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Accelerated Investment and Credit Risk under a Low Interest Rate Environment

: A Real Options Approach

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1-1. Motivation

- Recently, low interest rate environments

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  accelerate firm/households' investment and debt financing,

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  leading to non-performing loan problems (high credit risk) in many countries.

  Ex. US Subprime Mortgage Loan Problem

  Japanese Bubble Economy

  Why did these problems occur?

- We examine these problems by a real options perspective.

1-2. We consider…

- (Situation1): If there are expectations of continued low interest rate environment

  Question: Do firms increase investment and debt financing?

- (Situation2): If there are expectations of future interest rate increases

  Question: Do firms make "last-minute investments" while the financing cost are still favorable (low) ?

1-3. How to solve it?

- We need to analyze both investment and debt financing.
  - Real options can analyze

    the optimal investment timing
  - Corporate finance can analyze

    the optimal debt amount (capital structure)

+)

  - Real options + Corporate finance : Sundaresan&Wang[2007]

  We can analyze the relationship between investments and credit risks. $\rightarrow$ apply to **(Situation.1)**

- Furthermore,

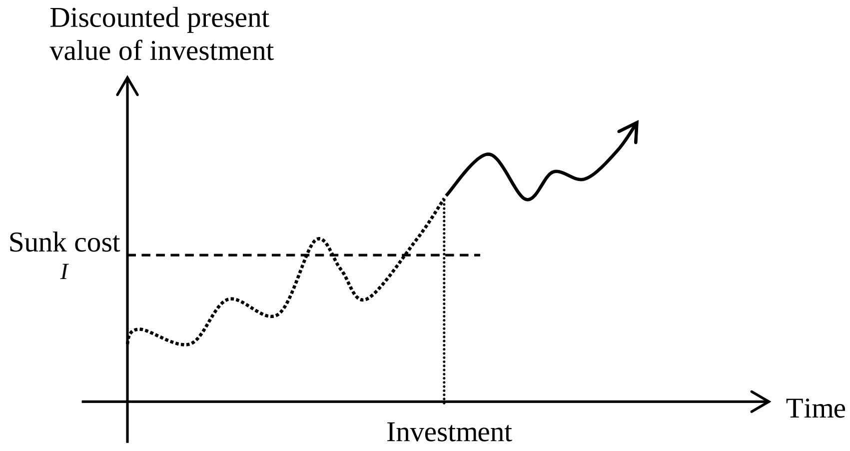
  We extend S&W[2007] to the model where the risk free rate changes using the technique of Grenadier&Wang[2007]

  $\rightarrow$ apply to **(Situation.2)**

2. Under the Expectations of Continued Low Interest Rate (Situation.1)

2-1.Standard real options model

- Dixit & Pindyck[1994]



- sunk cost I

  vs project cash flow X_t

$$dX_t = \mu X_t dt + \sigma X_t dW_t$$

- The firm decides the optimal investment time τ to maximize the net present value:

$$V(X_t) = \max_{\tau \in F_t} E_t \left[e^{-r(\tau - t)} \left(\int_{\tau}^{\infty} e^{-r(s-\tau)} X_s ds - I \right) \right]$$

