Optimal Housing, Consumption, and Investment Decisions over the Life-Cycle

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Outline

Motivation

- Labor income and housing decisions important for most individuals
- Some papers include labor income, some papers housing decisions
- The few papers including both aspects are restrictive [Campbell/Cocco (QJE03), Cocco (RFS05), Yao/Zhang (RFS05), Van Hemert (WP09)]
- Difficult optimization problem – typically solved by highly complex numerical methods

This paper

- Rich model: stochastic labor income, house price, interest rate, stock price
- Disconnect housing consumption and housing investment
- Closed-form “Excel-ready” solution
- Model generates life-cycle behavior with many realistic features
- Non-negligible welfare gains from “perfect” house price-linked financial contracts

Outline

Financial assets

- Short-term interest rate (= return on cash):

$$dr_t = \kappa\left(\bar{r} - r_t\right) dt - \sigma_r \, dW_{rt}$$

- Price $B_t = B(r_t, t)$ of bond (20Y used later):

$$\frac{dB_t}{B_t} = \left(r_t + \lambda_B \sigma_B(r_t, t)\right) dt + \sigma_B(r_t, t) \, dW_{rt}$$

- Stock price:

$$\frac{dS_t}{S_t} = \left(r_t + \lambda_S \sigma_S\right) dt + \sigma_S \left(\rho_{SB}, \sqrt{1 - \rho_{SB}^2}\right) \begin{pmatrix} dW_{rt} \\ dW_{St} \end{pmatrix}$$

Housing

"Unit" house price H_t: (unit $\approx$ 1 "average" sq. foot)

$$\frac{dH_t}{H_t} = \left(r_t + \lambda_H \sigma_H - r^{\text{imp}}\right) \, dt + \sigma_H \left(\rho_{HB}, \hat{\rho}_{HS}, \hat{\rho}_H\right) \begin{pmatrix} dW_{rt} \\ dW_{St} \\ dW_{Ht} \end{pmatrix}$$

Housing positions:

- owning φ_{ot} housing units
- renting φ_{rt} units at rental rate νH_t per unit
- investing in REITs, φ_{Rt} units, total return $\frac{dH_t}{H_t} + \nu \, dt$

Housing consumption: $\varphi_{Ct} = \varphi_{ot} + \varphi_{rt}$

Housing investment: $\varphi_{It} = \varphi_{ot} + \varphi_{Rt}$

Labor income and wealth

Income rate Y_t until retirement at $\tilde{T}$:

$$\frac{dY_t}{Y_t} = (\bar{\mu}_Y(t) + br_t)\ dt + \sigma_Y(t)\left(\rho_{YB}, \hat{\rho}_{YS}, \hat{\rho}_Y\right)\begin{pmatrix} dW_{rt} \\ dW_{St} \\ dW_{Ht} \end{pmatrix}$$

In retirement: $Y_t = \Upsilon Y_{\tilde{T}},\ t \in [\tilde{T}, T]$.

Human wealth/capital:

$$L_t = \mathrm{E}_t^{\mathbb{Q}}\left[\int_t^T e^{-\int_t^s r_u\,du} Y_s\,ds\right] = \begin{cases} Y_t F(t, r_t), & t < \tilde{T}, \\ Y_{\tilde{T}} F(t, r_t), & t \in (\tilde{T}, T], \end{cases}$$

where F is known in closed form.

Financial/tangible wealth: X_t. Total wealth: $X_t + L_t$.

The individual's optimization problem

$$J(t, X, r, H, Y) = \sup \mathrm{E}_t \left[\int_t^T e^{-\delta(u-t)} \frac{1}{1-\gamma} \left(c_u^{\beta} \varphi_{Cu}^{1-\beta} \right)^{1-\gamma} ds \right]$$

Choose:

- c_t perishable consumption rate
- φ_{Ct} housing units consumed
- $\hat{\pi}_{It}$ fraction of total wealth invested in house, $\hat{\pi}_{It} = \frac{H_t \varphi_{It}}{X_t + L_t}$
- $\hat{\pi}_{Bt}$ fraction of total wealth invested in bond
- $\hat{\pi}_{St}$ fraction of total wealth invested in stock

Selected parameter values

Individual	
Wealth	20,000
Risk aversion	4
Work life	30 Y
Retirement	20 Y
House	
Exp. return	1%
Volatility	12%
Imputed rent	5%
Rent	5%
Unit price	250

Excess stock return	5%
Income	
Initial	20,000
Avg. growth	2%
Volatility	7.5%
Retirement	60%
Correlations	
income/stock,bond	0
house/stock	0.5
house/bond	0.65
income/house	0.57

Outline

Solution to the HJB-equation...

