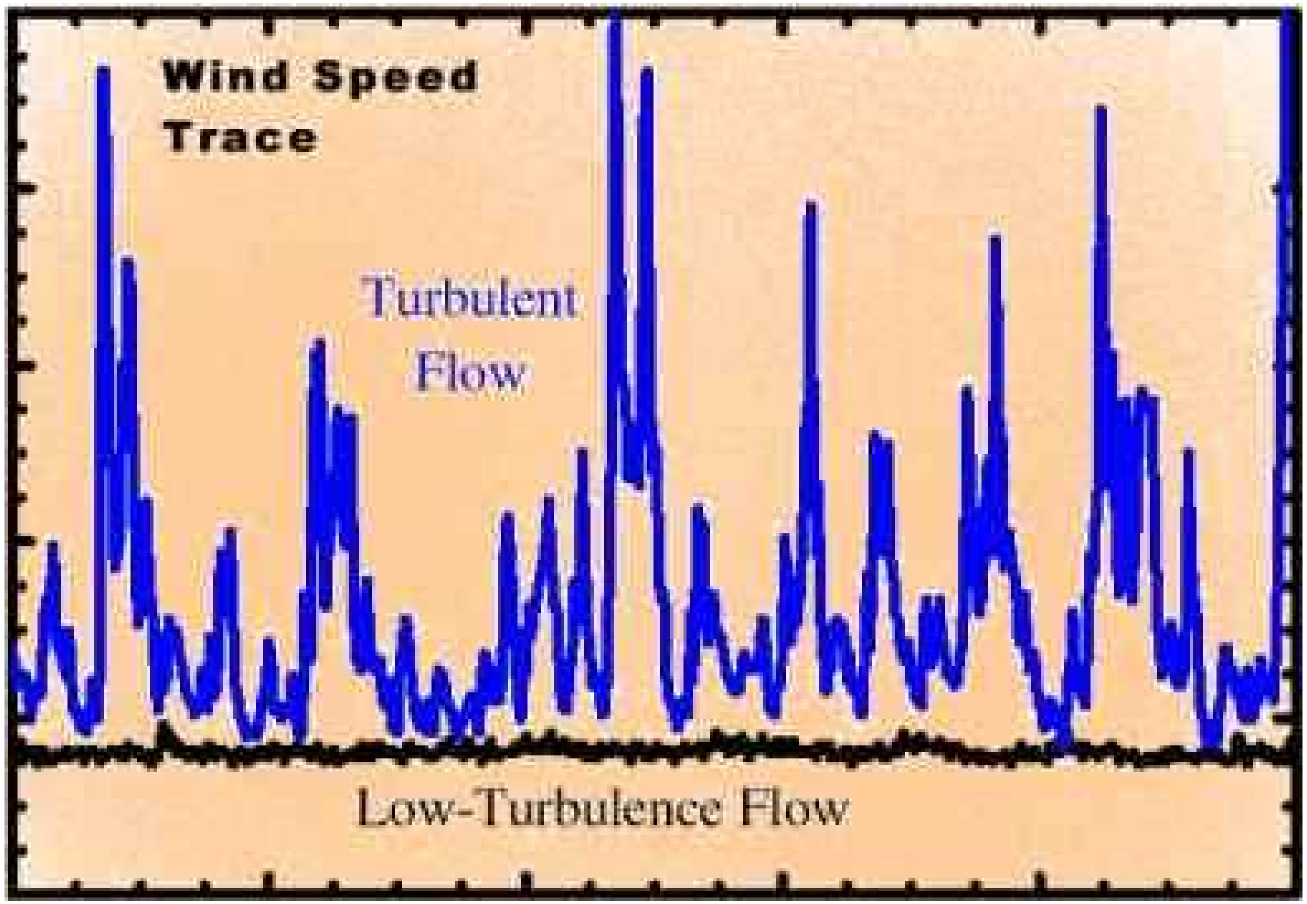
Excess volatility, with long range memory – looks a lot like endogeneous noise in complex systems

