Jump-Diffusion Risk-Sensitive Asset Management

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Outline

Risk Sensitive Control: A Definition

Risk-sensitive control is a generalization of classical stochastic control in which the degree of risk aversion or risk tolerance of the optimizing agent is explicitly parameterized in the objective criterion and influences directly the outcome of the optimization. In risk-sensitive control, the decision maker's objective is to select a control policy $h(t)$ to maximize the criterion

$$J(x, t, h; \theta) := -\frac{1}{\theta} \ln \mathbf{E} \left[e^{-\theta F(t,x,h)} \right] \quad (1)$$

where t is the time, x is the state variable, F is a given reward function, and the risk sensitivity $\theta \in (0, \infty)$ is an exogenous parameter representing the decision maker's degree of risk aversion.

An Intuitive View of the Criterion

A Taylor expansion of the previous expression around $\theta = 0$ evidences the vital role played by the risk sensitivity parameter:

$$J(x, t, h; \theta) = \mathbf{E}\left[F(t, x, h)\right] - \frac{\theta}{2}\mathbf{Var}\left[F(t, x, h)\right] + O(\theta^2) \qquad (2)$$

- $\theta \to 0$, "risk-null": corresponds to classical stochastic control;
- $\theta < 0$: "risk-seeking" case corresponding to a maximization of the expectation of a convex decreasing function of $F(t, x, h)$;
- $\theta > 0$: "risk-averse" case corresponding to a minimization of the expectation of a convex increasing function of $F(t, x, h)$.

Emergence of a Risk-Sensitive Asset Management (RSAM) Theory

- Jacobson [9], Whittle [14], Bensoussan [1] led the theoretical development of risk sensitive control.
- Risk-sensitive control first applied to finance by Lefebvre and Montulet [11] in a corporate finance context and by Fleming [6] in a portfolio selection context.
- Bielecki and Pliska [2]: first to apply continuous time risk-sensitive control as a practical tool to solve 'real world' portfolio selection problems.
- Major contribution by Kuroda and Nagai [10] who introduced an elegant solution method based on a change of measure argument which transforms the risk sensitive control problem in a linear exponential of quadratic regulator.

Extensions to a Jump-Diffusion Setting

The Risk-Sensitive Asset Management (RSAM) theory was developed based on diffusion models. In a jump-diffusion setting,

- Wan [15] briefy sketch an infinite time horizon problem with a single constant jump in each asset.
- Davis and Lleo [5] consider a finite time horizon problem with random jumps in the asset prices. They proved the existence of an optimal control and showed that the value function is a smooth (strong) solution of the Hamilton Jacobi Bellman Partial Differential Equation (HJB PDE).
- Davis and Lleo [6] consider a finite time horizon affine model with random jumps in both asset prices and factor levels. They proved the existence of an optimal control and showed that the value function is a continuous (weak) viscosity solution of the Hamilton Jacobi Bellman Partial Integro-Differential Equation (HJB PIDE).

The Risk-Sensitive Investment Problem - Setting

Let $(\Omega, \{\mathcal{F}_t\}, \mathcal{F}, \mathbb{P})$ be the underlying probability space. Take a market with a money market asset S_0 with dynamics

$$\frac{dS_0(t)}{S_0(t)} = a_0\left(t, X(t)\right) dt, \; S_0(0) = s_0$$

and m risky assets following jump-diffusion SDEs

$$\begin{aligned} \frac{dS_i(t)}{S_i(t^-)} &= \left[a\left(t, X(t^-)\right)\right]_i dt + \sum_{k=1}^{N} \sigma_{ik}(t, X(t)) dW_k(t) \\ &\quad + \int_{\mathbf{Z}} \gamma_i(t,z) \bar{N}_{\mathbf{p}}(dt,dz), \; S_i(0) = s_i, \; i = 1, \dots, m \end{aligned}$$

$X(t)$ is a n-dimensional vector of economic factors following

$$\begin{aligned} dX(t) &= b\left(t, X(t^-)\right) dt + \Lambda(t, X(t)) dW(t) \\ &\quad + \int_{\mathbf{Z}} \xi\left(t, X(t^-), z\right) \bar{N}_{\mathbf{p}}(dt,dz), \; X(0) = x \qquad (3) \end{aligned}$$

Note:

- $W(t)$ is a $\mathbb{R}^{m+n}$-valued $(\mathcal{F}_t)$-Brownian motion with components $W_k(t)$, $k = 1, \ldots, (m+n)$.
- $\bar{N}_{\mathbf{p}}(dt, dz)$ is a Poisson random measure (see e.g. Ikeda and Watanabe [8]) defined as

$$\bar{N}_{\mathbf{p}}(dt, dz) = \begin{cases} N_{\mathbf{p}}(dt, dz) - \nu(dz)dt =: \tilde{N}_{\mathbf{p}}(dt, dz) & \text{if } z \in \mathbf{Z}_0 \\ N_{\mathbf{p}}(dt, dz) & \text{if } z \in \mathbf{Z} \backslash \mathbf{Z}_0 \end{cases}$$

- the functions a_0, a, b, $\Sigma = [\sigma_{ij}]$, Λ are Lipschitz continuous, bounded with bounded derivatives in terms of the variables t and x.
- *Ellipticity condition*:

$$\Sigma\Sigma' > 0 \tag{4}$$

- the jump intensities $\xi(z)$ and $\gamma(z)$ satisfies appropriate well-posedness conditions.
- *Independence of systematic (factor-driven) and idiosyncratic (asset-driven) jump:* $\forall (t, x, z) \in [0, T] \times \mathbb{R}^n \times \mathbf{Z}$, $\gamma(t, z)\xi'(t, x, z) = 0$.

...

... plus an extra condition:

Assumption

The vector valued function $\gamma(t,z)$ satisfy:

$$\int_{\mathbf{Z}} |\xi(t,x,z)|\nu(dz) < \infty, \qquad \forall (t,x) \in [0,T] \times \mathbb{R}^n \tag{5}$$

Note that the minimal condition on ξ under which the factor equation (3) is well posed is

$$\int_{\mathbf{Z}_0} |\xi(t,x,z)|^2 \nu(dz) < \infty,$$

However, for this paper it is essential to impose the stronger condition (5) in order to connect the viscosity solution of HJB partial integro-differential equation (PIDE) with the viscosity solution of a related parabolic PDE.

Wealth Dynamics

The wealth, $V(t)$ of the investor in response to an investment strategy $h(t) \in \mathcal{H}$, follows the dynamics

$$\begin{aligned}\frac{dV(t)}{V(t^-)} &= \left(a_0\left(t, X(t)\right)\right) dt + h'(t)\hat{a}\left(t, X(t)\right) dt + h'(t)\Sigma(t, X(t)) dW_t \\ &\quad + \int_{\mathbf{Z}} h'(t)\gamma(t, z)\bar{N}_{\mathbf{p}}(dt, dz)\end{aligned} \tag{6}$$

with initial endowment $V(0) = v$, where $\hat{a} := a - a_0\mathbf{1}$ and $\mathbf{1} \in \mathbf{R}^m$ denotes the m-element unit column vector.

The objective is to maximize a function of the log-return of wealth

$$J(x, t, h; \theta) := -\frac{1}{\theta} \ln \mathbf{E}\left[e^{-\theta \ln V(t,x,h)}\right] = -\frac{1}{\theta} \ln \mathbf{E}\left[V^{-\theta}(t, x, h)\right] \tag{7}$$

Investment Constraints

We also consider $r \in \mathbb{N}$ fixed investment constraints expressed in the form

$$\Upsilon' h(t) \leq \upsilon \tag{8}$$

where $\Upsilon \in \mathbb{R}^m \times \mathbb{R}^r$ is a matrix and $\upsilon \in \mathbb{R}^r$ is a column vector.

Assumption

The system

$$\Upsilon' y \leq \upsilon$$

for the variable $y \in \mathbb{R}^m$ admits at least two solutions.

This assumptions guarantee that the feasible region is

- a convex subset of R^r,
- with no redundant or conflicting constraints, and
- has at least one interior point

which implies that there will be at least one investment policy satisfying the constraints.