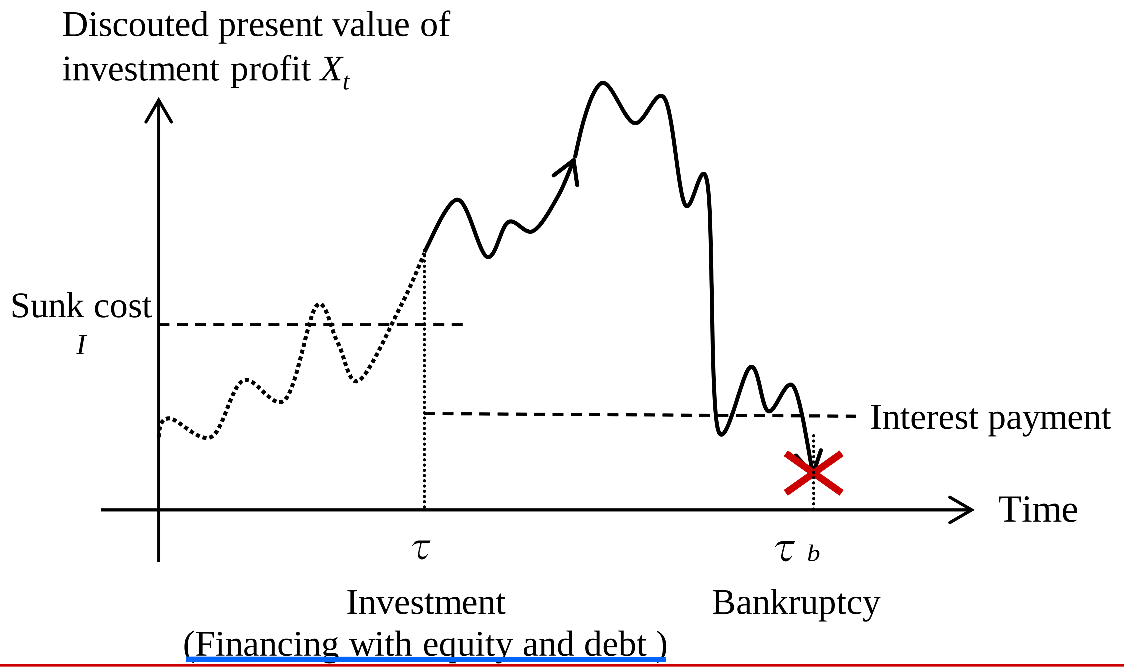
Discounted present value of the cash flow

Sunk cost(investment cost)

2-2. Real options + Corporate finance

- Sundaresan & Wang[2007]

The extended model where the sunk cost is financed with debt and equity.



2-3.Investment and Deft financing

- The firm decides the optimal investment time τ to maximize the net present value:

$$V(X_t) = \max_{\tau \in F_t} E_t\left[e^{-r(\tau - t)}\left(W(X_\tau) - I\right)\right]$$

Investment time — The Firm's value — Investment cost

- The firm decides the optimal debt amount b (interest payments) to maximize the firm's value:

$$W(X_t) \underset{\text{Max } b}{=} E(X_t, b) + D(X_t, b)$$

Equity value — Debt value

$$= W_a(X_t) + Tb(X_t, b) - Bc(X_t, b)$$

Tax befit — Bankruptcy cost

$b\uparrow \rightarrow TB\uparrow$ — $b\uparrow \rightarrow BC\uparrow$

2-4. The equity and the debt value

- Equity holder determines the optimal bankruptcy time τ_b to maximize the Equity Value:

$$E(X_t) = \max_{\tau_b \in F_t} E_t\left[\int_t^{\tau_b} e^{-r(s-t)} (1-\tau_{ax})(X_s - b) ds + 0 \right]$$

τ_{ax} tax rate

Bankruptcy time

Investment cash flow

– Interest payments

- The Debt value:

$$D(X_t) = E_t\left[\int_t^{\tau_b} e^{-r(s-t)} b\, ds + e^{-r(\tau_b - t)} (1-\alpha) W_a(X_{\tau_b}) \right]$$

α liquidation cost rate

Interest incomes

The liquidation value at the bankruptcy

$$W_a(X_{\tau_b}) = \int_{\tau_b}^{\infty} e^{-r(s-\tau)} (1-\tau_{ax}) X_s ds$$

2-5. The model solutions (1)

- The equity value

$$E(x) = (1-\tau_{ax})\left(\left(\frac{x}{r-\mu} - \frac{b}{r}\right) - \left(\frac{x_b}{r-\mu} - \frac{b}{r}\right)\left(\frac{x}{x_b}\right)^{\gamma}\right) \quad , x > x_b \qquad x_b = \frac{\gamma}{\gamma-1}\frac{b}{r}(r-\mu)$$

Discounted present value

The bunkurptcy option value > 0

The threshold of bankruptcy

- The debt value

$$D(x) = \begin{cases} \frac{b}{r} - \left(\frac{b}{r} - W_b(x_b)\right)\left(\frac{x}{x_b}\right)^{\gamma} & , x > x_b \\ W_b(x_b) \equiv (1-\alpha)\frac{(1-\tau_{ax})x_b}{r-\mu} & , x \le x_b \end{cases}$$

Future Interest payments

Exposure at default × PD = EL

PD=Probability of default

EL=Expected Loss

2-6.The model solutions (2)

