Electronic Market Making
Potential Profits and Research Opportunities

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Revised Title:

Electronic Market Making
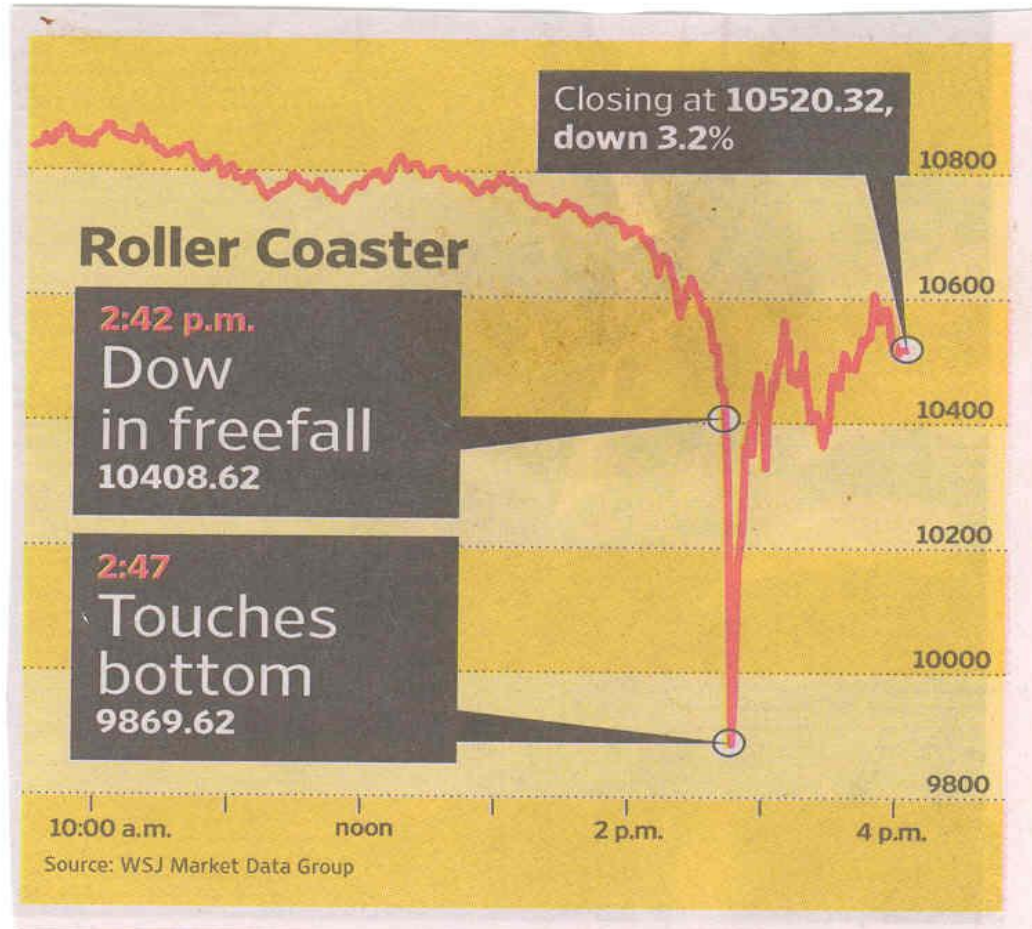## Potential DISASTER and Research Opportunities

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May 6, 2010

DJIA 10520.32 ▼ 347.80 −3.2% **NASDAQ** 2319.64 ▼ 3.4% **NIKKEI** 10695.69 ▼ 3.3% **STOXX 50** 2416.91 ▼ 1.7% **10-YR TREAS** ▲ 1 8/32, yield 3.401% **OIL** $77.11 ▼ $2.86 **GOLD** $1,196.90 ▲ $22.30 **EURO** $1.2629 **YEN** 9