By Itô,

$$e^{-\theta \ln V(t)} = v^{-\theta} \exp\left\{\theta \int_0^t g(X_s, h(s); \theta) ds\right\} \chi_t^h \qquad (9)$$

where

$$\begin{aligned} & g(t, x, h; \theta) \\ = \; & \frac{1}{2}(\theta+1) h' \Sigma\Sigma'(t, x) h - a_0(t, x) - h' \hat{a}(t, x) \\ & + \int_{\mathbf{Z}} \left\{ \frac{1}{\theta}\left[\left(1 + h'\gamma(t, z)\right)^{-\theta} - 1\right] + h'\gamma(t, z) \mathit{1}_{\mathbf{Z}_0}(z) \right\} \nu(dz) \end{aligned} \qquad (10)$$

and the Doléans exponential χ_t^h is given by

$$\begin{aligned}
\chi_t^h \quad := \quad & \exp\left\{-\theta \int_0^t h(s)'\Sigma(s, X(s))dW_s \right.\\
& -\frac{1}{2}\theta^2 \int_0^t h(s)'\Sigma\Sigma'(s, X(s))h(s)ds \\
& + \int_0^t \int_{\mathbf{Z}} \ln\left(1 - G(s, z, h(s); \theta)\right) \tilde{N}_{\mathbf{p}}(ds, dz) \\
& \left. + \int_0^t \int_{\mathbf{Z}} \left\{\ln\left(1 - G(s, z, h(s); \theta)\right) + G(s, z, h(s); \theta)\right\} \nu(dz)ds \right\},
\end{aligned} \tag{11}$$

and

$$G(t, z, h; \theta) \quad = \quad 1 - \left(1 + h'\gamma(t, z)\right)^{-\theta} \tag{12}$$

Change of Measure

This step is due to Kuroda and Nagai [10]. Let $\mathbb{P}_h^\theta$ be the measure on $(\Omega, \mathcal{F}_T)$ defined via the Radon-Nikodým derivative

$$\frac{d\mathbb{P}_h^\theta}{d\mathbb{P}} := \chi_T^h \tag{13}$$

For a change of measure to be possible, we must ensure that $G(z, h(s); \theta) < 1$, which is satisfied iff $h'(s)\gamma(z) > -1$ a.s. $d\nu$.

$$W_t^h = W_t + \theta \int_0^t \Sigma' h(s) ds$$

is a standard Brownian motion under the measure $\mathbb{P}_h^\theta$ and we have

$$\begin{aligned} & \int_0^t \int_{\mathbf{Z}} \tilde{N}_{\mathbf{p}}^h(ds, dz) \\ = \ & \int_0^t \int_{\mathbf{Z}} N_{\mathbf{p}}(ds, dz) - \int_0^t \int_{\mathbf{Z}} \left\{ \left(1 + h'\gamma(s, X(s), z)\right)^{-\theta} \right\} \nu(dz) ds \end{aligned}$$

As a result, $X(s),\ 0 \leq s \leq t$ satisfies the SDE:

$$\begin{aligned} dX(s) &= f(s, X(s), h(s); \theta) ds + \Lambda(s, X(s)) dW_s^\theta \\ &\quad + \int_{\mathbf{Z}} \xi\left(s, X(s^-), z\right) \tilde{N}_{\mathbf{p}}^h(ds, dz) \end{aligned} \tag{14}$$

where

$$\begin{aligned} f(t, x, h; \theta) &:= b(t, x) - \theta \Lambda \Sigma(t, x)' h(s) \\ &\quad + \int_{\mathbf{Z}} \xi(t, x, z) \left[\left(1 + h' \gamma(t, z)\right)^{-\theta} - 1_{\mathbf{Z}_0}(z) \right] \nu(dz) \end{aligned} \tag{15}$$

and b is the $\mathbb{P}$-measure drift of the factor process. Note that f is Lipschitz continuous in t and x.

Following the change of measure we introduce two auxiliary criterion functions under $\mathbb{P}_h^\theta$:

- the risk-sensitive control problem:

$$I(v,x;h;t,T;\theta) = -\frac{1}{\theta}\ln \mathbf{E}_{t,x}^{h,\theta}\left[\exp\left\{\theta\int_t^T g(X_s,h(s);\theta)ds - \theta\ln v\right\}\right] \tag{16}$$

  where $\mathbf{E}_{t,x}\left[\cdot\right]$ denotes the expectation taken with respect to the measure $\mathbb{P}_h^\theta$ and with initial conditions (t,x).

- the exponentially transformed criterion

$$\tilde{I}(v,x,h;t,T;\theta) := \mathbf{E}_{t,x}^{h,\theta}\left[\exp\left\{\theta\int_t^T g(s,X_s,h(s);\theta)ds - \theta\ln v\right\}\right] \tag{17}$$

How to Solve a Stochastic Control Problem

Our objective is to solve the control problem in a classical sense.