- The option value to invest:

$$V(x)=\begin{cases}\left(W(x_I^*)-I\right)\left(\dfrac{x}{x_I^*}\right)^{\beta} & ,x<x_I^* \\ W(x_I^*)-I & ,x\geq x_I^*\end{cases}$$

- The threshold of investment:

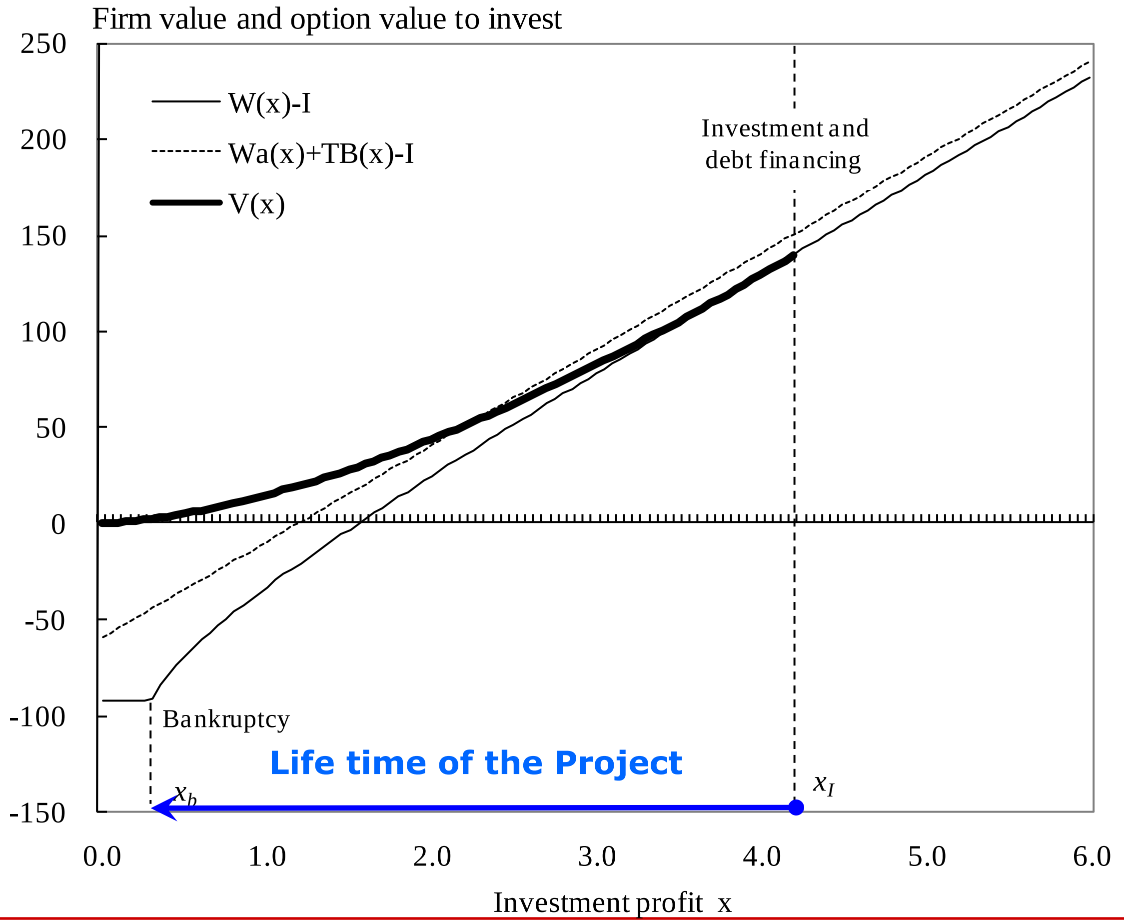
$$x_I^*=\psi\frac{\beta}{\beta-1}\frac{r-\mu}{(1-\tau_{ax})}I \qquad \psi\equiv\left(1+h^{\frac{1}{\gamma}}\left(\frac{\tau_{ax}}{1-\tau_{ax}}\right)\right)^{-1}\leq 1$$

- The optimal amount of debt (interest payments):

$$b^*=h^{\frac{1}{\gamma}}\frac{\gamma-1}{\gamma}\frac{\beta}{\beta-1}\frac{r\psi}{(1-\tau_{ax})}I \qquad h\equiv 1-\gamma\left(1-\alpha+\frac{\alpha}{\tau_{ax}}\right)\geqq 1$$