Market Plunge Baffles Wall Street

High-Speed Trading Glitch Costs Investors Millions

THE NEW YORK TIMES **BUSINESS** FRIDAY, MAY 7, 2010

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STOCKS & BONDS

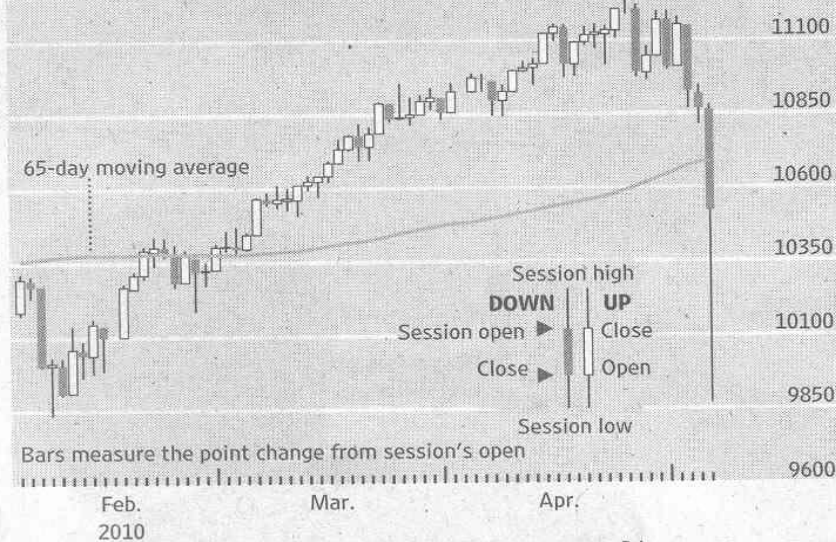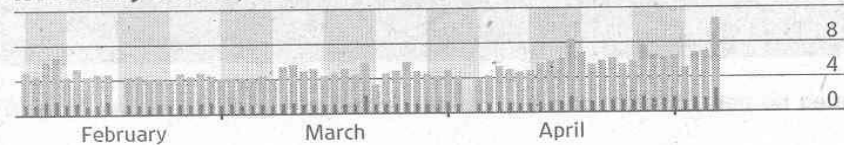# Dow Rapidly Plunges 1,000 Points, Then Makes a Partial Recovery

3 Month Graph

May 7, 2010 (New York Times)

The Trades of a Lifetime in 20 Minutes

Someone on Wall Street just made a killing.

That was the subject of so much chatter among professional investors once the smoke cleared from the sudden panic and recovery on Thursday that briefly knocked some stocks down to a penny or two a share. Who had kept his cool during those terrifying minutes and scooped up some dreamlike bargains?

The answer to that question is as elusive as the causes of the rout itself, because the shock rippled across so many markets in so short a time. Stock exchanges and regulators were still sorting through billions of transactions on Friday.

One thing, however, is certain: By luck, savvy, lightning speed or all three, there was money — gobs of it — to be made from the bargains that came and went in an instant.

"Somebody got [Accenture](#) at a penny. They're ready to announce their retirement," joked Daniel Seiver, a finance professor at San Diego State University.

For at least some of the winners, however, retirement may have to wait. On Friday, several large United States exchanges said that although their trading platforms functioned properly on Thursday, they were nonetheless canceling many trades made during the market's Big Bounce.

Some stocks traded for pennies
Apple traded for $100,000/share

Did High-Frequency Traders Exacerbate Market's Plunge?

markets out substantially," said Jamie Selway, managing director of New York broker White Cap Trading.

High-frequency firms have in recent years become central to how the market operates, acting as middlemen between investors who want to buy and sell. They post orders and attempt to flip their positions for a tiny profit.

The firms have said part of the value they add to markets is the liquidity they bring—being at the ready to swiftly complete a trade. Some of these firms have said that, were it not for them, the 2008 market declines would have been far worse.

That isn't what happened with some firms on Thursday. Market participants said a number of other high-frequency firms stopped trading as the speed and extent of the decline went outside their models, which are generally based on the market behaving in a normal fashion. To avoid the risk of big losses, the firms essentially turn off their computers.

"We did see several clients who stopped trading," said a broker who caters to high-frequency firms.

The firms, which trade large amounts of stock in a single day, often operate with a small amount of capital. They may have a big incentive to close down in chaotic times to protect it, said one trader at a high-frequency firm.

Technical factors that high-frequency firms and other quantitative funds use to trade likely also played a part as the selling accelerated.

When the market hits certain levels as it falls, these firms' computers are programmed to sell automatically as protection against further losses.

"This was a massive liquidation panic," said Bill Strazzullo, chief market strategist for Bell Curve Trading, a Freehold, N.J., technical-research firm.

RUTH FREMSON/THE NEW YORK TIMES

Visitors on the floor of the New York Stock Exchange watched monitors on Thursday as the Dow Jones index plunged by hundreds of points within minutes.