Multifractal fluctuations



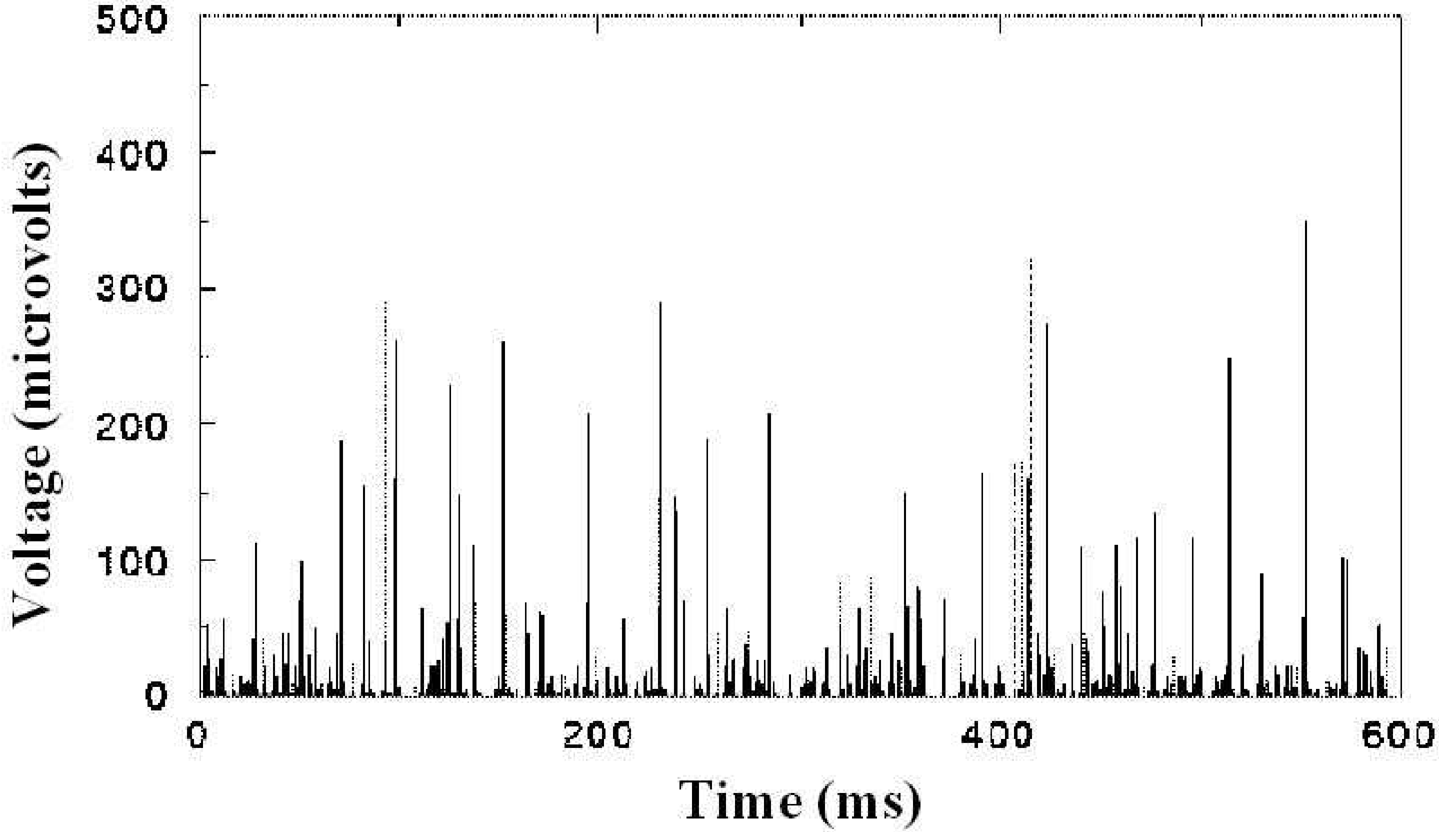
An exceptionally "good", parcimonious model:
Mandelbrot-Calvet-Fisher 1997; Bacry-Muzy 2000... – but why?

