Contagion and Confusion in Credit Markets

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June 24, 2010

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- Question: How do we measure dependence between two time series?

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- ρ is constant.

Linear Models in Finance

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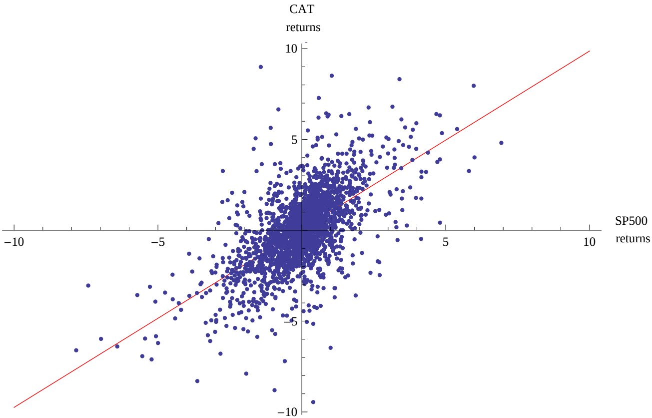
- α and β are constants
- ϵ_t is a sequence of independent, identically distributed, centered Gaussian random variables with variance σ^2
- X_t is, for example, the excess returns of the market (S&P 500)
- Y_t is, for example, the returns of Caterpillar stock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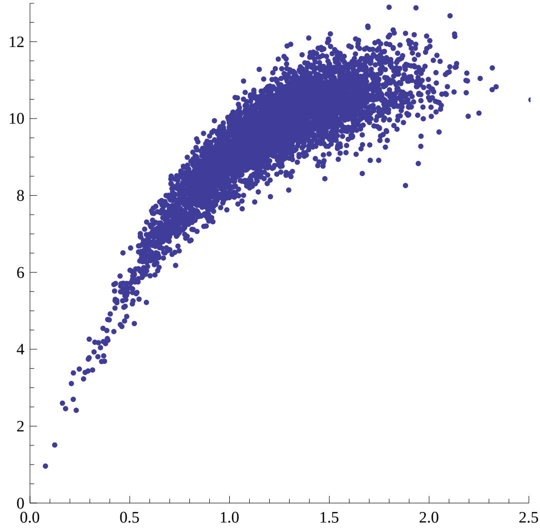
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Extending the Linear Model

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with regression slope $m'(x) = \beta$. It also follows that the regression slope $\beta = \rho\sigma_Y/\sigma_X$ and therefore that

$$\rho = \beta\sigma_X/\sigma_Y. \tag{2}$$

Extending the Linear Model

From linear regression theory, we know that we can write the variance σ_Y^2 of Y as a sum of the variance explained by the regression (namely, $\beta^2\sigma_X^2$) and the residual (unexplained) variance σ^2.

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and hence

$$\rho = \sigma_X\beta/(\sigma_X^2\beta^2 + \sigma^2)^{1/2}. \tag{4}$$

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and the usual correlation coefficient to

$$\rho(x) = \sigma_X \beta(x)/(\sigma_X^2 \beta(x)^2 + \sigma^2(x))^{1/2}, \tag{7}$$

where m and σ are smooth real-valued functions.

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- σ_X denotes the unconditional standard deviation of X
- $\beta(x) = m'(x)$ is the slope of the regression function $m(x)$
- $\sigma^2(x) = \mathrm{Var}(Y|X = x)$ is the scedastic function

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Moreover, let

- $x_L = F_X^{-1}(0.025)$ be a lower quantile of X; and
- $x_M = F_X^{-1}(0.50)$ be a median quantile of X.

Then we say that there is *contagion from X to Y* if $\rho(x_L) > \rho(x_M)$.

Developing the Hypothesis Test

We state the relevant hypothesis test:

$$H_0\colon\ \rho(x_L) \leq \rho(x_M) \text{ (no contagion)}$$
$$H_1\colon\ \rho(x_L) > \rho(x_M) \text{ (contagion).}$$

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which is facilitated by the fact that, under certain limiting conditions,

$$\widehat{\rho}(x) \xrightarrow{D} N(\rho(x), \widehat{\sigma}_{\widehat{\rho}(x)}). \tag{9}$$

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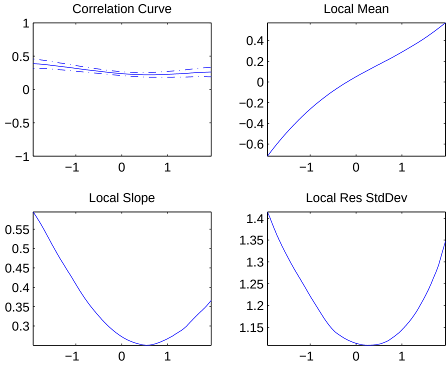
- We obtain, by approximating $\sigma_{\widehat{\rho}(x_M)}$ and $\sigma_{\widehat{\rho}(x_L)}$, a Studentized test statistic:

$$Z = \frac{\widehat{\rho}(x_L) - \widehat{\rho}(x_M)}{\sqrt{\widehat{\sigma}^2_{\widehat{\rho}(x_L)} + \widehat{\sigma}^2_{\widehat{\rho}(x_M)}}} \tag{10}$$

The Case of the U.S. and France

Take X_t and Y_t to be U.S. and French stock market returns, respectively.



