Optimal investment under relative performance concerns

GE.Espinosa Joint work with N.Touzi

June 26th, 2010

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- Classical portfolio optimization: maximization of one's utility with respect to one's personal wealth or consumption
- Economical literature: relative wealth concerns

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- Classical portfolio optimization: maximization of one's utility with respect to one's personal wealth or consumption
- Economical literature: relative wealth concerns

Aim: Try to derive a portfolio optimization theory with such relative wealth concerns.

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A simple case Influence of λ General framework

The market:

- a non-risky asset with 0 interest rate
- a d-dimensional risky asset S
- N agents

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The dynamics of S is given by:

$$dS_t = \operatorname{diag}(S_t)\sigma_t(\theta_t dt + dW_t)$$

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We will first assume that all agents are similar.

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A simple case Influence of λ General framework

We write X^i the wealth process of agent *i* and π^i the portfolio of agent *i*. Investment horizon *T*.

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We write X^i the wealth process of agent *i* and π^i the portfolio of agent *i*. Investment horizon T.

Characteristics of agent *i*:

- exponential utility function with risk preference $\eta > 0$
- relative performance preference $\lambda \in [0,1]$
- average wealth of other agents $ar{X}^i = rac{1}{N-1}\sum_{j
 eq i}X^j$

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Thus agent *i* wants to maximize upon admissible π^i :

$$-\mathbb{E}e^{-\frac{1}{\eta}\left[(1-\lambda)X_{T}^{i}+\lambda(X_{T}^{i}-\bar{X}_{T}^{i})\right]}$$

given other π^j $(j \neq i)$

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A simple case Influence of λ General framework

By symmetry, at the equilibrium, it is the same as:

$$\sup_{\pi^{i}} - \mathbb{E}e^{-\frac{1-\lambda}{\eta}X_{T}^{i}}$$

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FrameworkA simple caseGeneral caseInfluence of λ ExamplesGeneral frame

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Same as in the classical case but $\eta \to \frac{\eta}{1-\lambda}$

So the optimal portfolio is (for deterministic θ , $\lambda < 1$):

$$\hat{\pi}_t^i = \frac{\eta}{(1-\lambda)} \sigma_t^{-1} \theta_t$$

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A simple case Influence of λ General framework

Influence of λ :

- $|\hat{\pi}^i|$ is increasing w.r.t λ
- if $\lambda
 ightarrow 1$, $|\hat{\pi}^i|
 ightarrow \infty$ a.s.

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Specific parameters:

- risk preference $\eta_i > 0$
- relative performance preference $\lambda_i \in [0,1]$

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A simple case Influence of λ General framework

Specific parameters:

- risk preference $\eta_i > 0$
- relative performance preference $\lambda_i \in [0,1]$

Portfolio constraints:

Each agent has an area of investment. π^i must stay in a certain A_i that will be assumed to be a vector sub-space of \mathbb{R}^d .

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So finally agent *i*'s criterion:

$$\sup_{\pi^{i}\in\mathcal{A}_{i}}-\mathbb{E}e^{-\frac{1}{\eta_{i}}[X_{T}^{i,\pi^{i}}-\lambda_{i}\bar{X}_{T}^{i}]}$$

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A simple case Influence of λ General framework

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And we look for Nash equilibria between the N agents.

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Using the ideas of El Karoui-Rouge or Hu-Imkeller-Muller for optimal investment in incomplete markets, we derive a (quadratic) BSDE:

$$dY_t^i = \left(\frac{\eta|\theta_t|^2}{2} - \frac{1}{2\eta} \left| Z_t^i + \eta \theta_t - P_{\sigma_t A_i} (Z_t^i + \eta \theta_t) \right|^2 \right) dt + Z_t^i . dB_t$$
$$Y_T^i = \lambda(\bar{X}_T^i - \bar{x}_i) = \frac{\lambda}{N-1} \sum_{j \neq i} \int_0^T \pi_u^j . \sigma_u dB_u$$

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And the optimal portfolio is given by:

$$\sigma_t \hat{\pi}_t^i = P_{\sigma_t A_i} (Z_t^i + \eta \theta_t)$$

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Framework	Idea
General case	Case σ and θ deterministic
Examples	Limit as $N \to \infty$
Examples	Influence of λ