$$J(t,X,r,H,Y) = \frac{1}{1-\gamma} g(t,r,H)^{\gamma} (X + YF(t,r))^{1-\gamma},$$

$$g(t,r,H) = \frac{\eta\nu}{1-\beta} H^{k} \int_{t}^{T} e^{-d_1(u-t) - \beta\frac{\gamma-1}{\gamma}\mathcal{B}_{\kappa}(u-t)r}\, du,$$

$$c = \eta \frac{\beta\nu}{1-\beta} H^{k} \frac{X + YF}{g},$$

$$\varphi_C = \eta H^{k-1} \frac{X + YF}{g},$$

$$\hat{\pi}_S, \hat{\pi}_S, \hat{\pi}_S : \quad \text{see below}$$

Investments – fractions of total wealth

Stocks $$\hat{\pi}_S = \underset{4\%}{\frac{1}{\gamma}\frac{\xi_S}{\sigma_S}} \underset{0 \leftrightarrow 33\%}{- \frac{\sigma_Y \zeta_S}{\sigma_S}\frac{L}{X+L}},$$

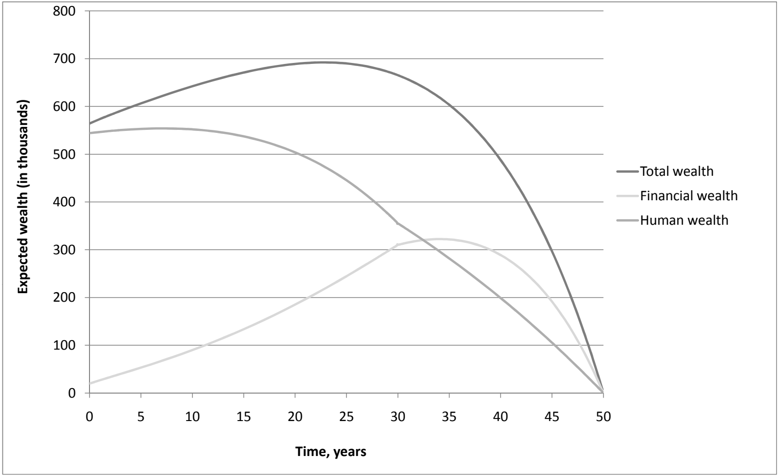
Bonds $$\hat{\pi}_B = \underset{-63\%}{\frac{1}{\gamma}\frac{\xi_B}{\sigma_B}} - \left(\underset{0 \leftrightarrow 116\%}{\frac{\sigma_Y \zeta_B}{\sigma_B}} - \underset{0 \leftrightarrow -42\%}{\frac{\sigma_r}{\sigma_B}\frac{F_r}{F}}\right)\frac{L}{X+L} \underset{49\%}{- \frac{\sigma_r}{\sigma_B}\frac{g_r}{g}},$$

House $$\hat{\pi}_I = \underset{91\%}{\frac{1}{\gamma}\frac{\xi_I}{\sigma_H}} \underset{0 \leftrightarrow -109\%}{- \frac{\sigma_Y \zeta_I}{\sigma_H}\frac{L}{X+L}} \underset{15\%}{+ \frac{H g_H}{g}}$$

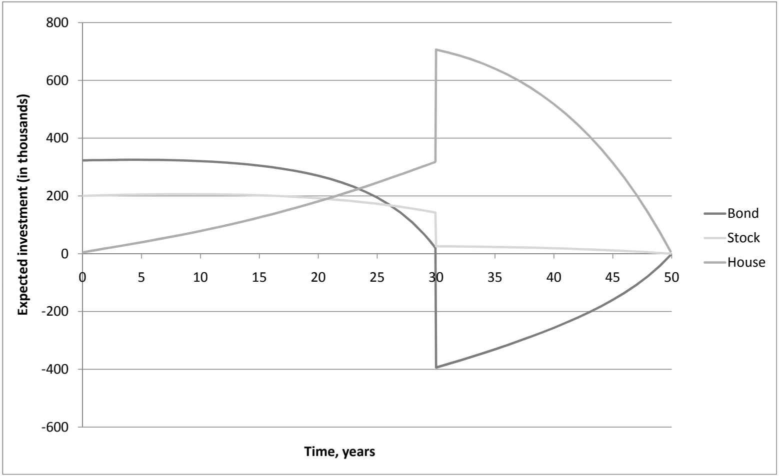
speculative — adjust for human wealth — hedge