Turbulence: intermittency



Slow, regular and featureless exogeneous drive but intermittent endogeneous dynamics

Barkhausen noise



Slow, regular and featureless exogeneous drive but intermittent endogeneous dynamics

B) Are markets in “equilibrium?”

- UHF data allows one to understand the microscopics of order flow and price formation

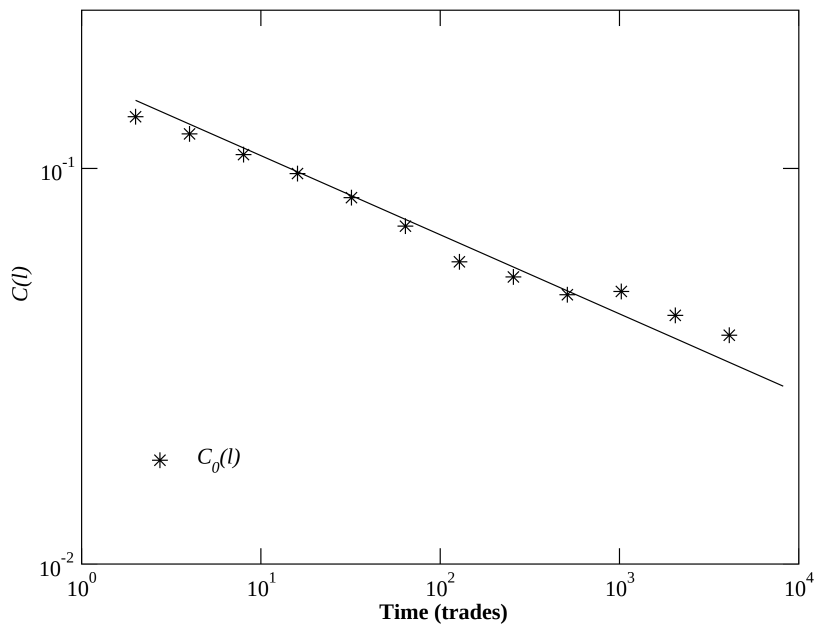
- One can distinguish buy orders from sell orders $\epsilon = \pm 1$

- Surprise: the autocorrelation of the sign of trades is very long-range correlated over several days or weeks (see also Lillo-Farmer)

$$\mathcal{C}(\ell) = E[\epsilon_n \epsilon_{n+\ell}] \propto \ell^{-\gamma} \qquad \gamma < 1$$

- A beautiful paradox: Sign of order flow very predictable and orders impact the price – but no predictability in the sign of price changes ?? – see below

Trade correlations



Correlations extend to several days! $- p_s(10000) = 53\%$

B) Are markets in "equilibrium?"

- Even "liquid" markets offer a *very small immediate liquidity* (10^{-5} for stocks) – buyers/sellers have to fragment their trades over days, weeks or even months

- "Information" can only be *slowly incorportated into prices*, latent demand does not match latent supply

- Markets are hide and seek games between "icebergs" of buyers and sellers and are *not* in instantaneously in equilibrium

Some empirical facts

- A) Are news really the main determinant of volatility?
  - No, endogenous dynamics more likely, markets are complex systems that generate rich endogenous dynamics
  - However exogenous news provide “stirring” of the system

- B) Are price really such that supply instantaneously equals demands?
  - No, “information” can only be very slowly incorportated into prices

Impact

- Using high frequency data, one can measure impact accurately:

$$\mathcal{I}_+ = E[p_{n+1} - p_n | \epsilon_n = +1], \qquad \mathcal{I}_- = -E[p_{n+1} - p_n | \epsilon_n = -1]$$

- Empirical finding (1): impact is proportional to spread

$$\mathcal{I}_+ = \mathcal{I}_- \approx 0.3\, S$$

- Trading, even uninformed and with relatively small volumes in usual market conditions, strongly influences prices and leads to measurable effects – even "liquid" markets are not that liquid

  (1% of the daily volume moves the price by 5% of the daily volatility!!)

What is impact?