$\beta>0,\ \gamma<0$ $\qquad \frac{1}{2}\sigma^2 x(x-1)+\mu x-r=0$

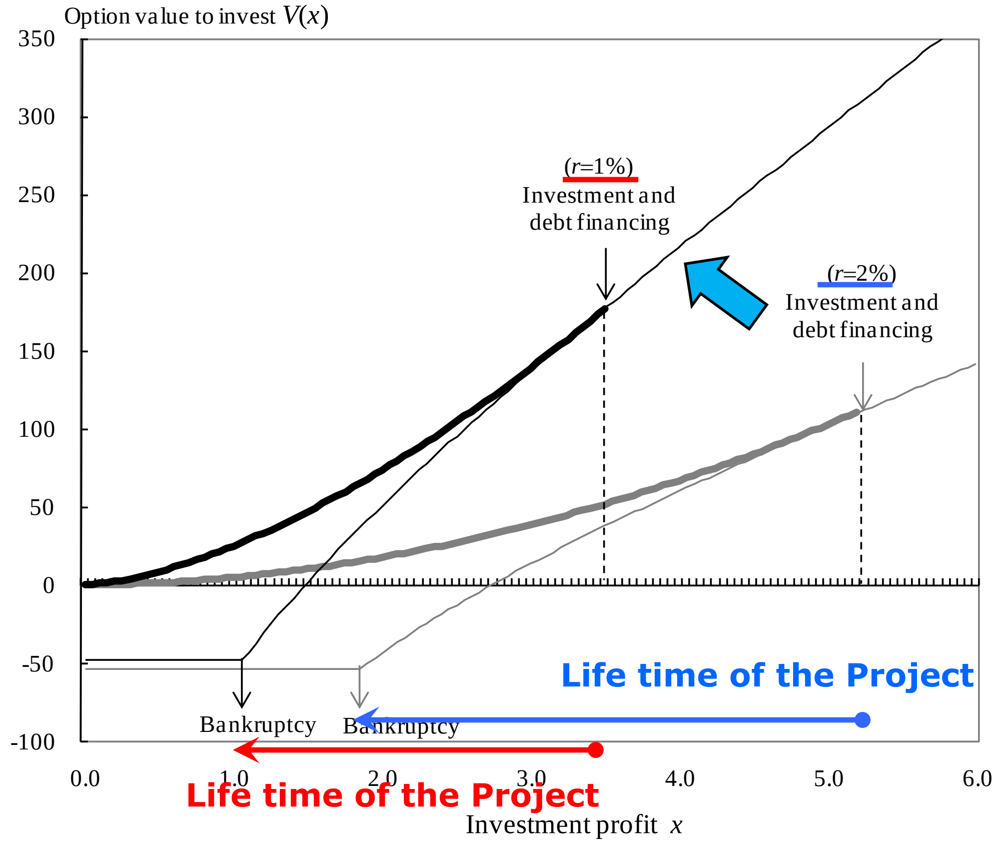
2-7. The model solutions (3)



Parameters: I=100,α=50%, τ_{ax}=50%, r=1%, μ=0%,σ=15%,

2-8. Comparative Statics on risk free rates r (1)

- As risk free rate is lower, the shorter the life time of the project.



Parameters: I=100,α=30%, τ_{ax}=30%, μ=0%,σ=15%,

2-9. Comparative Statics on risk free rates r (2)