Note: σ_Y drops to zero at retirement, but $L/(X+L) > 0 \rightsquigarrow$ jump

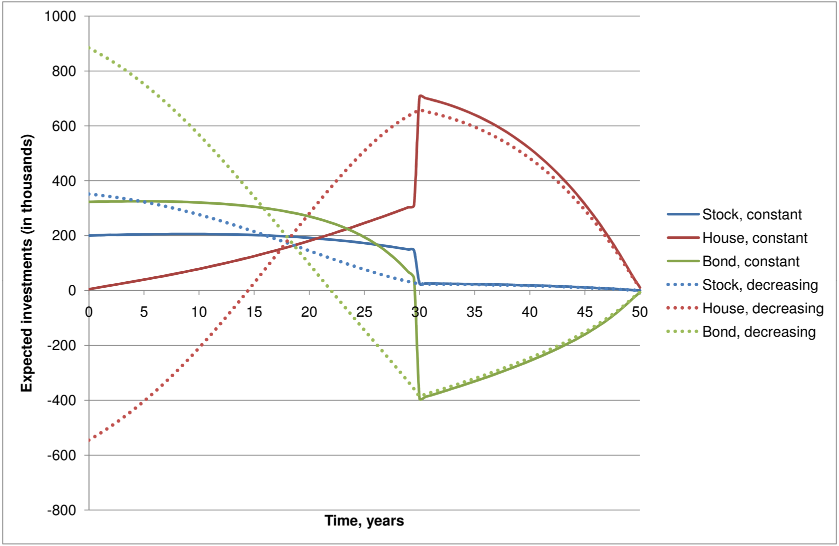
Expected wealth over the life-cycle



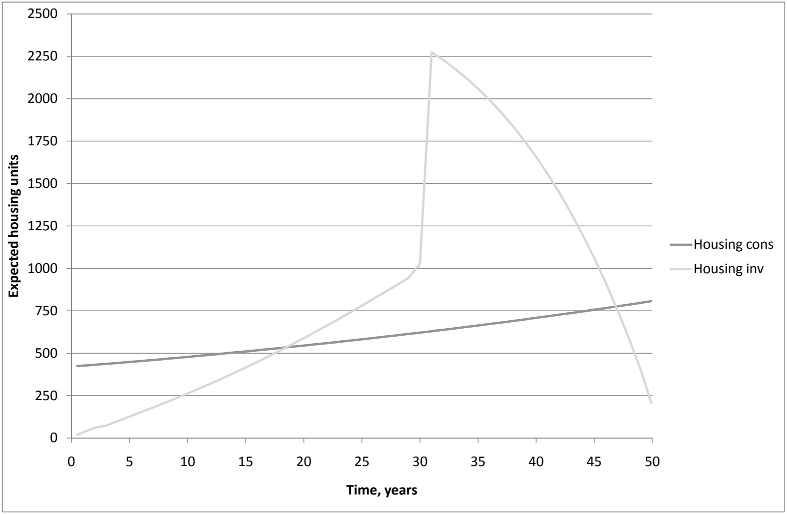
Expected investments over the life-cycle



