Risk Preferences and their Robust Representation

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Risk Orders, Risk Measures and Risk Acceptance Families

Motivation

- The goal is to understand "risk" in a context (setting) independent manner, focusing on some crucial invariant features:
  - **"diversification should not increase the risk"**
  - **"the better for sure, the less risky"**

- We consider a structural approach to risk which is motivated by the former theory on preferences and risk
  - von Neumann and Morgenstern and their theory on preference comparison and utility for lotteries.
  - Artzner, Delbaen, Eber and Heath; Föllmer and Schied and their theory of monetary risk measures for random variables.

  and three recent preprints by Cerreia-Vioglio, Maccheroni, Marinacci and Montrucchio on
  - Quasiconvex risk measures
  - Complete quasiconvex duality theory
  - Uncertainty averse preferences

- Based on the concept of acceptance families we will give a robust representation for a huge class of risk orders.

Outline

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Risk Orders, Risk Measures and Risk Acceptance Families

Setting, Definitions

Risky positions in $\mathcal{X}$ are ordered $\cdots \succcurlyeq x \succcurlyeq y \succcurlyeq z \cdots$ according to a total preorder $\succcurlyeq$.

The relation $x \succcurlyeq y$ means "x *is riskier than* y".

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The relation $x \succcurlyeq y$ means "*x is riskier than y*".

In this framework, the notions of "diversification should not increase the risk" and "the better for sure the less risky" translate into

Definition (Risk Order)

A total preorder $\succcurlyeq$ is a risk order *if it is*

- **Quasiconvex:** $x \succcurlyeq \lambda x + (1-\lambda) y$ *whenever* $x \succcurlyeq y$,
- **Monotone:** $x \succcurlyeq y$ *whenever* $y \trianglerighteq x$.

- Diversification imposes $\mathcal{X}$ convex. (in fact a mixture space)
- $\trianglerighteq$ is a preorder expressing a kind of "better than . . . for sure".

Risk Orders, Risk Measures and Risk Acceptance Families

A Setting Dependant Interpretation of Risk

Possible settings by the specification of the convex set $\mathcal{X}$ and the monotonicity preorder $\trianglerighteq$.

- **Random variables** on $(\Omega, \mathscr{F}, P)$ with as preorder $\trianglerighteq$ the "$\geq$ P-almost surely".
- **Stochastic processes** modeling cumulative wealth processes $X = X_0, X_1, \ldots X_T$ with as preorder $\trianglerighteq$ the cash flow monotonicity "$X_t - X_{t-1} := \Delta X_t \geq \Delta Y_t$".
- **Probability distributions** (lotteries) $\mathcal{M}_1$ is a convex set with standard monotonicity preorders $\trianglerighteq$ either the first or second stochastic order.
- **Cumulative consumption streams** are right continuous non decreasing functions $c : [0,1] \to \mathbb{R}^+$ building a convex cone. Here $c^{(1)}$ is "better for sure" than $c^{(2)}$ if $c^{(1)} - c^{(2)}$ is still a cumulative consumption stream.
- **Stochastic kernels** are probability distributions subject to uncertainty, that is, mappings $\tilde{X} : \Omega \to \mathcal{M}_1$. Possible preorders $\trianglerighteq$ are either the P-almost sure first or second stochastic order.
- ...

Risk Orders, Risk Measures and Risk Acceptance Families

Definitions

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Total preorders can (separability) be represented by functions $F : \mathcal{X} \to [-\infty, +\infty]$

$$x \succcurlyeq y \qquad \Longleftrightarrow \qquad F(x) \geq F(y)$$

Numerical representations of risk orders inherit their properties and belongs to the following class:

Definition (Risk Measure)

A function $\rho : \mathcal{X} \to [-\infty, +\infty]$ *is a* risk measure *if it is*

- **Quasiconvex:** $\rho(\lambda x + (1-\lambda)\, y) \leq \max\{\rho(x), \rho(y)\}$.
- **Monotone:** $\rho(x) \leq \rho(y)$ *whenever* $x \trianglerighteq y$.