The process involves

1. deriving the HJB PIDE;
2. identifying a (unique) candidate optimal control;
3. proving existence of a $C^{1,2}$ (classical) solution to the HJB PIDE.
4. proving a verification theorem;

The HJB PIDEs

The HJB PIDE associated with the risk-sensitive control criterion (16) is

$$\frac{\partial \Phi}{\partial t}(t,x) + \sup_{h \in \mathcal{J}} L_t^h \Phi(t,x) = 0, \qquad (t,x) \in (0,T) \times \mathbb{R}^n \tag{18}$$

where

$$\begin{aligned} & L_t^h \Phi(t,x) \\ = \; & f(t,x,h;\theta)' D\Phi + \frac{1}{2} \mathrm{tr}\left(\Lambda\Lambda'(t,x) D^2 \Phi \right) \\ & - \frac{\theta}{2} (D\Phi)' \Lambda\Lambda'(t,x) D\Phi - g(t,x,h;\theta) \\ & + \int_{\mathbf{Z}} \left\{ -\frac{1}{\theta} \left(e^{-\theta[\Phi(t,x+\xi(t,x,z)) - \Phi(t,x)]} - 1 \right) - \xi(t,x,z)' D\Phi \right\} \nu(dz) \end{aligned}$$

and subject to terminal condition $\Phi(T,x) = \ln v$.

To remove the quadratic growth term, we consider the PIDE associated with the exponentially-transformed problem (17):

$$\begin{aligned}
& \frac{\partial \tilde{\Phi}}{\partial t}(t,x) + \frac{1}{2}\mathrm{tr}\left(\Lambda\Lambda'(t,x)D^2\tilde{\Phi}(t,x)\right) + H(t,x,\tilde{\Phi},D\tilde{\Phi}) \\
& + \int_{\mathbf{Z}} \left\{ \tilde{\Phi}\left(t, x+\xi(t,x,z)\right) - \tilde{\Phi}(t,x) - \xi(t,x,z)' D\tilde{\Phi}(t,x) \right\} \nu(dz) \\
= \; & 0 \qquad (19)
\end{aligned}$$

subject to terminal condition $\tilde{\Phi}(T,x) = v^{-\theta}$ and where

$$H(s,x,r,p) = \inf_{h\in\mathcal{J}} \left\{ f(s,x,h;\theta)'p + \theta g(s,x,h;\theta) r \right\} \qquad (20)$$

for $r \in \mathbb{R}$, $p \in \mathbb{R}^n$.

In particular $\tilde{\Phi}(t,x) = \exp\left\{-\theta\Phi(t,x)\right\}$.

Identifying a (Unique) Candidate Optimal Control

The supremum in (18) can be expressed as

$$
\begin{aligned}
& \sup_{h \in \mathcal{A}} L_t^h \Phi \\
= \quad & b'(t,x) D\Phi + \frac{1}{2} \mathrm{tr}\left(\Lambda\Lambda'(t,x) D^2\Phi\right) - \frac{\theta}{2} (D\Phi)'\Lambda\Lambda'(t,x)' D\Phi \\
& + a_0(t,x) \\
& + \int_{\mathbf{Z}} \left\{ -\frac{1}{\theta} \left(e^{-\theta(\Phi(t,x+\xi(t,x,z)) - \Phi(t,x))} - 1 \right) - \xi(t,x,z)' D\Phi 1_{\mathbf{Z}_0}(z) \right\} \\
& + \sup_{h \in \mathcal{J}} \left\{ -\frac{1}{2} (\theta + 1) h'\Sigma\Sigma'(t,x)' h - \theta h'\Sigma\Lambda'(t,x) D\Phi + h'\hat{a}(t,x) \right. \\
& -\frac{1}{\theta} \int_{\mathbf{Z}} \left\{ \left(1 - \theta\xi(t,x,z)' D\Phi\right) \left[\left(1 + h'\gamma(t,z)\right)^{-\theta} - 1 \right] \right. \\
& \left. \left. + \theta h'\gamma(t,z) 1_{\mathbf{Z}_0}(z) \right\} \nu(dz) \right\}
\end{aligned}
$$

- Because $\Sigma\Sigma' > 0$ and because systematic jumps are independent from idiosyncratic jumps, this corresponds to the maximization of a concave function on a convex set of constraints.
- By the Lagrange Duality (see for example Theorem 1 in Section 8.6 in [12]), we conclude that the supremum in (22) admits a unique maximizer $\hat{h}(t; x; p)$ for $(t; x; p) \in [0; T] \times R^n \times R^n$.
- By measurable selection, $\hat{h}$ can be taken as a Borel measurable function on $[0; T] \times R^n \times R^n$.