... with age-dependent income volatility

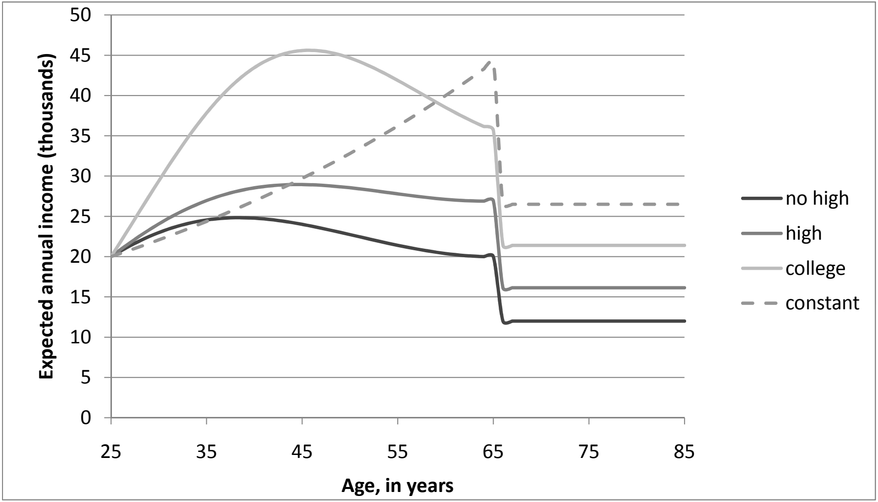


Housing consumption and investments

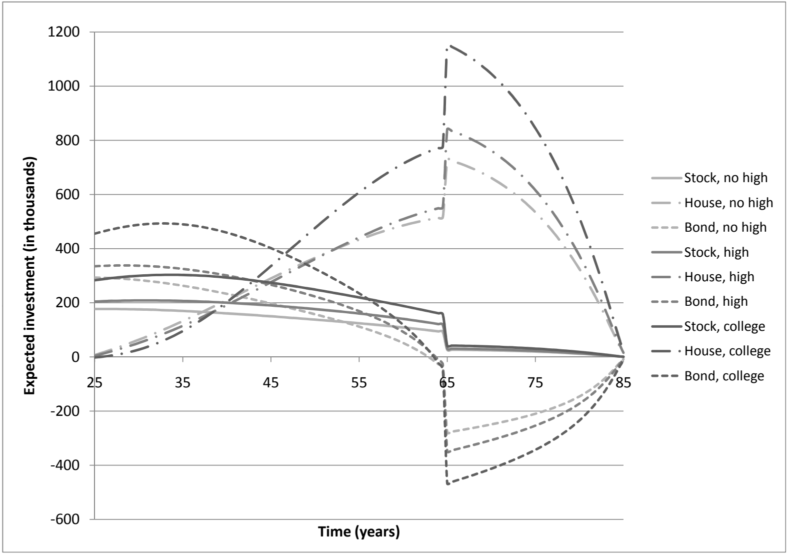


Outline

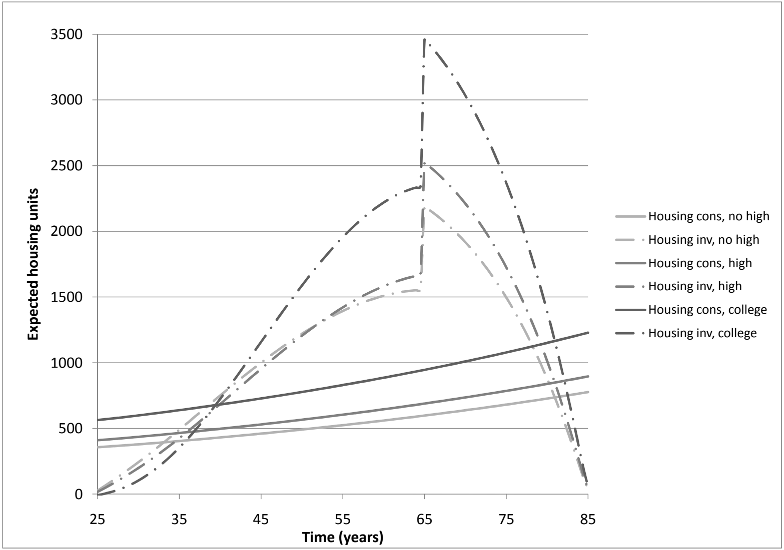
Empirical income profiles



Expected investments again



Housing consumption and investments again

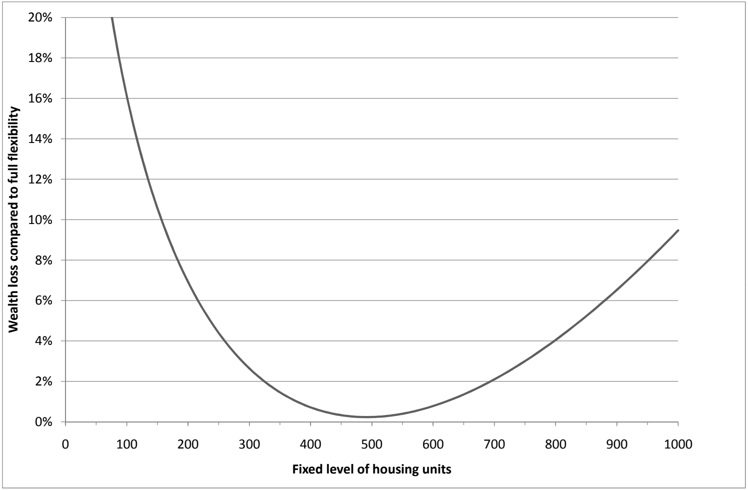


Outline

Unspanned labor income

- Our solution requires market completeness, i.e., spanned labor income
- Labor income is much closer to being spanned when housing assets are included – high income-house price correlation
- If labor income is unspanned, the implementation of our consumption/investment strategy is sub-optimal
- Bick, Kraft & Munk (presented Thursday): the welfare loss is relatively small (magnitude $\leq 3\%$)

Constant housing consumption



Note: minimum certainty-equivalent wealth loss is only 0.24%

Infrequent rebalancing of housing positions

	Welfare loss	
Adjustment frequency	2 years	5 years
Infrequent φ_C, frequent φ_I	0.03%	0.07%
Infrequent φ_I, frequent φ_C	0.43%	1.85%
Infrequent φ_C and φ_I	0.46%	1.96%

- suggests moderate welfare gains from market for REITs or CSI housing contracts
- suggests moderate effects of housing transactions costs

Outline

Summary

- Framework for consumption, housing, and investment decisions over the life-cycle
- High income/house correlation $\leadsto$ life-cycle patterns in optimal decisions, in particular housing investment
- Calibrated model has many realistic features
- Lots of comparative statics in the paper
- Need to know more about typical life-cycle pattern in income volatility and income/house price correlation
- Our model is a benchmark for numerical solutions with portfolio constraints and transaction costs