Risk Orders, Risk Measures and Risk Acceptance Families

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Any risk measure defines at any level of risk $m \in \mathbb{R}$ a risk acceptance set

$$\mathcal{A}^m = \{x \mid \rho(x) \leq m\}$$

of those positions with a risk below m. Here again, the family, called *risk acceptance family*, gets properties from the risk measure and belongs to the following class.

Risk Orders, Risk Measures and Risk Acceptance Families

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Definition (Risk Acceptance Family)

A family $\mathcal{A} = (\mathcal{A}^m)_{m \in \mathbb{R}}$ *of subset of* $\mathcal{X}$ *is a* risk acceptance family *if it is*

- **Convex:** $\mathcal{A}^m$ *is convex,*
- **Monotone:** $\mathcal{A}^m \subset \mathcal{A}^n$ *and* $x \trianglerighteq y$ *for some* $y \in \mathcal{A}^m$ *implies* $x \in \mathcal{A}^m$,
- **Right-Continuous:** $\mathcal{A}^m = \bigcap_{n>m} \mathcal{A}^n$.

Risk Orders, Risk Measures and Risk Acceptance Families

One-to-One Relation between Risk Orders, Risk Measures and Risk Acceptance Families

Theorem (Risk Orders↔Risk Measures↔Risk Acceptance Families)

- Any numerical representation ρ of a risk order $\succcurlyeq$ is a risk measure. Any risk measure ρ defines a risk order $\succcurlyeq$ through

$$x \succcurlyeq y \quad \Longleftrightarrow \quad \rho(x) \geq \rho(y)$$

- Risk measures and risk acceptance families are related one to one through

$$\mathcal{A}^m := \{x \in \mathcal{X} \mid \rho(x) \leq m\} \quad \text{and} \quad \rho(x) = \inf\left\{m \,\middle|\, x \in \mathcal{A}^m\right\}$$

Axioms of monotonicity or quasiconvexity for the risk orders are global!

Economic Index of Riskiness (Aumann and Serrano; Forster and Hart)

For a loss function l, consider

$$\lambda(X) = \sup\{\lambda > 0 \mid E[l(-\lambda X)] \leq c\}$$

which represents the maximal exposure to a position X provided that the expected loss remains below and a threshold. The **economic index of riskiness** is then defined as

$$\rho(X) := 1/\lambda(X) \quad \Longrightarrow \quad \mathcal{A}^m = \left\{X \middle| c \geq E[l(-X/m)]\right\}$$

Risk Orders, Risk Measures and Risk Acceptance Families

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Certainty Equivalent

- **Probability Distributions:** $l : \mathbb{R} \to [-\infty, +\infty[$ nondecreasing loss function

$$\rho(\mu) = l^{-1}\left(\int l(-x)\,\mu(dx)\right) \quad \Longrightarrow \quad \mathcal{A}^m = \left\{\mu \,\middle|\, \int l(-x)\,\mu(dx) \leq l(m)\right\}$$

- **Random Variables:** $l : \mathbb{R} \to [-\infty, +\infty[$ is a nondecreasing convex loss function.

$$\rho(X) = l^{-1}\left(E\left[l(-X)\right]\right) \quad \Longrightarrow \quad \mathcal{A}^m = \left\{X \,\middle|\, E\left[l(-X)\right] \leq l(m)\right\}$$

Risk Orders, Risk Measures and Risk Acceptance Families

Further Properties : Convexity, Positive Homogeneity and Scaling Invariance

Proposition (Convexity, Positive Homogeneity and Scaling Invariance)

(i) ρ is convex iff $\lambda \mathcal{A}^m + (1-\lambda)\,\mathcal{A}^{m'} \subset \mathcal{A}^{\lambda m + (1-\lambda) m'}$.

(ii) ρ is positive homogeneous iff $\lambda \mathcal{A}^m = \mathcal{A}^{\lambda m}$ for $\lambda > 0$.

(iii) ρ is scaling invariant iff $\lambda \mathcal{A}^m = \mathcal{A}^m$ for $\lambda > 0$.