PREVIOUS CLOSE: 10,868.10

Dow industrials

Close
10,520.32
−3.2%
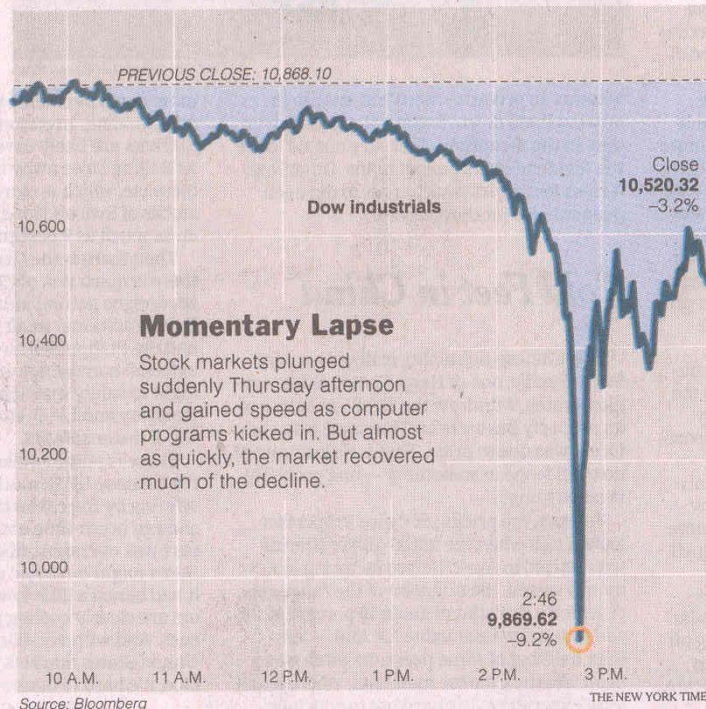
Momentary Lapse

Stock markets plunged suddenly Thursday afternoon and gained speed as computer programs kicked in. But almost as quickly, the market recovered much of the decline.

10,600

10,400

10,200

10,000

2:46
9,869.62
−9.2%

10 A.M.   11 A.M.   12 P.M.   1 P.M.   2 P.M.   3 P.M.

Source: Bloomberg

THE NEW YORK TIMES

When Machines Take Control

By NELSON D. SCHWARTZ
and LOUISE STORY

The glitch that sent markets tumbling Thursday was years in the making, driven by the rise of computers that transformed stock trading more in the last 20 years than in the previous 200.

The old system of floor traders matching buyers and sellers has been replaced by machines that process trades automatically, speeding the flow of buy and sell orders but also sometimes facilitating the kind of unexplained volatility that roiled markets Thursday.

"We have a market that responds in milliseconds, but the humans monitoring respond in minutes, and unfortunately billions of dollars of damage can occur in the meantime," said James Angel, a

High-Speed Glitch In Trading Costs Investors Millions

professor of finance at Georgetown University's McDonough School of Business.

In recent years, what is known as high-frequency trading — rapid computerized buying and selling — has taken off and now accounts for 50 to 75 percent of daily trading volume. At the same time, new electronic exchanges have taken over much of the volume that used to be handled by the New York

Stock Exchange.

In fact, more than 60 percent of trading in stocks listed on the New York Stock Exchange takes place on other, computerized exchanges.

Many questions were left unanswered even hours after the end of the trading day. Who or what was the culprit? Why did markets spin out of control so rapidly? What needs to be done to prevent this from happening again?

The Nasdaq exchange said it would cancel trades that moved shares more than 60 percent up or down at 2:40 p.m., when stocks like Accenture plummeted to a penny a share, for seemingly no reason. Exelon, the utility operator, fell to a hun-

A defense of high-frequency trading (NY Times)

May 8, 2010
Op-Ed Contributor

Fixing Wall Street's Autopilot

By MICHAEL DURBIN

Chapel Hill, N.C.

ON Thursday afternoon, the Dow plunged 1,000 points within a few minutes, followed by an equally sudden recovery. We don't know all the details about the drop, but it was almost certainly the result of computer or human error in a high-speed trading program.

Among the many arcane corners of the financial world highlighted by the Wall Street crisis, high-frequency trading — in which computers scan billions of bits of market data for trading opportunities that may exist for mere fractions of a second — has generated a surprising amount of discussion. Alongside the risk of expensive errors like what happened Thursday, critics say, these programs facilitate insider trading and overwhelm regulators' access to critical information.

These are fair criticisms. Fortunately, they can also be easily addressed without undermining the positive role that high-frequency trading plays in the market.

