On the non-linear relationship between default intensity and leverage

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Introduction

- Credit risk: potential losses due to
  - Default;
  - Downgrade;
- Recent bankruptcies and technical defaults:
  - General Motors (2009), Lehman Brothers (2008);
  - AIG (2008), Fannie Mae (2008) and Freddie Mac (2008);
- Need tools/models to estimate the distribution of losses due to credit risk;

Introduction

- Credit risk comes from two sources:
  - Moment of default;
  - Amount of losses given default;
- Literature has large focus on modeling moment of default:
  - Structural, reduced-form, hybrid models;
- Altman et al. (2004) among others: recovery rate is inversely proportional to default probability;
- Recovery rate modeling has been overlooked until very recently
  - Bakshi, Madan & Zhang (2006b), Pan & Singleton (2008), Das & Hanouna (2009);

Key contributions of the paper

- **Hybrid model** where default intensity is a non-linear transformation of leverage;
  - Bakshi, Madan, Zhang (2006a) is a linear function of leverage;
  - Duffie, Saita, Wang (2007) and Bharath & Shumway (2006) use Cox proportional hazard approach to find determinants of default;
- Observed leverage at default determines **recovery rate** upon default;
  - Assets available after liquidation are used to compute recovery rate;
  - Gives rise to a term structure of recovery rate;
  - Bakshi, Madan, Zhang (2006b) and Das & Hanouna (2009) use arbitrary functions of default intensity to build recovery rate;
  - Pan & Singleton (2008) discuss the identification problem between recovery rate and default intensity;
    - Solved by the use of recovery of face value and the recovery rate model proposed;

Key contributions of the paper

- Estimation of parameters account for **trading noise**;
  - Maximum likelihood approach using term structure of CDS prices;
  - Consistent with Duan & Fulop (2009);
- **Empirical study** on non-linearity of default intensity with respect to leverage;
  - Estimation performed on a firm-by-firm basis;
  - Investment-grade companies vs non-investment grade;
  - Stability over time, impact of credit crisis;

Outline

1. Introduction;
2. Hybrid model;
   1. Moment of default;
   2. Amount of losses;
3. Estimation;
4. Empirical study;
5. Conclusion;

Moment of default

- Hybrid models: combine elements of structural and reduced-form credit risk models;
  - Incomplete information models: Duffie & Lando (2001), Çetin, Jarrow, Protter & Yildirim (2004), Giesecke (2004);
  - Other important contributions: Bakshi, Madan & Zhang (2006a), Madan & Unal (2000);
- **Structural component**: define a model for the evolution of assets of the company $\{A_t, t \geq 0\}$ and its liabilities (or of a default barrier) $\{L_t, t \geq 0\}$;
- Debt ratio or leverage $\{X_t, t \geq 0\}$: $X_t = \frac{L_t}{A_t}$;
- **Reduced-form component**: default occurs at the first jump of a Cox process;
- Default intensity $\{H_t, t \geq 0\}$: $H_t = h\left(X_t\right)$;

Moment of default

- Transformation of leverage h :
  - Increasing (when leverage increases, default intensity should increase as well);
  - Convex (a small change in leverage has more impact on default intensity when the latter is large);
  - 2 illustrations
- **Assumption**:

$$h(x) \equiv \frac{\alpha}{\theta}\left(\frac{x}{\theta}\right)^{\alpha-1}, \alpha > 0, \theta > 0.$$

- **Interpretation**:
  - α determines the sensitivity of default with respect to debt ratio;
  - θ determines a critical level of leverage after which default and liquidation seriously accelerate;

Illustration # 1

- Default probability for various h functions when X_t is constant;
- **Examples**: $h(x) = c$ (constant), $h(x) = cx^2$, $h(x) = cx^{10}$.