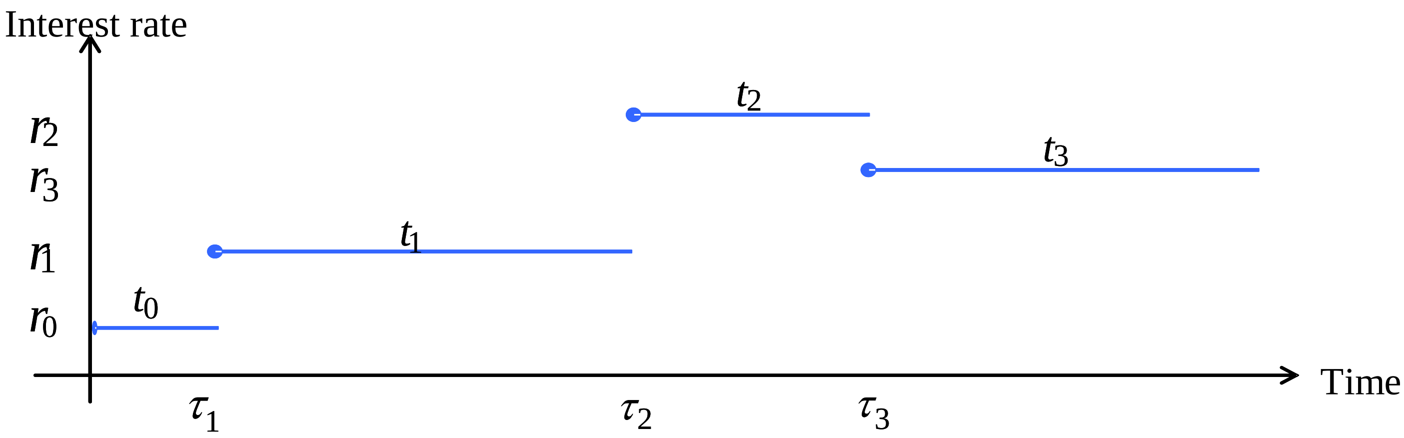
- As the risk free rate is lower
  The firm does not only (a) invest earlier (1)
  but also (b) take a higher leverage (2)/(1)
  ⇒ Therefore, the credit risk(PD,EL) increases

risk free rate r	investment timing (profit x at time of investment) (1)	amount of debt (interest payments b) (2)	(2)/(1)	PD	EL
5%	9.6	6.9	0.71	26%	18%
2%	5.2	3.9	0.75	39%	30%
1%	3.5	2.9	0.84	51%	42%
0.5%	2.6	2.7	1.04	64%	56%
0.1%	1.7	4.6	2.68	88%	84%

3. Under the Expectations of Future Interest Rates Increases (Situation.2)

3-1.The interest rate process

- Risk free rate changes following a Poisson jump process

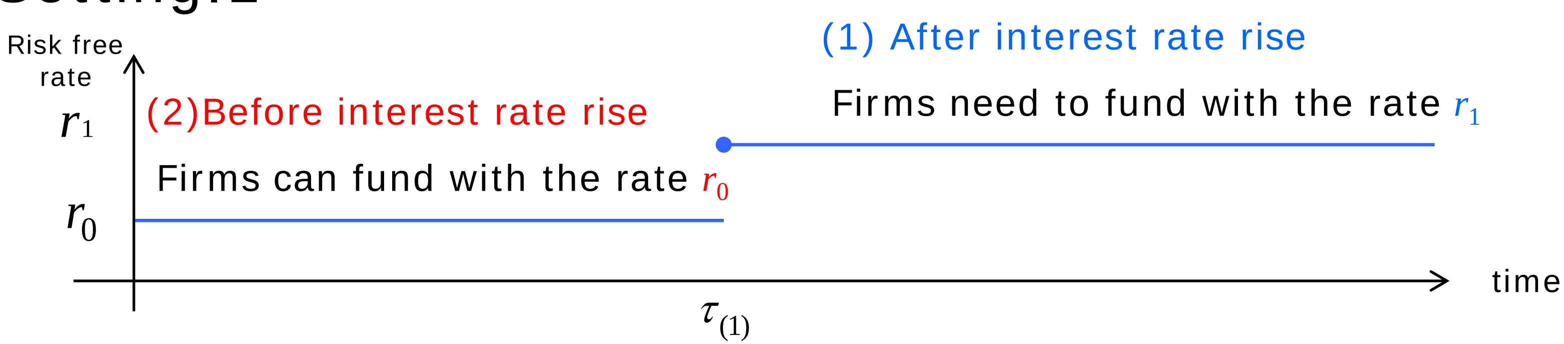


- n-th jump: $r_{n-1} \rightarrow r_n$
- intensity $\lambda_n\, dt$

3-2.Investment problem(1)

Question. Which “model settings” would you choose ?