- Efficient market story: Informed agents successfully forecast short term price movements and trade accordingly. This results in correlations between trades and price changes, but uninformed trades have no price impact – prices stick to "Fondamental Values"

- A more plausible story: since there is no easy way to distinguish "informed" from "non informed" traders, all trades statistically impact prices since other agents believe that some of these trades might contain useful information – a mechanism leading to feedback loops and avalanches

Impact & volatility

- Empirical finding (1): impact is proportional to spread

$$\mathcal{I}_+ = \mathcal{I}_- \approx 0.3\,S$$

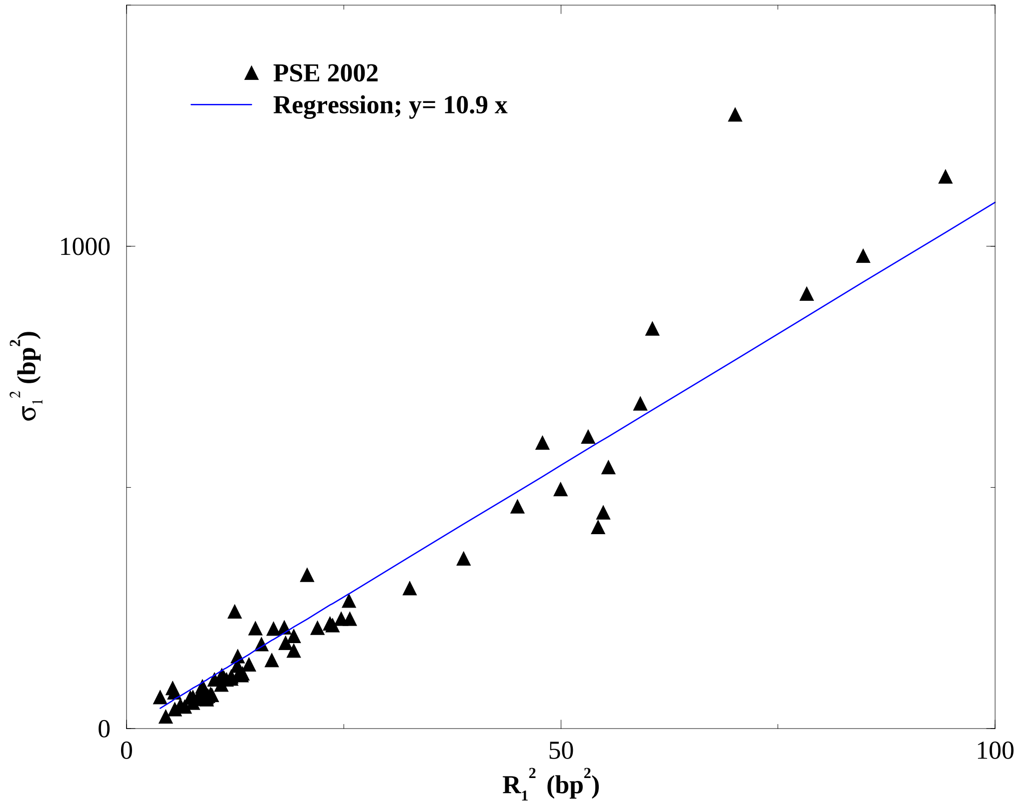
- Empirical finding (2): volatility per trade is proportional to impact

$$\sigma_1^2 = A\mathcal{I}^2 + BJ^2, \qquad B \approx 0$$

(impact component + "news" component)

- Volatility is indeed mostly due to impact of trades – very little to quote jumps J without trades ("news")

Volatility: impact + news?



Very little contribution from quote jumps J without trades ("news")
– with J. Kockelkoren, M. Potters, M. Wyart

Impact: non linear and transient

- Impact is both non-linear and non local in time

$$p_t = p_{-\infty} + \lambda \sum_{t'=-\infty}^{t} G(t-t')\, \epsilon_{t'} \cdot S_{t'} \cdot V_{t'},$$

- $\psi \approx 0.2$: very concave impact – trades are more important than volume (Hasbrouck, Jones)