(iv) ρ is affine iff $\succcurlyeq$ is *independent* and *archimedian*.

These properties are no longer global!

Examples (Savage; Markowitz; Sharpe; von Neumann and Morgenstern)

- **Savage representation:** $\rho(X) := E_Q\left[l(-X)\right]$ convex RM if l is a convex loss function.
- **Mean Variance:** $\rho(X) = E[-X] + \frac{\gamma}{2} Var(X)$ convex RM monotone w.r.t. trivial order.
- **Sharpe Ratio:** $\rho(X) = E[-X] / \sqrt{E\left[X^2 - E[X]^2\right]}$ scaling invariant RM monotone w.r.t. trivial order
- **von Neumann and Morgenstern:** $\rho(\mu) = \int l(-x)\,\mu(dx)$ affine RM monotone w.r.t. the first stochastic order if l is a loss function.

Risk Orders, Risk Measures and Risk Acceptance Families

Further Properties : Monetary Risk Measures (Artzner, Delbaen, Eber and Heath; Föllmer and Schied)

Existence of a numéraire π. $\mathcal{X}$ is a vector space and $\trianglerighteq$ a vector order.

Definition (Cash Additive and Subadditive Risk Measures)

A risk measure ρ is

- **Cash Additive** *if $\rho(x + m\pi) = \rho(x) - m$ for any $m \in \mathbb{R}$.*
- **Cash Subadditive** *if $\rho(x + m\pi) \geq \rho(x) - m$ for any $m > 0$.*

Proposition

- ρ is cash additive iff $\mathcal{A}^0 = \mathcal{A}^m + m\pi$.
- $\succcurlyeq$ is cash additive iff
  - (i) $y \succ x \succ z$ implies the existence of a unique $m \in \mathbb{R}$ such that $x \sim m\pi$;
  - (ii) $x \succcurlyeq y$ implies $x + m\pi \succcurlyeq y + m\pi$.
- A cash additive risk measure ρ is automatically convex.

Classical cash additive monetary risk measures

- **Average Value at Risk:** $AV@R_q(X) = \sup_Q \left\{ E_Q[-X] \,\middle|\, dQ/dP < 1/q \right\}$.
- **Entropie:** $\rho(X) = \ln\left(E\left[e^{-X}\right]\right)$.
- **Optimized Certainty Equivalent:** $\rho(X) = -\sup_m \{m + E[f(X - m)]\}$.

Outline

Robust Representation of Risk Orders

Setup, Lower Semicontinuous Risk Orders

- $\mathcal{X}$ is a locally convex topological vector space with dual $\mathcal{X}^*$.
- $\trianglerighteq$ is a vector order: $x \trianglerighteq y$ iff $x - y \in \mathcal{K}$ closed convex cone with polar cone $\mathcal{K}^\circ$.

Examples

- $\mathbb{L}^\infty$, $\mathcal{K} = \mathbb{L}^\infty_+$, weak topology $\sigma\left(\mathbb{L}^\infty, \mathbb{L}^1\right)$, dual $\mathbb{L}^1$, polar cone $\mathcal{K}^\circ = \mathbb{L}^1_+$.
- $\mathcal{M}_{1,c} \subset ca_c = \mathcal{X}$, weak topology $\sigma\left(ca_c, C\right) \Longrightarrow \mathcal{X}^* = C$.
  - First stochastic order: $\mathcal{K}^1 = \{\mu \mid \int f\, d\mu \geq 0,$ for all nondecreasing $f\}$.
  - Second stochastic order: $\mathcal{K}^2 = \{\mu \mid \int f\, d\mu \geq 0,$ for all nondecreasing concave $f\}$.

Definition (Lower Semicontinuous Risk Orders)

A risk order $\succcurlyeq$ is lower semicontinuous if $\mathcal{L}(x) = \{y \in \mathcal{X} \mid x \succcurlyeq y\}$ is $\sigma(\mathcal{X}, \mathcal{X}^*)$-closed for any $x \in \mathcal{X}$.