Setting.1



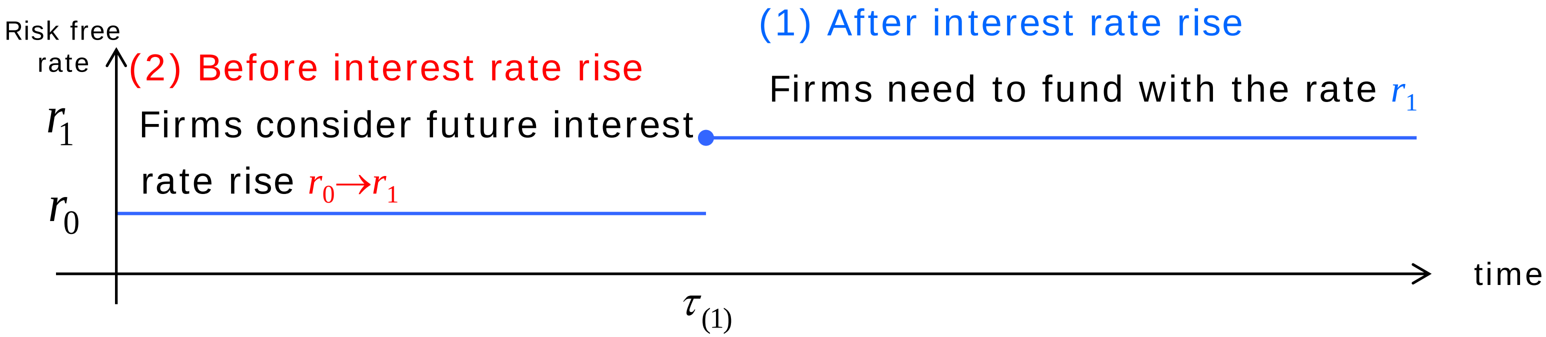
(1) invest after interest rate rise

$$V(X_t) = \max_{\tau \in F_t} E_t \left[e^{-r_1(\tau-\tau_1)-r_0(\tau_1-t)} \left(\int_\tau^\infty e^{-r_1(s-\tau)} X_s ds - I \right) 1_{\{\tau>\tau_1\}} + e^{-r_0(\tau-t)} \left(\int_\tau^\infty e^{-r_0(s-\tau)} X_s ds - I \right) 1_{\{\tau\le\tau_1\}} \right]$$

(2) invest before interest rate rise

3-3.Investment Problem(2)

Setting.2



(1) invest after interest rate rise

$$V(X_t) = \max_{\tau \in F_t} E_t \left[e^{-r_1(\tau-\tau_1)-r_0(\tau_1-t)} \left(\int_\tau^\infty e^{-r_1(s-\tau)} X_s ds - I \right) 1_{\{\tau>\tau_1\}} + e^{-r_0(\tau-t)} \left(\int_\tau^{\tau_1} e^{-r_0(s-\tau)} X_s ds + \int_{\tau_1}^\infty e^{-r_1(s-\tau)} X_s ds - I \right) 1_{\{\tau \le \tau_1\}} \right]$$

(2) invest before interest rate rise

3-4. The two different discount rates

The Answer.

- In Standard finance theory ⇒ setting.2 is correct!
  We need no-arbitrage conditions between short and long rates
  ⇒ time-consistent discount rate