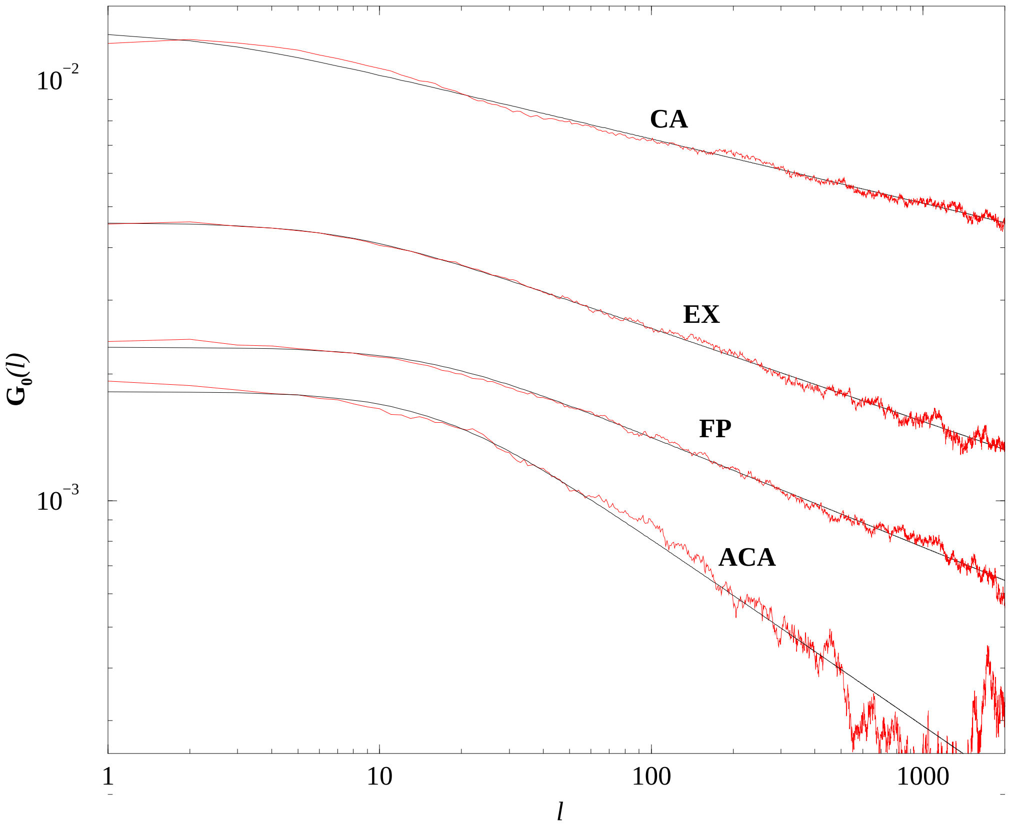
- The impact function $G(\ell)$ must decay as $\ell^{-\beta}$ to exactly offset the correlation of trades and remove predictability of returns!

$$\beta = \frac{1-\gamma}{2}, \qquad 0 \le \gamma \le 1$$

Impact: non linear and transient

- Is there a coarse-graining time scale beyond which impact is linear and permanent?, like assumed in most models (e.g. Kyle, agent based, etc.)

Critically resilient markets



Decay of $G(\ell)$ for different stocks: impact is transient – with J. Kockelkoren, M. Potters

Impact: non linear and transient

- Bachelier's legacy: the random walk nature of prices results from a subtle balance between trending order flow (liquidity takers) and mean-reverting impact (liquidity providers)

- This dynamical equilibrium can be locally broken $\longrightarrow$ micro-liquidity crises and endogeneous jumps (??)

Transient impact: more technicalities

- $u_\ell = (p_{n+\ell} - p_n).\varepsilon_n$ is the bare profit of a trade after ℓ trades

- The full distribution of u_ℓ is nearly symmetrical around its mean:

  $\longrightarrow$ Very few trades can be qualified as 'informed' on the short run