July 24, 2009 (New York Times)

Stock Traders Find Speed Pays, in Milliseconds

By [CHARLES DUHIGG](#)

It is the hot new thing on Wall Street, a way for a handful of traders to master the stock market, peek at investors' orders and, critics say, even subtly manipulate share prices.

It is called high-frequency trading — and it is suddenly one of the most talked-about and mysterious forces in the markets.

Powerful computers, some housed right next to the machines that drive marketplaces like the [New York Stock Exchange](#), enable high-frequency traders to transmit millions of orders at lightning speed and, their detractors contend, reap billions at everyone else's expense.

These systems are so fast they can outsmart or outrun other investors, humans and computers alike. And after growing in the shadows for years, they are generating lots of talk.

Nearly everyone on Wall Street is wondering how hedge funds and large banks like [Goldman Sachs](#) are making so much money so soon after the financial system nearly collapsed. High-frequency trading is one answer.

A Warning Last Year
The SEC is Worried!

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        * BUSINESS
        * APRIL 7, 2010


SEC Weighs New Rules on Fast Trading
        *
By JACOB BUNGE
U.S. regulators are moving toward a new rule that would track
transactions by high-frequency trading firms to improve oversight of
their activity, according to people familiar with the matter.
The plan would see the Securities and Exchange Commission give the
firms unique identifiers, allowing the agency to keep closer tabs on
traders that aren't registered market makers or broker-dealers.

The SEC and other global regulators are intensifying scrutiny of
computer-driven trading, which has expanded rapidly and is estimated to
account for two-thirds of daily U.S. stock volume. It also is a key
source of liquidity for listed derivatives markets.

The agency is undertaking a cost-benefit analysis on tagging high-
frequency firms' trades and is expected to move forward with a proposal
in the near future, according to several people familiar with the
process. The move would make it simpler for regulators to track
activity, avoiding the need to follow lengthy audit trails from
exchanges. "The SEC can get any data they want, period, but right now
it's kind of cumbersome because they have to go through the clearing
firm," said one securities lawyer who had discussed the matter with SEC
officials. "This would make it more automated."
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Another warning!

Hurrying Into the Next Panic?

By PAUL WILMOTT

So, is trading faster than any human can react truly worrisome? The answers that come back from high-frequency proponents, also rather too quickly, are "No, we are adding liquidity to the market" or "It's perfectly safe and it speeds up price discovery." In other words, the traders say, the practice makes it easier for stocks to be bought and sold quickly across exchanges, and it more efficiently sets the value of shares.

Those responses disturb me. Whenever the reply to a complex question is a stock and unconsidered one, it makes me worry all the more.

Thus the problem with the sudden popularity of high-frequency trading is that it may increasingly destabilize the market. Hedge funds won't necessarily care whether the increased volatility causes stocks to rise or fall, as long as they can get in and out quickly with a profit. But the rest of the economy will care.

Buying stocks used to be about long-term value, doing your research and finding the company that you thought had good prospects. Maybe it had a product that you liked the look of, or perhaps a solid management team. Increasingly such real value is becoming irrelevant. The contest is now between the machines — and they're playing games with real businesses and real people.

Now Senator Dodd wants the SEC to look at high-speed trading

Nation briefs

Dodd urges SEC to look at high-speed trading

WASHINGTON — Federal regulators have got to address the "casino environment" on Wall Street where computerized high-frequency trading can trigger market-shaking turmoil, Senate Banking Committee Chairman Chris Dodd said Sunday.

Dodd, D-Conn., pointed to the new phenomena of computers buying and selling stock in nanoseconds as a possible cause of Thursday's meltdown. The market fell nearly 1,000 points within minutes before rebounding.

Dodd said his committee will hold hearings on Thursday's events. But he said for now the priority is for the Securities and Exchange Commission and the Commodities Futures Trading Commission to come up quickly with answers for dealing with high-frequency trading marked by a lack of marketwide circuit breakers to prevent the market from spiraling out of control.

The Agenda

1. The Economics
2. The Statistics
3. The Returns
4. The Risks
5. The Strategies
6. The Optimization
7. The End

Electronic Security Markets

- Stocks, options, futures,…
- Markets are computers that process and continuously report
  - current best bid and ask (offer) prices
  - bid size (quantity people want to buy at bid price)
  - ask size (quantity offered for sale at ask price)
  - trade prices and sizes when trades occur
- Market makers post quotes (both sides, price and quantity)
- In return for this service, their transaction costs are nil
- Market makers manually posting quotes on a computer terminal can manage more securities than when on floor
- But since quotes are posted and reported electronically, why not replace the human market maker with a computer algorithm?