Proposition (Metha)

A risk order $\succcurlyeq$ is separable and lower semicontinuous if and only if there exists a corresponding lower semicontinuous risk measure ρ.
Moreover, the class of corresponding lower semicontinuous risk measures is stable under lower semicontinuous increasing transformations.

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Robust Representation of Risk Orders

Representation Theorem

Main robust representation result:

Theorem

Any lower semicontinuous risk measure $\rho : \mathcal{X} \to [-\infty, +\infty]$ has a robust representation:

$$\rho(x) = \sup_{x^* \in \mathcal{K}^\circ} R\left(x^*, \langle x^*, -x \rangle\right)$$

for a unique maximal risk function $R \in \mathcal{R}^{max}$.

Definition

$\mathcal{R}^{max}$ denotes the set of maximal risk functions

$$R : \mathcal{K}^\circ \times \mathbb{R} \to [-\infty, +\infty]$$

- *nondecreasing and left-continuous in the second argument*
- *R is jointly quasiconcave,*
- *$R\left(\lambda x^*, s\right) = R\left(x^*, s/\lambda\right)$ for any $\lambda > 0$,*
- *R has a uniform asymptotic minimum, $\lim_{s \to -\infty} R\left(x^*, s\right) = \lim_{s \to -\infty} R\left(y^*, s\right)$,*
- *its right-continuous version, $R^+\left(x^*, s\right) := \inf_{s' > s} R\left(x^*, s\right)$ is upper semicontinuous*

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Example: The cash additive case on $\mathbb{L}^\infty$

$$\rho(X) = \sup_{Q} \left\{ E_Q\left[-X\right] - \alpha_{min}(Q) \right\}$$

In this case:

$$R\left(Q, s\right) = s - \alpha_{min}(Q).$$

Moreover, ρ and α_{min} are one-to-one.

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for a unique maximal risk function $R \in \mathcal{R}^{max}$.
Conversely, for any risk function $R : \mathcal{K}^\circ \times \mathbb{R} \to [-\infty, +\infty]$ which is nondecreasing and left-continuous in the second argument

$$\rho(x) = \sup_{x^* \in \mathcal{K}^\circ} R\left(x^*, \langle x^*, -x \rangle\right)$$

is a lower semicontinuous risk measures

- The one-to-one relation between ρ and the risk function $R \in \mathcal{R}^{max}$ is crucial!
  - makes comparative statics meaningful
  - Cerreia-Vioglio, Maccheroni, Marinacci and Montrucchio introduced the notion of complete quasiconvex duality.
- Cerreia-Vioglio et al. provide complete quasiconvex duality results on M-spaces with unit under further assumptions on the monotonicity $\to (\mathbb{L}^\infty, \|\cdot\|_\infty)$, $\mathcal{K} = \mathbb{L}^\infty_+$

Robust Representation of Risk Orders

Representation Theorem: Modifications

- In case the order is **regular**, i.e., there is π with $\langle x^*, \pi\rangle > 0$ for all $x^* \in \mathcal{K}^\circ \setminus \{0\}$, one gets a robust representation:

$$\rho(x) = \sup_{x^* \in \mathcal{K}^\circ_\pi} R\left(x^*, \langle x^*, -x\rangle\right)$$

  where $\mathcal{K}^\circ_\pi = \{x^* \in \mathcal{K}^\circ \mid \langle x^*, \pi\rangle = 1\}$.
  - In case of random variables with $\pi = 1$, $\mathcal{K}^\circ_1$ is a set of probability measures.
  - The first and second stochastic order are not regular
- Similar robust representation results hold on open/closed convex sets rather than vector spaces
- The setup is general and includes the following risk orders/preferences:
  - Expected utilities (Von Neumann and Morgenstern)
  - Mean variance preferences (Markowitz)
  - Coherent and convex risk measures (Artzner et al. and Föllmer/Schied and Frittelli/Gianin)
  - Performance measures such as the Sharpe ratio and their monotone versions (Cherny and Madan)
  - Economic index of riskiness (Aumann and Serrano)
  - Value at risk
  - Intertemporal preference functionals (Hindy, Huan and Kreps)
  - Multiprior maxmin expected utilities (Gilboa and Schmeidler)
  - Variational preferences (Maccheroni et al.)
  - Uncertainty averse preferences (Cerreia-Vioglio et al.)
  - ...