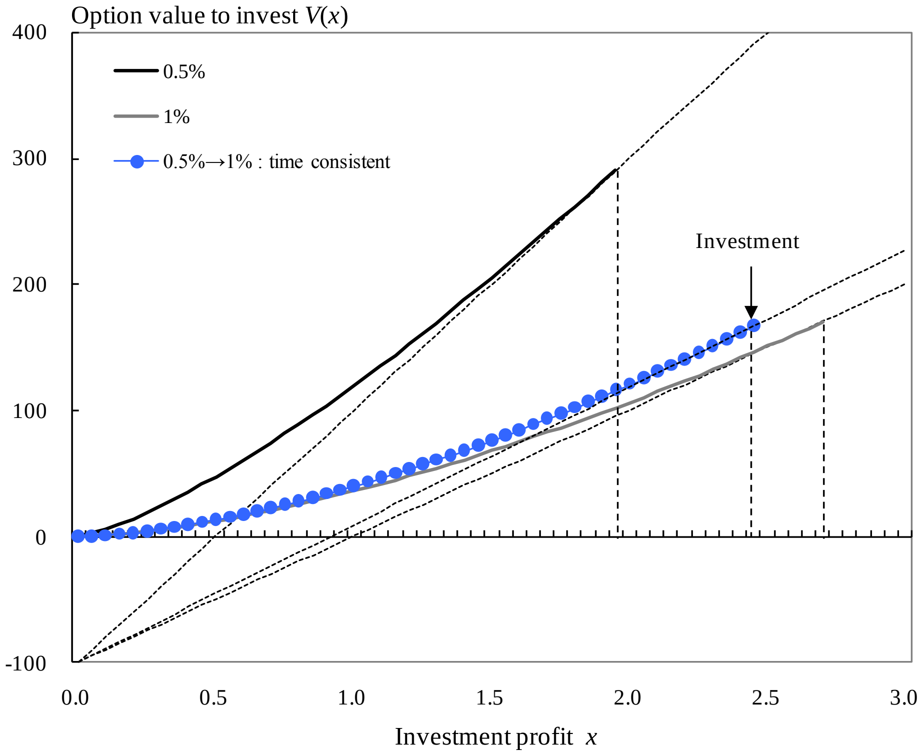
- Setting.1 is time-inconsistent discount(Behavioral Economics)
  Especially, hyperbolic discount rate explains "short-sighted" behavior

⇒ Firms investment timing vary depending on how firms incorporate the possibility of future interest rate rises.

⇒ We examine the both two cases and compare those results.

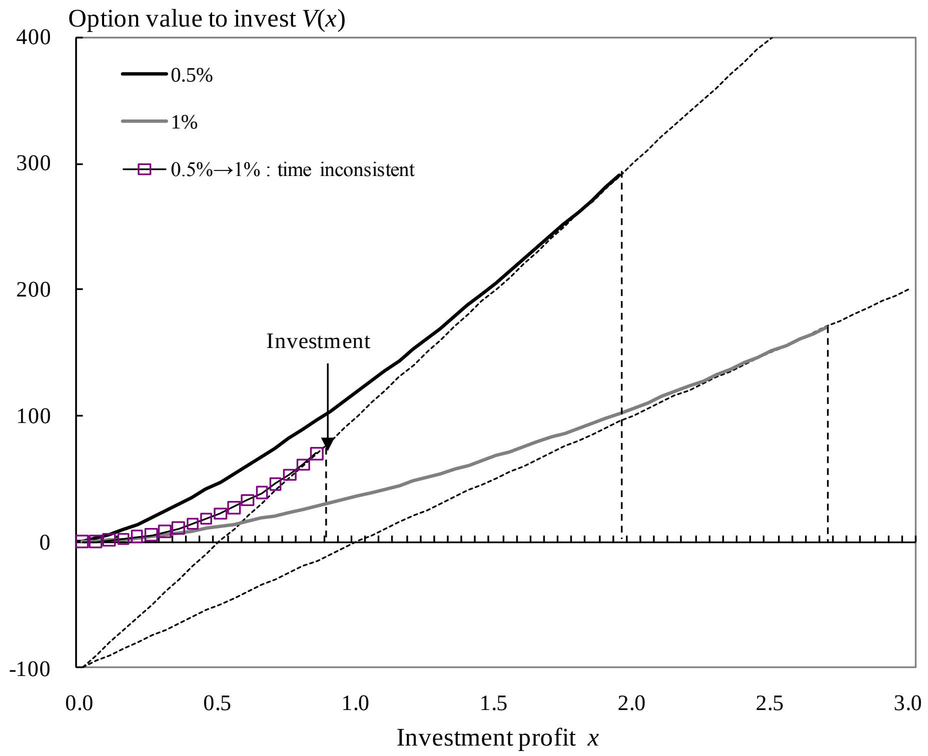
3-5.The optimal investment timing (time-consistent)

- Firms decrease their investments, carefully considering the likelihood of future interest rate hikes



3-6.The optimal investment timing (time-inconsistent)