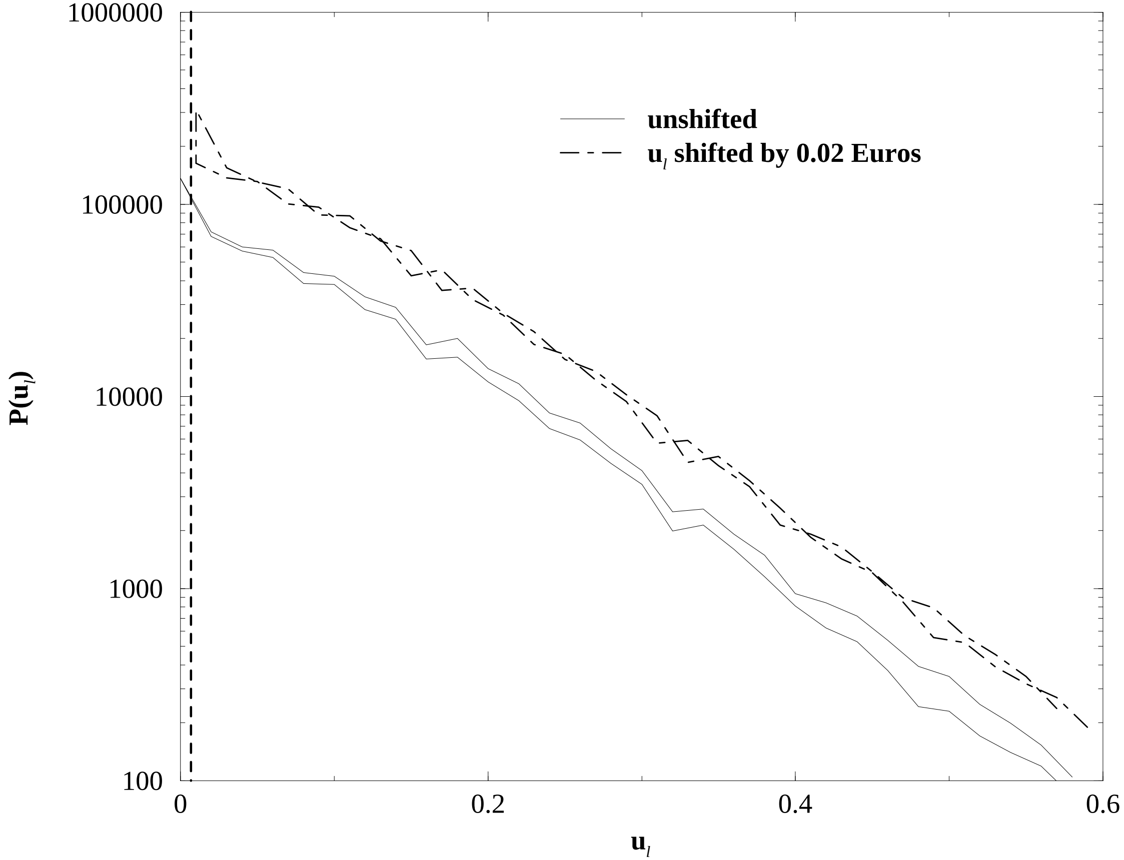
- Average response function:

$$\mathcal{I}(\ell) = E[\Big(p_{n+\ell} - p_n\Big) \cdot \varepsilon_n], \qquad \mathcal{I} = \mathcal{I}(\ell = 1)$$

- Mid-point fluctuations in trade time: diffusion

$$\mathcal{D}(\ell) = E[\Big(p_{n+\ell} - p_n\Big)^2] \approx \sigma_1^2 \ell \qquad \longrightarrow \beta = \frac{1-\gamma}{2}$$

Impact distribution



$\ell = 128$: where are the 'informed' trades??