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- Let $x_M = F_X^{-1}(0.50)$ be a median quantile of X and let x_T be a tail quantile of X_t associated with crisis.

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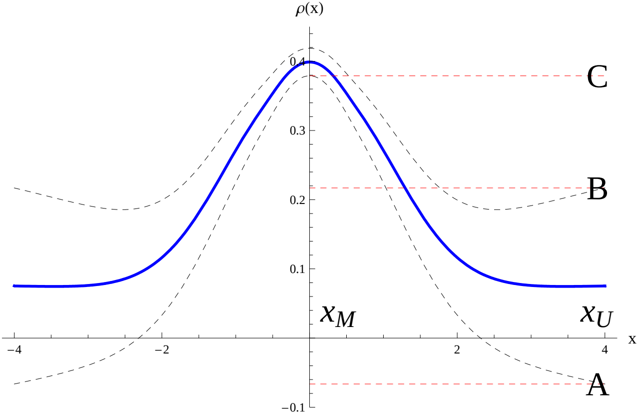
- We say there is *confusion* from X to Y if
  - $\rho(x_M) > \rho(x_T)$ and
  - $\rho(x_T) = 0$.

Intuition for Confusion



A Hypothesis Test For Confusion?

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- We call this approach the *asymptotic approach*, because it uses the asymptotic behavior of $\widehat{\rho}(x)$.

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are dependent.

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- For each bootstrap B_i, we ultimately generate estimates

$$\left(\widehat{\rho}_i(x_M), \widehat{\rho}_i(x_T), \widehat{\sigma}_{i,\widehat{\rho}(x_M)}, \widehat{\sigma}_{i,\widehat{\rho}(x_T)}\right) \tag{14}$$

A Bootstrapping Approach

We count, over all N bootstraps, the number of times in which

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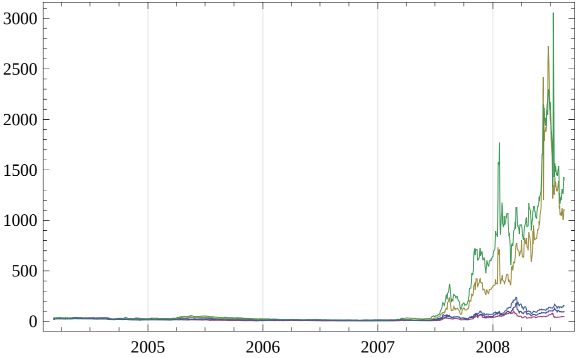
We call the proportion of bootstraps satisfying these two conditions an empirical estimate of the *probability of confusion*.

7 Years of Credit Default Swap History

Historical credit default swap premia for Bear Stearns, Ambac, Citigroup, J.P. Morgan Chase, and Freddie Mac.

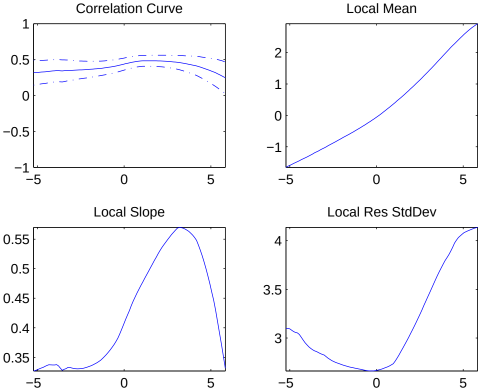


Results

Covariate X is the daily percentage change in Bears Stearns CDS.

Dependent	$\widehat{\rho}(x_M)$	$\widehat{\rho}(x_U)$	$\widehat{\sigma}_{\widehat{\rho}(x_M)}$	$\widehat{\sigma}_{\widehat{\rho}(x_U)}$	$Z_{\widehat{\rho}(x_U)-\widehat{\rho}(x_M)}$	$\mathbb{P}$(Confusion)
Deutsche Bank (Subordinated)	0.3438	0.2744	0.0378	0.0942	-0.6832	0.005
J.P. Morgan Chase	0.6880	0.5382	0.0213	0.0802	-1.8040	0.059
Fannie Mae	0.4147	0.3044	0.0396	0.1037	-0.9934	0.078
Freddie Mac	0.3978	0.2671	0.0406	0.1075	-1.1375	0.099
Countrywide	0.5956	0.4146	0.0259	0.0858	-2.0314*	0.002
Bank of America	0.5794	0.3793	0.0296	0.1017	-1.8885	0.009
Ambac Assurance	0.3628	0.3900	0.0400	0.0880	0.2818	0.000
Ambac Financial Group	0.3709	0.2797	0.0401	0.1008	-0.8413	0.095
Lehman Brothers	0.8731	0.7204	0.0074	0.0583	-2.5981*	0.000
Citigroup	0.5797	0.4260	0.0296	0.0955	-1.5372	0.001

Confusion from Countrywide CDS to Ambac CDS?



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- There is limited evidence of a condition stronger than the absence of contagion, which we call *confusion*.
- Diversified bond and fixed-income derivative investors do not have to worry about "all correlations going to one" during crises.

References

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