Putting them together it brings:

$$Y_0^i = -\eta \ln \frac{d\mathbb{Q}}{d\mathbb{P}} + \frac{1}{2\eta} \int_0^T |Q_t^i(Z_t^i)|^2 dt - \int_0^T (Z_t^i - \frac{\lambda}{N-1} \sum_{j \neq i} P_t^j(Z_t^j)) dB_t$$

where P_i = orthogonal projection on σA_i , $Q_i = I - P_i$, \mathbb{Q} = the martingale probability and B a Brownian motion under \mathbb{Q} .

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It can be rewritten as:

$$Y_0^i = -\eta \ln \frac{d\mathbb{Q}}{d\mathbb{P}} + \frac{1}{2\eta} \int_0^T |Q_t^i([\psi_t(\zeta_t)]^i)|^2 dt - \int_0^T \zeta_t^i dB_t$$

where $Y \in \mathbb{R}^N$, $\zeta \in M_{N,d}(\mathbb{R})$ and $\psi \in GL(M_{N,d}(\mathbb{R}))$.

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Framework General case Examples	Idea Case σ and θ deterministic Limit as $N \to \infty$ Influence of λ
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It can be rewritten as:

$$Y_0^i = -\eta \ln \frac{d\mathbb{Q}}{d\mathbb{P}} + \frac{1}{2\eta} \int_0^T |Q_t^i([\psi_t(\zeta_t)]^i)|^2 dt - \int_0^T \zeta_t^i dB_t$$

where $Y \in \mathbb{R}^N$, $\zeta \in M_{N,d}(\mathbb{R})$ and $\psi \in GL(M_{N,d}(\mathbb{R}))$.

 \rightarrow N-dimensional system of coupled quadratic BSDEs.

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 Limit as $N \to \infty$

Assume σ and θ are deterministic:

Theorem: There exists a Nash equilibrium and the equilibrium portfolio for agent *i* is given by:

$$\pi^{i} = \eta \sigma^{-1} P^{i} \left[I - \frac{\frac{\lambda}{N-1}}{1 + \frac{\lambda}{N-1}} \sum_{j \neq i} P^{j} \left(I + \frac{\lambda}{N-1} P^{i} \right) \right]^{-1} \theta$$

 $(P^i \text{ is the orthogonal projection on } \sigma A_i)$

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Under the assumption:

$$\lambda < 1 ext{ or } igcap_{i=1}^{N} A_i = \{0\}$$

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If *d* is fixed:

Theorem: Let *d* be fixed, and assume moreover that $\frac{1}{N}\sum_{i=1}^{N}P^{i} \to U \text{ in } \mathcal{L}(\mathbb{R}^{d}) \text{ with } ||\lambda U|| < 1. \text{ Then } \pi_{N}^{i} \to \pi_{\infty}^{i} \text{ where:}$

$$\pi^{i}_{\infty} = \eta \sigma^{-1} P^{i} [(I - \lambda U)^{-1} \theta]$$

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Market index:
$$ar{X}_t^N = rac{1}{N}\sum_{i=1}^N X_t^i$$

We find:

$$d\bar{X}_t^{\infty} = \eta U(I - \lambda U)^{-1} \theta_t [\theta_t dt + dW_t]$$

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Framework General case Examples	Idea Case σ and θ deterministic Limit as $N \to \infty$ Influence of λ
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We find:

$$d\bar{X}_t^{\infty} = \eta U (I - \lambda U)^{-1} \theta_t [\theta_t dt + dW_t]$$

Moreover, $U(I - \lambda U)^{-1}$ is diagonalizable with eigenvalues

$$0 < \frac{\mu_1}{1 - \lambda \mu_1} < \ldots < \frac{\mu_d}{1 - \lambda \mu_d} < 1$$

and with the same orthonormal eigenvectors as U (independent of λ).