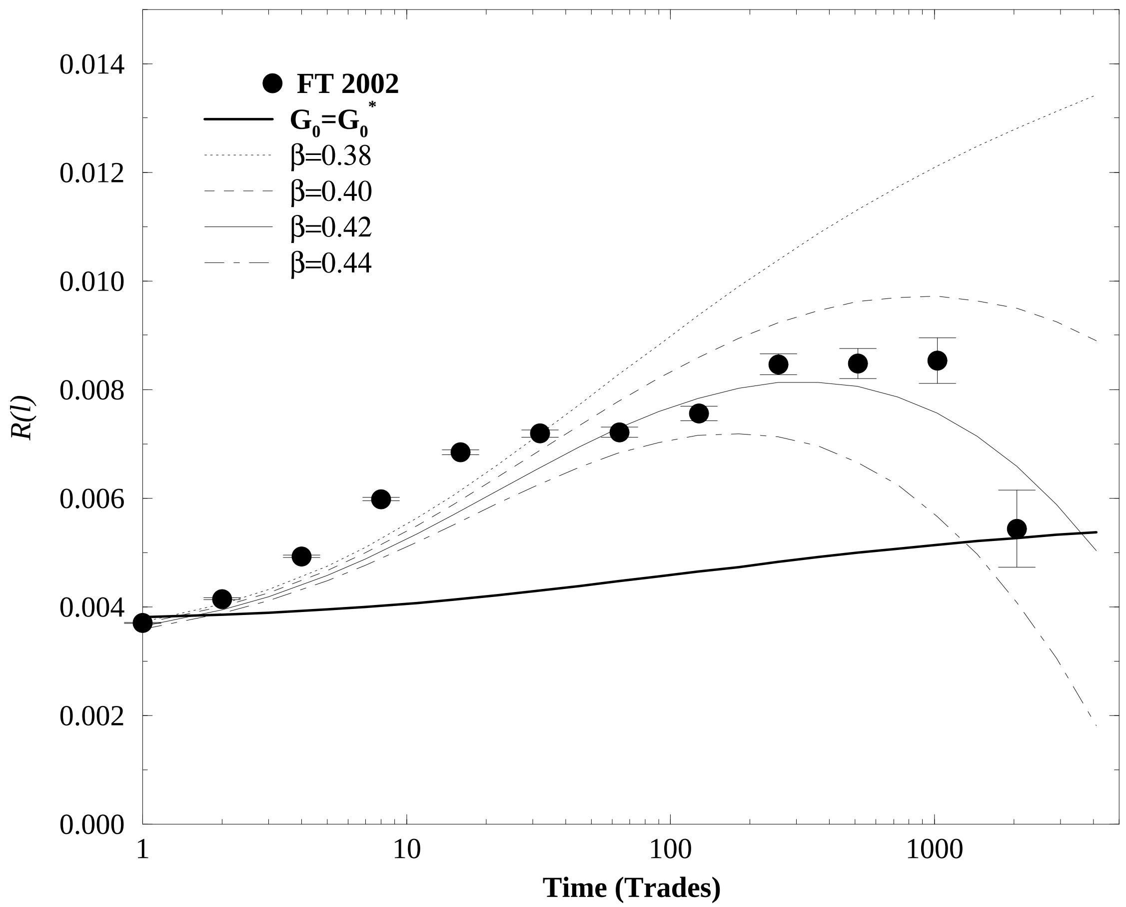
Transient impact: more technicalities

- An exact relation that allows to measure $G(\ell)$:

$$\mathcal{I}(\ell) = K \left[G(\ell) + \sum_{0<n<\ell} G(\ell - n)\mathcal{C}(n) + \sum_{n>0} \left[G(\ell + n) - G(n) \right] \mathcal{C}(n) \right]$$

(and a more complicated equation for $\mathcal{D}(\ell)$).

Theoretical and empirical response function



Second generation models

- Markets are complex systems (i.e. made of *heterogeneous, interacting elements*) $\longrightarrow$ rich endogenous dynamics

- "Second generation" models should start from:
  - agent based models (what do traders do?),
  - high frequency microstructure data,
  - a proper theory of *impact* (non-linear, transient,...)
  - identify interactions, feedback loops and contagion mechanisms

Second generation models

- Coarse-graining *should* lead to the emergence of some universality, power-laws and intermittency (but how, precisely?)

- We should be ambitious and try to predict (at least qualitatively) the value and dynamics of the parameters (volatility, correlations, etc.)

- Help identify systemic instabilities and liquidity (micro-) crises

  (e.g. spread $\rightarrow$ vol. $\rightarrow$ spread and May 6th "flash crash")

- Think about rules and regulations that endogenize stabilisation mechanisms

  (e.g. mark-to-market with liquidity discount, dynamic make/take fees, etc.)