Advantages of Computer Algorithms for Market Making

- The quotes can be revised more quickly in response to new information, which typically updates too fast for a human to process (many times a second)
- One trader can manage many more securities, so costs per security are lower
- Better quality quoting decisions, so more profitable
- This results in smaller bid-ask spreads and better liquidity

Some Typical $ Numbers for Stock Market Making

- Assume
  - One cent bid-ask spread (thus profit per share)
  - 3 billion shares per day trading volume
  - Market makers involved with two-thirds of trades
- Then
  - Market makers do 1 billion round-trips per day
  - So market makers extract $10 million per day
- Example – a small firm with 1% market share and 13 employees
  - Revenue is $100,000 per day ➔ $26,000,000/year
  - If fixed costs are 50%, this leaves $1M per employee

Example: Global Electronic Trading Co.

Forbes.com

On The Cover/Top Stories
The New Masters of Wall Street
Liz Moyer and Emily Lambert 09.21.09, 12:00 AM ET

Daniel Tierney and Stephen Schuler share a lot of traits with many other enigmatic traders
populating the financial world. Their firm, Global Electronic Trading Co., is tucked behind a
nondescript door on the second floor of the Chicago Board of Trade's art deco building. Until this
summer, when it added some company specifics, its Web site contained little more than a reading
list with recommendations like *Reminiscences of a Stock Operator*. Not a single photo is publicly
available of either of its principals.

What distinguishes Tierney and Schuler is that Getco, as their firm is known, currently buys and
sells 15% of all the stocks traded in the U.S., ranking it among the likes of Goldman Sachs and
Fidelity Investments. Getco was reportedly valued at $1 billion two years ago and is rumored to
have earned roughly half as much as that in net profit last year alone. Tierney, 39, and Schuler, 47,
are among Wall Street's super-nouveau-riche.

"…buys and sells 15% of all the stocks traded in the U.S."
"…valued at $1 billion two years ago…"
"…rumored to have earned half…in net profit last year…"

Profit Calculations for Getco

Assumptions:
- All of Getco's business is market making
- My earlier assumptions are valid

Then:
- Getco trades 450 M shares per day
- 225 M round-trips per day
- $2.25 M revenue per day
- $560 M revenue per year
- $280 M net profit per year

Final Remarks About the Economics

- The importance of and cost of information technology are huge
- This is a big fixed cost, both labor and equipment
- In the firm where I worked, half the employees were in the information technology category
- The next slide shows an article which highlights how the software can be extremely valuable

August 24, 2009

Arrest Over Software Illuminates Wall St. Secret

At 9:20 p.m. on July 3, Mr. McSwain arrested Mr. Aleynikov, 39, at Newark Liberty Airport, accusing him of stealing software code from Goldman Sachs, his old employer. At a bail hearing three days later, a federal prosecutor asked that Mr. Aleynikov be held without bond because the code could be used to "unfairly manipulate" stock prices.

This case is still in its earliest stages, and some lawyers question whether Mr. Aleynikov should be prosecuted criminally, or whether a civil suit may be more appropriate. But the charges, along with civil cases in Chicago and New York involving other Wall Street firms, offer a glimpse into the turbulent world of ultrafast computerized stock trading.

Little understood outside the securities industry, the business has suddenly become one of the most competitive and controversial on Wall Street. At its heart are computer programs that take years to develop and are treated as closely guarded secrets.

"…turbulent world of ultrafast computerized stock trading."
"…computer programs that take years to develop and are treated as closely guarded secrets."

Sample Tick Data (Advanced Micro Devices)

time (secs)	bid size	bid price	ask price	ask size	trade price	trade size
661.8152	1400	13.02	13.04	2300		
661.8815	1400	13.02	13.04	2700		
661.8863	1400	13.02	13.04	2300	13.04	200
661.8977	1400	13.02	13.04	2700		
661.8978	1400	13.02	13.04	2300		
662.8941	700	13.03	13.04	3000		
663.7779	2800	13.02	13.04	2000		
663.8497	1300	13.02	13.03	100		
664.1635	1400	13.02	13.04	6100		
664.5565	1300	13.02	13.03	1000	13.02	100
664.7779	2700	13.02	13.03	1200		
664.8563	1100	13.02	13.03	2330		