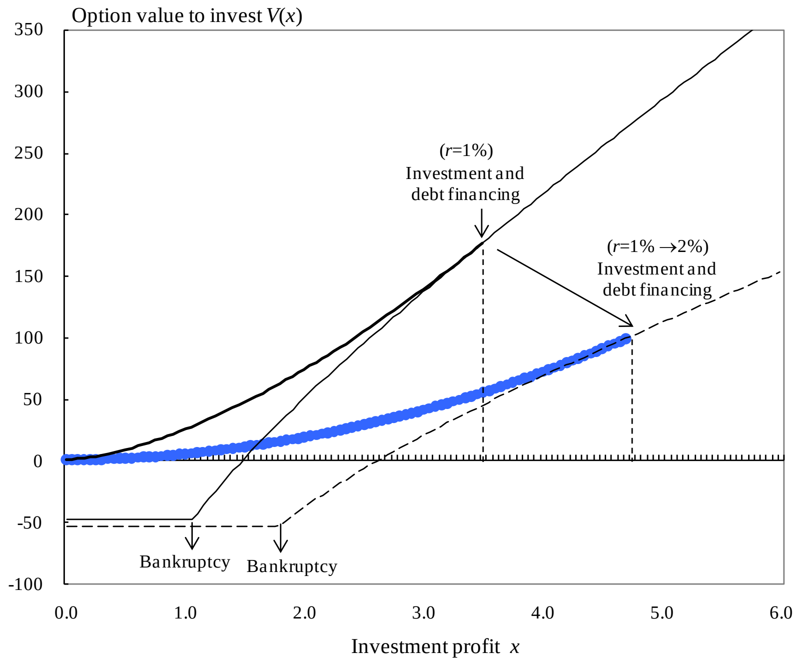
- Firms make "last-minute investments" while the financing costs are favorable (low).



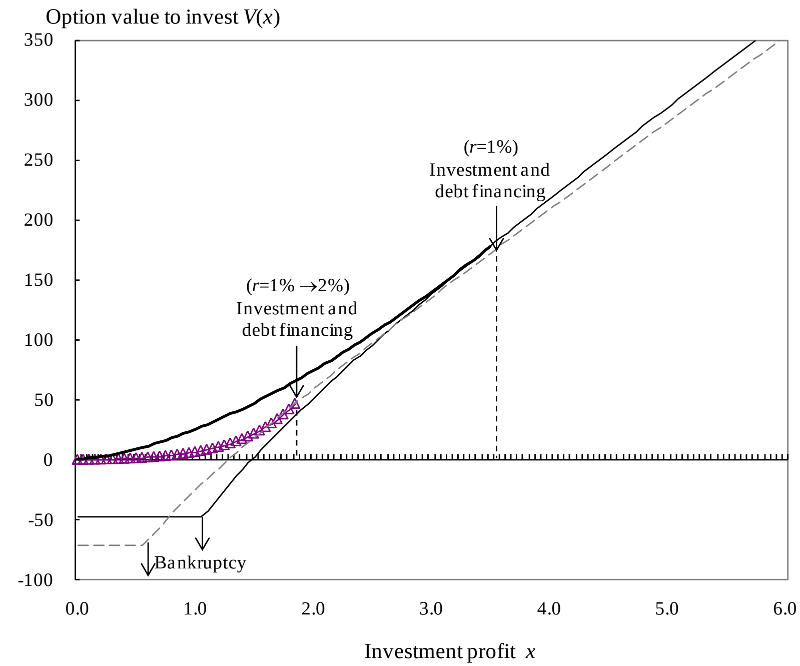
This behavior is the opposite which the central bank expects (that is, decreasing investment).

3-7.The case considering debt financing

Time consistent



Time inconsistent



(0.5%const)

(0.5%→1%)

(0.5%const)

(0.5%→1%)

Last-minute investment (time inconsistent) increases the credit risk

Thank you for your attention!

References

Dixit, Avinash and Robert Pindyck, *Investment under uncertainty*, Princeton University Press, 1994.

Grenadier, Steven and Neug Wang, “Investment under uncertainty and time-inconsistent preferences,” *Journal of Financial Economics*, 84(1), 2007, pp. 2-39.

Sundaresan, Suresh and Neng Wang, “Investment under uncertainty with strategic debt service,” *American Economic Review*, 97(2), 2007, pp. 256-261.

Tetsuya, Yamada, “Accelerated investment and credit risk under a low interest rate environment,” IMES Discussion Paper Series No. 2010-E-8,2010, Institute for Monetary and Economic Studies, Bank of Japan.