# Overprized options on variance swaps in local vol models

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# Outline

1. Setting:
    - Stochastic Volatility
    - Local Volatility - Gyöngy - Dupire

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3. Counterexample

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3. time horizon: $[0, T]$.

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local vol:

$$d\tilde{S}_t(\omega) = \tilde{S}_t(\omega)\tilde{\sigma}(t, \tilde{S}_t(\omega))dB_t(\omega)$$

$\sigma = \sigma(t,s)$ is deterministic.

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$$\text{explicit representation:} \quad \tilde{\sigma}^2(t,s) = \mathbb{E}[\sigma^2(t,\omega)|S_t = s].$$

Price of European call $C = C(t,K)$ depends solely on $law(S_t)$.
$\Longrightarrow (S_t)$ and $(\tilde{S}_t)$ generate the same call prices $C = C(t,K)$.

# Setting 3

## Dupire's formula:

Assume that for $s > 0, t \in [0, T]$ call prices $C(t, K)$ are known. Define

$$\tilde{\sigma}^2(t, s) = 2\frac{\partial_t C(t, s)}{s^2 \partial_{KK} C(t, s)}.$$

Then $\tilde{S}$, $d\tilde{S}_t = \tilde{S}_t \tilde{\sigma}(t, \tilde{S}_t) dB_t$ reproduces $C(t, K)$.

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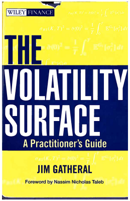
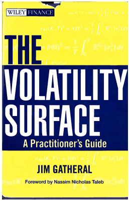



by known prices of European options.

Returning to the lower bound, **it has been conjectured† that the minimum possible value of an option on variance is the one generated from a local volatility model fitted to the volatility surface.** Clearly options on variance have value even in a local volatility model because realized variance depends on the realized path of the stock price from inception to expiration. Given that local variance is a risk-neutral conditional expectation of instantaneous variance, it seems obvious that any other model would generate extra fluctuations of the local volatility surface relative to its initial state.

Between these model independent upper and lower bounds, it seems

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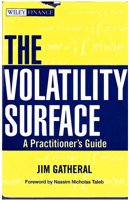
recall: $V = \int_0^T \sigma^2(t)\,dt \qquad \tilde{V} = \int_0^T \tilde{\sigma}^2(t, \tilde{S}(t))\,dt$

Conjecture: $\mathbb{E}[(V-K)^+] \geq \mathbb{E}[(\tilde{V}-K)^+]$ for all $K > 0$.

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- $law(V) \succcurlyeq_c law(\tilde{V})$ in the convex order.

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Fair coin flip $\epsilon = \pm 1$ (independent of $B$), $\sigma^2 = \sigma^2_\epsilon$,

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$\implies V = \int_0^3 \sigma^2(t)\, dt \equiv 6.$

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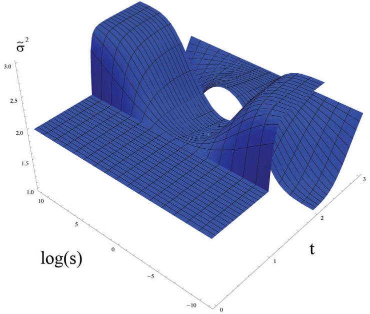

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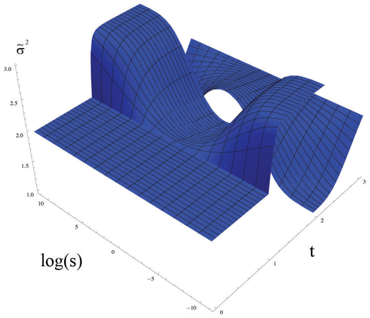

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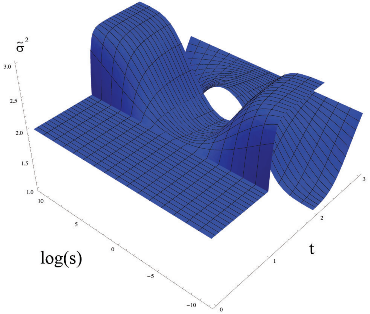


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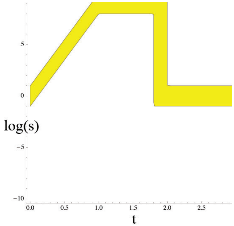

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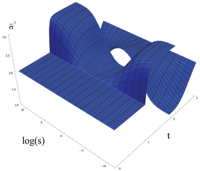


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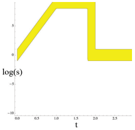


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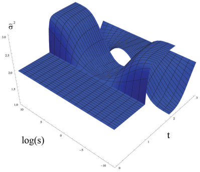


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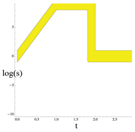


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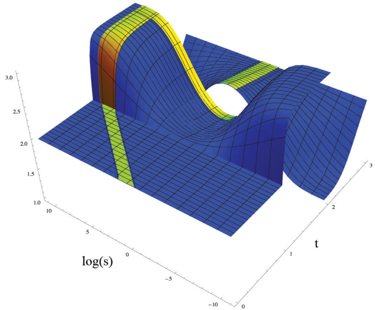

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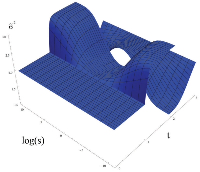


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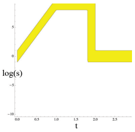


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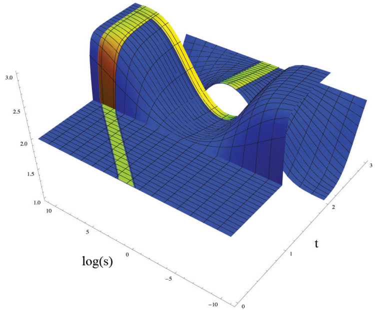


for yellow paths: $\int_0^3 \tilde{\sigma}^2(t, \tilde{S}_t(\omega))\, dt > 6$

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$$\mathbb{E}[(V-6)^+] = 0$$

# Counterexample

$$\left.\begin{array}{l} V = \int_0^3 \sigma(t, \tilde{S}_t)\, dt \equiv 6, \text{ but} \\[2ex] \tilde{V} = \int_0^3 \tilde{\sigma}^2(t, \tilde{S}_t)\, dt \text{ is } \textit{not} \text{ deterministic} \end{array}\right\} \Rightarrow \; V \not\preceq_c \tilde{V}$$

More specific, consider call with strike 6, i.e. $f(v) := (v-6)^+$:

$$\mathbb{E}[(V-6)^+] = 0 < \mathbb{E}[(\tilde{V}-6)^+].$$

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   $\Longrightarrow$ generalized Black-Scholes-model, in particular complete.
2. $\sigma(., \omega)$ can be chosen in a continuous/smooth way.
3. Using Gyöngy's result in two dimensions, one obtains a counterexample of (time-inhomogenous) Markovian structure.

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2. Further assumptions are necessary to rigorously prove

$$V \succcurlyeq_c \tilde{V}.$$