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 \rightarrow Both the drift and the risk (volatility) of the market increase with λ

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Framework General case ExamplesIdea Case σ and θ deter imit as $N \to \infty$ Influence of λ	
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- \rightarrow Both the drift and the risk (volatility) of the market increase with λ
- \rightarrow encourages financial bubbles

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Investment on the whole market Investment on a specific asset Investment on hyperplanes

Each agent can invest in the whole market:

$$\forall i, A_i = \mathbb{R}^d$$

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Investment on the whole market Investment on a specific asset Investment on hyperplanes

Each agent can invest in the whole market:

$$\forall i, A_i = \mathbb{R}^d$$

Under the assumption $\prod_{j=1}^N \lambda_j < 1$, there is an equilibrium.

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First case: $\forall i, \lambda_i = \lambda$, then:

$$\hat{\pi}_t^i = \left[\frac{N-1}{N+\lambda-1} + \frac{\lambda N}{(1-\lambda)(N+\lambda-1)}\frac{\eta^N}{\eta_i}\right]\pi_t^{0,i}$$

 η^N is the average of the η_j 's.

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As $N \to \infty$, if $\eta^N \to \eta > 0$ then the equilibrium portfolio of agent *i* converges to:

$$\hat{\pi}_t^{\infty,i} = \left(1 + \frac{\lambda}{1 - \lambda} \frac{\eta}{\eta_i}\right) \pi_t^{0,i}$$

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Same conclusions as in the beginning.

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Second case:
$$\forall j \neq i_0, \ \lambda_j = 1, \ \lambda_{i_0} < 1 \ (\forall i, \ \eta_i = \eta)$$
, then:

$$\hat{\pi}_t^{i_0} = \left[rac{1}{1-\lambda_{i_0}} + rac{\lambda_{i_0}(N-1)}{1-\lambda_{i_0}}
ight]\pi_t^0$$

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As $N \to \infty$, even if $\lambda_{i_0} < 1$, $|\pi_t^{i_0}| \to \infty$ a.s (except for $\lambda_{i_0} = 0$).

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As $N \to \infty$, even if $\lambda_{i_0} < 1$, $|\pi_t^{i_0}| \to \infty$ a.s (except for $\lambda_{i_0} = 0$).

 \rightarrow Impact of surrounding "stupidity".

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$$d = N$$
, $A_i = \mathbb{R}e_i$

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$$d = N$$
, $A_i = \mathbb{R}e_i$
- $\sigma^2 = \sigma^2 \begin{pmatrix} 1 & \rho^2 \\ & \ddots & \\ \rho^2 & & 1 \end{pmatrix}$ with $\rho \in (-1, 1)$ and $\sigma > 0$

Framework	Investment on the whole market
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$$d = N$$
, $A_i = \mathbb{R}e_i$
- $\sigma^2 = \sigma^2 \begin{pmatrix} 1 & \rho^2 \\ & \ddots & \\ \rho^2 & & 1 \end{pmatrix}$ with $\rho \in (-1, 1)$ and $\sigma > 0$

- we also assume $\forall i, \ \theta_i = \theta$.

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As $N \to \infty$ we find:

$$\hat{\pi}^{i} = \frac{\eta\theta}{\sigma} \frac{1}{1 - \lambda\rho^{2}} e_{i}$$

GE.Espinosa Joint work with N.Touzi Optimal investment under relative performance concerns

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So:

- the more you look at other agents (λ close to 1)
- the more correlated the assets are (ρ^2 close to 1) the more risk you take.

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So:

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For independent investments ($\rho = 0$), we find the classical optimal portfolio: no impact of λ .

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- Here again d = N. But $A_i = (\mathbb{R}e_i)^{\perp}$

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- Here again
$$d = N$$
. But $A_i = (\mathbb{R}e_i)^{\perp}$

-
$$\sigma = \sigma I$$
 and $\forall i, \theta_i = \theta$.

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We find:



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We find:

$$\hat{\pi}_t^i = rac{\eta heta}{\sigma} rac{1}{1-\lambda+rac{\lambda}{N-1}} \sum_{j
eq i} e_j$$

Same kind of conclusions as for investment on the whole market, but smaller impact of λ , especially for small N.

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Special thanks to J.Lebuchoux - Reech Aim

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