Existence of a $C^{1,2}$ Solution to the HJB PDE

Proving the existence of a strong, $C^{1,2}$, solution is the most difficult and intricate step in the process.

However, this is a necessary step if we want to use the Verification Theorem to conclusively solve our optimal investment problem. First, we explore the properties of the value functions Φ and $\tilde{\Phi}$

- The exponentially transformed value function $\tilde{\Phi}$ is positive and bounded;
- The value function $\tilde{\Phi}$ is Lipschitz continuous in the state variable x.

Existence of a $C^{1,2}$ Solution in 6 Steps

A promising methodology originally proposed by Pham [13] for PIDEs and extended to impulse-control problems by Davis, Guo and Wu [4] relies on

- the interaction between weak viscosity solutions and strong classical solutions;
- the ability, under some circumstances, to rewrite a PIDE as a PDE.

Our approach innovates in that respect by tackling a full fledged HJB PIDE (i.e. with an embedded optimization) rather than a more 'traditional' PIDE.

The process involves 6 steps.

Step 1: $\tilde{\Phi}$ is a Lipschitz continuous viscosity solution (VS-PIDE) of (19)

(i). we already know that $\tilde{\Phi}$ is Lipshitz continuous.

(ii). change notation and rewrite the HJB PIDE as

$$
\begin{aligned}
&-\frac{\partial \tilde{\Phi}}{\partial t}(t,x) + H_v(t,x,\tilde{\Phi},D\tilde{\Phi}) - \frac{1}{2}\mathrm{tr}\left(\Lambda\Lambda'(t,x)D^2\tilde{\Phi}(t,x)\right) \\
&-\mathcal{I}[t,x,\tilde{\phi}] = 0
\end{aligned}
\tag{21}
$$

subject to terminal condition $\tilde{\Phi}(t,x) = v^{-\theta}$ and where

$$
\begin{aligned}
H_v(s,x,r,p) &= -H(s,x,r,p) \\
&= \sup_{h\in\mathcal{A}} \left\{ -f_v(s,x,h;\theta)'p - \theta g(s,x,h;\theta) r \right\}
\end{aligned}
$$

for $r \in \mathbb{R}$, $p \in \mathbb{R}^n$...

$$
\begin{aligned}
f_v(t,x,h;\theta) \quad &:= \quad f(t,x,h;\theta) - \int_{\mathbf{Z}\setminus\mathbf{Z}_0} \xi(t,x,z)\nu(dz) \\
&= \quad b(t,x) - \theta\Lambda\Sigma(t,x)'h(s) \\
&\quad\quad + \int_{\mathbf{Z}} \xi(t,x,z)\left[\left(1+h'\gamma(t,z)\right)^{-\theta} - 1\right]\nu(dz)
\end{aligned}
$$

and

$$
\begin{aligned}
&\mathcal{I}[t,x,\tilde{\Phi}] \\
:= \quad &\int_{\mathbf{Z}} \left\{\tilde{\Phi}\left(t,x+\xi(t,x,z)\right) - \tilde{\Phi}(t,x) - \xi(t,x,z)'D\tilde{\Phi}(t,x)1_{\mathbf{Z}_0}\right\}\nu(dz)
\end{aligned}
$$

(iii). show that $\tilde{\Phi}$ is *a* (discontinuous) viscosity solution of (21).

Step 2: From PIDE to PDE

Change notation and rewrite the HJB PIDE as the parabolic PDE *à la* Pham [13]:

$$\begin{aligned} & \frac{\partial \tilde{\Phi}}{\partial t}(t,x) + \frac{1}{2}\mathrm{tr}\left(\Lambda\Lambda'(t,x)D^2\tilde{\Phi}(t,x)\right) + H_a(t,x,\tilde{\Phi},D\tilde{\Phi}) + d_a^{\tilde{\Phi}}(t,x) \\ = \quad & 0 \end{aligned} \tag{22}$$

subject to terminal condition $\tilde{\Phi}(T,x) = v^{-\theta}$ and with

$$H_a(s,x,r,p) \quad = \quad \inf_{h\in\mathcal{U}} \left\{ f_a(x,h)'p + \theta g(x,h;\theta)r \right\} \tag{23}$$