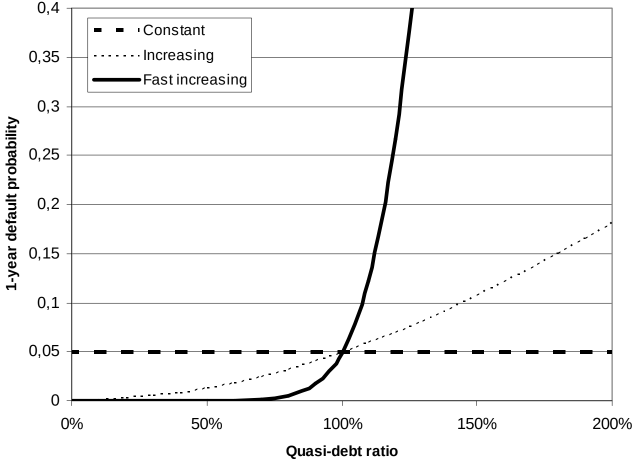
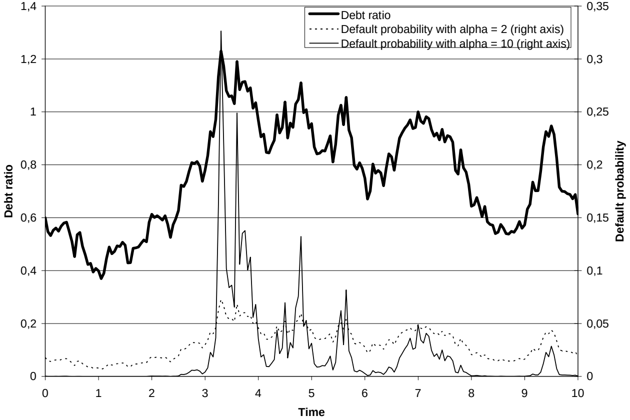


Illustration # 2

- Joint evolution of debt ratio and default intensity
- **Examples**: $h(x) = cx^2$, $h(x) = cx^{10}$



Amount of loss upon default

- Debt ratio at the moment of default (τ) determines amount of loss;
- Assets available to debtholders: subtract legal and liquidation fees (fraction κ of assets);
- Proposed recovery rate model:

$$R_\tau = \min\left(\frac{A_\tau\left(1-\kappa\right)}{L_\tau}; 1\right).$$

- Recovery rate model and recovery of face value assumption: no identification problem;
- Historical facts about recovery rates:
  - Between 40% and 70%;
  - Inversely proportional to default probability;
  - Decrease during recessions;

Unobservability of market values

- Market value of assets and liabilities: not observable (Jarrow & Turnbull (2000), Jarrow & Protter (2004))
- Solutions:
    - Change model: incomplete information models;
    - Estimation approach: MLE (Duan (1994))
        - Using equity or other derivatives price, find the corresponding asset value;
- Equity (or other derivatives' prices) are noisy;
- Duan & Fulop (2009) use an adaptation of the Auxiliary Particle Filter to the estimation of Merton (1974) using equity prices;
    - More precise estimates of the asset volatility;

State-space representation

- Unobserved variable (state equation): evolution of **market** debt ratio $\{X_t, t \geq 0\}$;
- Observed variables (measurement equation): prices of derivatives, equity, bonds, etc. given by $\left\{Y_t^{(i)}, t \geq 0\right\}, i = 1, 2, 3, ..., N$
    - Can integrate N sources of information: equity, term structure of CDS, bonds, etc.
- All prices depend on the evolution of market debt ratio using the function $g^{(i)}(X_t)$;
- **Idea**: observed prices are noisy non-linear transformations of **market** debt ratio i.e.

$$Y_t^{(i)} = g^{(i)}(X_t)\, e^{\nu_k}$$

  where ν_k is a Gaussian noise.
- We use the unscented Kalman filter (UKF) since $g^{(i)}$ is non-linear;
    - The standard Kalman filter would be inappropriate;

Purposes

- Understand relationship between default intensity and leverage for investment-grade and non-investment grade companies;
- Stationarity of this relationship with respect to the occurrence of the 2007-2010 credit crisis;
- Behavior of the recovery rate given default
  - Term structure of recovery rate;
  - Impact of 2007-2010 credit crisis;