Histogram of Spreads

Spread (cents)	Count	Percentage
1	14,258	66.53%
2	5,779	26.97%
3	1,277	5.96%
4	86	0.40%
5+	30	0.14%

Histogram of Time (seconds) Between Ticks

From	To	Count	Percentage
0	.01	3020	14.09%
.01	.02	857	4.00%
.02	.06	2063	9.63%
.06	.1	1806	8.43%
.1	.2	2902	13.54%
.2	.4	3301	15.40%
.4	.6	1701	7.94%
.6	.8	1100	5.13%
.8	1	714	3.33%
1	2	1685	7.86%
2	10	1825	8.52%
10	98.76	455	2.12%

Correlations Between One-Second Bid Price Changes and One-Second Ask Price Changes (Last prices used to convert tick data to evenly spaced, one-second time series)

Bid's lead in seconds	Correlation between bid price change and ask price change
0	0.628
1	0.094
2	0.036
3	0.008
4	0.007

Autocorrelation of one-second bid price changes (Last prices used to convert tick data to evenly spaced, one-second time series)

Lag in Seconds	Autocorrelation
0	-0.023
1	0.012
2	-0.003
3	0.004
4	0.016

Wanted: A Useful Statistical Model For Forecasting the Bid and Ask Prices

- Forecast bid and ask prices, or bid-ask midpoint and spread
- But how far in the future? 1 second? 1 minute?
- Fast, simple, to be utilized in real time …
- A number of factors have a bearing:
  - current bid and ask prices
  - recent bid and ask prices
  - current bid and ask sizes
  - recent bid and ask sizes
  - recent transaction prices and sizes
  - recent behavior of similar securities
  - hedge funds doing volatility trading, pair trading, etc.
- But conventional models inappropriate, since neither continuous time nor evenly spaced discrete time

Some Relevant References

- GARCH for irregularly spaced financial data: the ACD-GARCH Model, E. Ghysels & J. Jasiak, *Studies in Nonlinear Dynamics and Econometrics,* 1998
- A partially observed model for micromovement of asset prices with Bayes estimation via filtering, Y. Zeng, *Mathematical Finance,* 2003
- A discrete-state continuous-time model of financial transactions prices and times: the autoregressive conditional multinomial-autoregressive conditional duration model, J. Russell & R. Engle, *J. Business & Economic Statistics,* 2005
- Modeling financial high frequency data using point processes, L. Bauwens & N. Hautsch, *Handbook of Financial Time Series, 2006,* Springer
- Evaluating models of autoregressive conditional duration, M. Meitz & T. Teräsvirta, *J. Business & Economic Statistics,* 2006

What Traders Use: the <u>Microprice</u>

- A convex combination of the bid and ask prices, with weights based on the bid and ask sizes
- Sort of a forecast, or maybe an estimate of the "value"
- Example
  - Bid price = 13.02
  - Ask price = 13.04
  - Bid size = 200
  - Ask size = 800
  - Microprice = (200/1000)* 13.04 + (800/1000)*13.02
                = .2*13.04 + .8*13.02 = 13.024

A More Sophisticated Approach

- Forecast the bid-ask midpoint and spread 5 seconds (say) ahead
- Use linear regression with the independent variables being different microprice models and "adjustments" like "velocity" of bid size
- Update regression coefficients in real-time using Kalman filtering (see reference below)

Flexible least squares for temporal data mining and statistical arbitrage, G. Montana, K. Triantafyllopoulos, & T. Tsagaris, *Expert Systems with Applications,* 2009

OK, so you think you have a good forecasting model and a good strategy expected to generate profits.

What's holding you back?

Strategies need to deal with risks

- Ignoring risks is like a trader who sells exotic derivatives for more than their theoretical values but never hedges
- The trading strategy might have too much volatility given the expected return
- Traders like to go home flat, so at end of the day they might buy at ask and sell at bid, thereby not only losing out on the profit but also incurring a transaction cost
- Every now and then big trades eat through the book, so the bid and ask prices move adversely
- Hedge funds doing volatility trading, pair trading, etc.
- May 6, 2010

Risks While Market Making in Stocks

- Positions in individual stocks might be too big
- The overall portfolio might be too long or too short

Measuring These Risks

- Look at the variance of the portfolio's change in value over a 24 hour (or whatever) period, assuming positions are constant

Managing These Risks

- Pursue quotes that will lead to a reduced variance, and avoid quotes that will lead to an increase in the variance