Robust Representation of Risk Orders

Sketch of the proof and computation of the risk function

- Start with the risk acceptance family $\mathcal{A} = (\mathcal{A}^m)_{m \in \mathbb{R}}$ corresponding to ρ.

- For any risk level m holds by means of the theory of **cash-additive risk measures**

$$X \in \mathcal{A}^m \quad \Longleftrightarrow \quad E_Q[-X] \leq \alpha_{min}(Q, m) \quad \text{for all } Q$$

where $\alpha_{min}(Q, m) = \sup_{X \in \mathcal{A}^m} E_Q[-X]$ is a penalty function.

- Then

$$\begin{aligned} \rho(X) &= \inf \{m \in \mathbb{R} \mid X \in \mathcal{A}^m\} \\ &= \inf \{m \in \mathbb{R} \mid E_Q[-X] - \alpha_{min}(Q, m) \leq 0 \text{ for all } Q\} \\ &= \sup_Q \inf_{m \in \mathbb{R}} \{m \mid E_Q[-X] \leq \alpha_{min}(Q, m)\} \\ &= \sup_Q R\left(Q, E_Q[-X]\right) \end{aligned}$$

where $R(Q, \cdot)$ is the generalized left-inverse of $m \mapsto \alpha_{min}(Q, m)$

- The difficult part of the proof is to show that the duality is complete.

Outline

Illustrative Settings

Random Variables

Proposition

Any lower semicontinuous risk measure $\rho : \mathbb{L}^\infty \to [-\infty, +\infty]$ has a robust representation:

$$\rho(X) = \sup_{Q \in \mathcal{M}_{1,f}(P)} R(Q, E_Q[-X]), \quad X \in \mathbb{L}^\infty,$$

for a unique $R \in \mathcal{R}^{max}$. In particular, if ρ is cash additive,

$$\rho(X) = \sup_{Q \in \mathcal{M}_{1,f}(P)} \{E_Q[-X] - \alpha_{min}(Q, 0)\}.$$

- In case that ρ has the Fatou property it is well known (Delbaen) that ρ is $\sigma(\mathbb{L}^\infty, \mathbb{L}^1)$-lower semicontinuous and the previous representations hold in terms of $\mathcal{M}_1(P)$.
- Under adequate lower semicontinuity conditions, robust representations of risk measures on $\mathbb{L}^p$ and Orlicz spaces work analogously.

Illustrative Settings

Random Variables

Robust Representation of the Certainty Equivalent $\rho(X) = l^{-1}(E[l(-X)])$.

- **Quadratic Function:** $l(s) = s^2/2 - s$ for $s \geq -1$ and $l(s) = -1/2$ elsewhere.

$$\rho(X) = \sup_{Q \in \mathcal{M}_1(P)} \left\{ \frac{E_Q[-X] + 1}{\|dQ/dP\|_{\mathbb{L}^2}} - J(E_Q[-X]) \right\}, \qquad X \in \mathbb{L}^\infty,$$

whereby $J(s) = 1$ if $s > 1$ and $J(s) = +\infty$ elsewhere.

- **Logarithm Function:** If $l(s) = -\ln(-s)$ for $s < 0$, then

$$\rho(X) := -\exp(E[\ln(X)]) = \sup_{Q \in \mathcal{M}_1(P)} \left\{ \frac{E_Q[-X]}{\exp\left(E\left[\ln\left(\frac{dQ}{dP}\right)\right]\right)} \right\}, \quad X \in \mathbb{L}^\infty.$$

Economic Index of Riskiness (Aumann and Serrano)

$$\rho(X) = \sup_Q \frac{E_Q[-X]}{E_Q[\ln(c_0 dQ/dP)]}, \quad X \in \mathbb{L}^\infty.$$

Conclusion

THANK YOU FOR YOUR ATTENTION!