Data and methodology

- Capital structure of the companies: debt ratio is a geometric Brownian motion with drift $\mu_X^{\mathbb{P}}$ ($\mu_X^{\mathbb{Q}}$ for pricing purposes) and diffusion σ_X;
  - Approach presented in this paper **is not** limited by this assumption;
- 225 companies of the CDX NA IG and CDX NA HY indices;
- CDS prices: DATASTREAM
  - Term structure of prices: 1-5 years used, 1-10 years available;
  - Observed each month from January 2004 to May 2008;
  - Approximately 50 000 observations;
- Interest rates: FRED
- Credit rating: S&P
- Parameters are estimated for each company using time series of monthly CDS prices;

Relationship with credit ratings

- Market debt ratio mean return μ_X :
  - 1.85% (IG) vs 3.98% (non-IG): **non significant**
- Market debt ratio volatility σ_X :
  - 10.31% (IG) vs 13.02% (non-IG): **significant**
- Convexity of the transformation α :
  - 12.01 (IG) vs 16.7 (non-IG): **significant**
- Critical level of debt ratio θ :
  - 1.59 (IG) vs 1.42 (non-IG): **significant**
- Liquidation and legal costs κ :
  - 51.74% (IG) vs 45.82% (non-IG): **non significant**
- Initial market debt ratio $\widehat{X}_0$:
  - 64.69% (IG) vs 76.55% (non-IG): **significant**

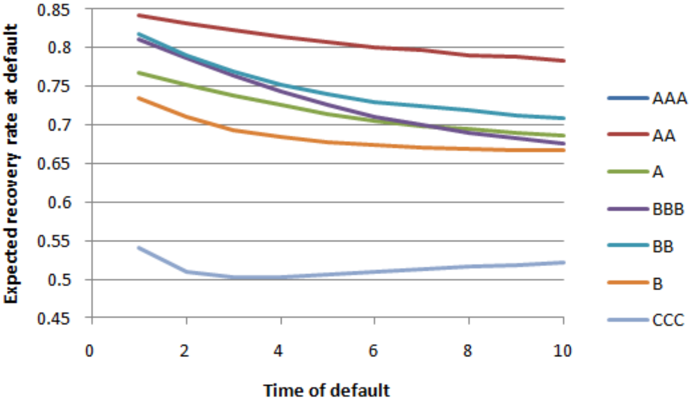
Effect of credit crisis

- Shown for investment-grade companies;
- For parameters not shown, effect of credit crisis is small;

	2004-2006	2006-2008
μ_X	-0.16%	2.59%
σ_X	5.97%	11.54%
α	16.3951	11.4695
θ	1.3862	1.6814

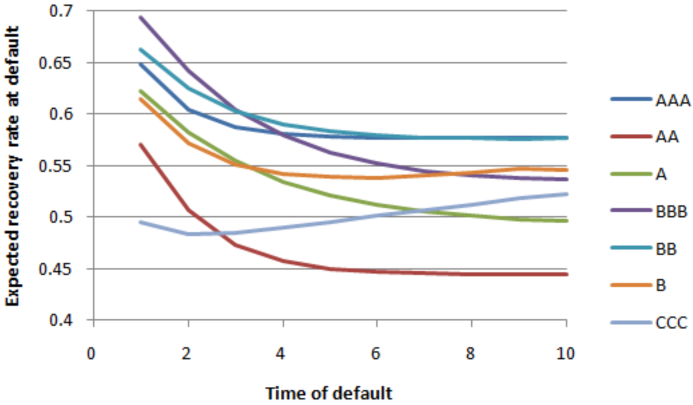
Term structure of recovery rate

- Time period: 2004-2006
- Values consistent with literature;



Term structure of recovery rate

- Time period: 2006-2008
- Important drop: approximately 15%



Conclusion

- Hybrid credit risk model;
  - Non-linear transformation of leverage;
  - Recovery rate inversely proportional to default probability;
- Empirical study;
  - Investment-grade companies have lower convexity: default has a greater amount of surprise;
  - Investment-grade companies have higher default threshold: greater leverage is tolerated;
  - Effect of credit crisis is to increase share of surprises in default;
  - Term structure of recovery rate: increasing and decreasing as with credit spread curves;
    - Decreased importantly in the second part of the sample;