Risks While Market Making in Options

- Delta risk
- Volatility risk

- Other Greeks
- Sticky strikes

Measuring These Risks

- Same kind of thing: the variance of the change over a coming period of the dollar value of the portfolio consisting of all the options on a symbol and the underlying symbol itself

Managing These Risks

- Delta hedging (and maybe hedging with interest rates)
- As with stocks, manage quotes so as to keep variance, which is largely due to vega risk, from becoming too large

A Simple Trading Strategy (for bid quotes)

- Parameters
    - size-to-cancel > 0 (# shares)
    - size-to-join > size-to-cancel
    - edge-to-cancel > 0 ($)
    - edge-to-join > edge-to-cancel
    - max-position (# shares)

- Data
    - bid price
    - bid size
    - microprice
    - current-position

- Computed values: edge-to-buy = microprice – bid price
                       quote-size = max-position – current position

Rule for Joining Best Bid
- we have no quote
- edge-to-buy > edge-to-join
- bid size > size-to-join

Rule for Canceling Our Quote
- our quote exists at best bid
- bid size < size-to-cancel, or
- edge-to-buy < edge-to-cancel

Remark About May 6, 2010 "Flash Crash"

- Market makers must post both bid and ask quotes, but they do not need to be at the best prices
- To avoid buying a stock, for example, the algorithm can post a bid quote at 1¢
- If all the algorithms are sufficiently conservative, then when the market is dropping precipitously nobody will post a quote near what a reasonable bid price might be
- Human, common sense was missing

A More Sophisticated Trading Strategy

- Recall that the <u>Variance</u> of a dollar change of a portfolio can be taken as the measure of risk
- Define the <u>Marginal Risk</u> (MR) for a security as the change of Variance if the position in the security is increased by one
- Introduce: <u>Risk Adjustment Parameter</u> (RAP)
- Introduce: <u>Risk Adjusted Edge to Buy</u> (RAETB)

$$RAETB = \text{edge-to-buy} - RAP * MR$$

- Then use the criterion RAETB > edge-to-join instead of the criterion edge-to-buy > edge-to-join when deciding whether to post a quote at the best bid price

Desirable: an Optimal Control Model For Guiding Decisions

- Unfortunately:
  - Large number of relevant variables make Markovian approaches challenging if not impossible
  - Cannot observe all relevant, useful information
  - Difficult to accurately estimate transition probabilities, etc.
  - Probabilities are dynamic and changing with time
- Needed: a model that combines decision making with learning
- Some relevant technologies:
  - Reinforcement learning
  - Adaptive control
  - Statistical/Bayesian design
  - Dynamic programming with learning

Three of the Rare, Relevant Papers

- Reinforcement learning for optimized trade execution by Y. Feng and M. Kearns (U. Penn) and Y. Nevmyvaka (Carnegie Mellon)
  - Actually for proprietary trading rather than market making
- Electronic market making: initial investigations by Y. Nevmyvaka, K. Sycara, and D. Seppi (all Carnegie Mellon)
  - Somewhat qualitative, simple strategy with some *ad hoc* quantitative rules; worries about market maker's inventory
- An electronic market-maker by N. Chan and C. Shelton, MIT
  - Markov decision chain where transition probabilities are unknown and learned (reinforcement learning framework)
  - 3 states: inventory, order imbalance, and spread
  - Actions: quote sizes and locations
  - Reward: a measure of profitability
  - Result: algorithm produces sequence of actions that converges to what's optimal if true probabilities are constant and known

Another Approach: Choose a Strategy Involving One or More Parameters, then Maximize Long-Run Profits by Adjusting Parameters as You Learn by Doing

But how do you choose the sequence of parameter values?

Idea: formulate as a multi-armed bandit problem

- Some aspects of the sequential design of experiments, H. Robbins *Bulletin of the American Mathematical Society,* 1952
- A dynamic allocation index for the sequential allocation of experiments, J. Gittins and D. Jones, *Progress in Statistics,* 1974
- Multi-armed bandits and the Gittins index, P. Whittle, *J. Royal Statist. Soc. Ser. B,* 1980
- Extensions of the multi-armed bandit problem: the discounted case, P. Varaiya *et al., IEEE Trans. Auto. Control,* 1985
- An experimental analysis of the bandit problem, J. Banks *et al., Economic Theory,* 1997