for $r \in \mathbb{R}$, $p \in \mathbb{R}^n$ and where

$$
\begin{aligned}
f_a(x,h) &:= f(x,h) - \int_{\mathbf{Z}} \xi(t,x,z)\nu(dz) \\
&= b(t,x) - \theta\Lambda\Sigma(t,x)'h(s) \\
&\quad + \int_{\mathbf{Z}} \xi(t,x,z)\left[\left(1+h'\gamma(t,z)\right)^{-\theta} - 1_{\mathbf{Z}_0}(z) - 1\right]\nu(dz)
\end{aligned}
\tag{24}
$$

and

$$
d_a^{\tilde{\Phi}}(t,x) = \int_{\mathbf{Z}} \left\{\tilde{\Phi}\left(t, x + \xi(t,x,z)\right) - \tilde{\Phi}(t,x)\right\}\nu(dz) \tag{25}
$$

Step 3: Viscosity solution to the PDE (22)

Consider a viscosity solution (**VS-PDE**) u of the semi-linear PDE (23) (always interpreted as an equation for 'unknown' u with the last term prespecified, with $\tilde{\Phi}$ defined as the value function $\tilde{\Phi}$)

$$\frac{\partial \check{\phi}}{\partial t}(t,x) + \frac{1}{2}\mathrm{tr}\left(\Lambda\Lambda'(t,x)D^2\check{\phi}(t,x)\right) + H_a(t,x,\check{\phi},D\check{\phi}) + d_a^{\tilde{\Phi}}(t,x) = 0 \tag{26}$$

$\tilde{\Phi}$ **is a viscosity solution of the PDE** (22) - this is due to the fact that by choosing $\tilde{\Phi}$, PIDE (21) and PDE (22) are in essence the same equation $\rightarrow$ if $\tilde{\Phi}$ solves one of them, then it solves both.

Step 4: Uniqueness of the Viscosity solution to the PDE (22)

If a function u solves the PDE (22) it does not mean that u also solves the PIDE (21) because the term d_a in the PDE (22) depends on $\tilde{\Phi}$ regardless of the choice of u.

Thus, if we were to show the existence of a classical solution u to PDE (22), we would not be sure that this solution is the value function $\tilde{\Phi}$ unless we can show that PDE (22) admits a unique solution.

This only requires applying a "classical" comparison result for viscosity solutions (see Theorem 8.2 in Crandall, Ishii and Lions [3]) *provided appropriate conditions on f_a and d_a are satisfied*.

Step 5: Existence of a Classical Solution to the HJB PDE (22)

We use the argument in Appendix E of Fleming and Rishel [7] to show the existence of a a classical solution to the PDE (22).

Step 6: Any classical solution is a viscosity solution

Observe that a classical solution is also a viscosity solution[1]

Hence, the classical solution to PDE (22) is also the unique viscosity solution of PDE (22) and the viscosity solution of PIDE (21).

This shows $\tilde{\Phi}$ is $C^{1,2}$ and satisfies PIDE (19) in the classical sense.

[1] Broadly speaking the argument is that if the solution of the PDE is smooth, then we can use it as a test function in the definition of viscosity solutions. If we do this, we will recover the classical maximum principle and therefore prove that the solution of the PDE is a classical solution.

Verification Theorem

Broadly speaking, the verification theorem states that if we have

- a $C^{1,2}([0,T]\times\mathbb{R}^n)$ bounded function ϕ which satisfies the HJB PDE (18) and its terminal condition;
- the stochastic differential equation

$$\begin{aligned} dX(s) &= f(s,X(s),h(s);\theta)ds + \Lambda(s,X(s))dW_s^\theta \\ &\quad + \int_{\mathbf{Z}} \xi\left(s, X(s^-), z\right) \tilde{N}_{\mathbf{p}}^\theta(ds,dz) \end{aligned}$$

  defines a unique solution $X(s)$ for each given initial data $X(t)=x$; and,
- there exists a Borel-measurable maximizer $h^*(t,X_t)$ of $h \mapsto L^h\phi$ defined in (19);

then Φ is the value function and $h^*(t,X_t)$ is the optimal Markov control process.

. . . and similarly for $\tilde{\Phi}$ and the exponentially-transformed problem.

Concluding Remarks

- In this article, we reformulated the risk-sensitive investment management problem to allow jumps in both factor levels and asset prices, stochastic volatility and investment constraints.
- We want to extend this approach to cover credit risk, for which we needed asset price processes with jumps.
- Further research is needed to determine both the extend of the jump-diffusion problems the 6-step approach proposed in this article can be used to solve and how much further it can be extended.

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