Some Features of the Bandit Problem

- Intuition: think of a room of Las Vegas type slot machines
  - Machines have unknown, and different, payoff distributions
  - The gambler wants to play the machines in some sequence with objective like maximizing expected discounted payoffs
  - The sequence should converge to the best machine, but not too quickly because it might converge to the wrong machine
  - What's the optimal strategy, that is, sequence of machines?
- Can often be formulated as dynamic programs, but the state must identify the history of actions and payoffs, so the state space is enormous and such programs can be impossible to solve
- For some versions there is an important simplification: for each slot machine, indexed by k, there is an index $I(k,H)$, with H a sufficient statistic based on the payoffs AT JUST SLOT MACHINE k
- Optimal strategy: next play the machine with highest index value

Application to the Market Making Problem

- Divide time into equal periods; 1 period = 1 play of slot machine
- Focus on one parameter
- Assume finitely many parameter values; 1 value = 1 slot machine
- Focus on P, the profit per period (slot machine payoff)
- Each parameter value (slot machine) is associated with a different probability distribution of P
- The objective is to choose a sequence of parameter values with an objective like maximizing the expected discounted value of the sequence of P's
- You want to converge quickly to the best parameter value, but not so quickly that you fail to learn the best value

Variation: A Continuum of Slot Machines

- Assume a concave profit function:

$$P = X_1 u^2 + X_2 u + X_3 + N,$$

  where u is the parameter value, the X's are unobservable model constants, and N is "noise" with $E[N] = 0$

- Given sequences $\{u_t\}$ and $\{P_t\}$ of utilized parameter values and respective profits, one can use regression to estimate the X's
- $E[P]$ is maximized by

$$u = -X_2/(2X_1),$$

  so it's tempting to always choose u accordingly, substituting the current estimates of X_1 and X_2.

- However, you can learn the true values of the X's more quickly, and thus be better off, if you cleverly vary the choices of the strategy parameter u

Using Kalman Filtering to Update Estimates of the X's

- Assume

  $$X_i(t+1) = X_i(t) + W_i(t), \qquad i = 1, 2, 3; \qquad t = 0, 1, \ldots$$

  where $\{W_i(t)\}$ is a sequence of IID Normal random variables having mean zero and standard deviation σ_i

- The estimates of the X's, denoted $Y_1(t)$, $Y_2(t)$, and $Y_3(t)$, are updated using recursive calculations, without having to retain the historical choices of the strategy parameter or the corresponding one-period profit observations

- The recursive calculations involve a 3 x 3 covariance matrix, denoted V_t, which measures the uncertainty/accuracy of the recursive estimates

Recursive Equation for the Covariance Matrix

$$V_{t+1} = V_t + \Sigma - L_t,$$

where

- $L_t = (C_t V_t)^T (C_t V_t) / [(\sigma_N)^2 + C_t V_t (C_t)^T]$
- C_t is the row-vector $[(u_t)^2, u_t, 1]$
- Σ is the diagonal matrix having diagonal elements $(\sigma_1)^2$, $(\sigma_2)^2$, and $(\sigma_3)^2$
- σ_N is the standard deviation of the Normally distributed noise N
- This equation gives some insight on how the choice of *u* affects the rate of learning
- By choosing *u* to maximize a diagonal element of L_t one maximizes the rate one learns the true value of the corresponding *X*
- Note these functions of *u* are ratios of two polynomials, each of which has degree 4
- But we also want to maximize profits, so we have a trade-off

Implementing the Trade-Off: Do Like Rolling Markowitz

- For scalar $\lambda > 0$, each period choose u to maximize

$$E[P_t] + \lambda \mathbf{1}L_t\mathbf{1^T} = Y_1(t)u^2 + Y_2(t)u + Y_3(t) + \lambda \mathbf{1}L_t\mathbf{1^T}$$

  where $\mathbf{1}$ denotes a 3-dimensional row vector of ones

- By adjusting λ we can control the trade-off between learning and short-run profits
- This model/system has been evaluated with simulation and seems to converge nicely for realistic cases
- (But the traders remain skeptical….)

Concluding Remarks

- The subject area pertaining to electronic market making is a rich, fertile ground for academic research
- Desperately needed are good statistical models for forecasting bid and ask prices (a two-dimensional process)
  - very high frequency data (and many different kinds)
  - not discrete time with evenly spaced time points
  - suitable for real-time implementation
- Also needed are good algorithms for posting quotes
  - must be profitable and deal with risks
  - suitable for real-time implementation
  - ideally, optimization techniques can be devised for turning good algorithms into better ones

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