1. Ex 36 Here $Z_t = JN_t - Jct$ N_t Poisson with rate c.

$$d\lambda_t = (k(\theta-\lambda) - Jc)dt + J dN_t$$

Generator: $[Af](\lambda) = (k(\theta-\lambda) - Jc)\partial_\lambda f + c(f(\lambda+J) - f(\lambda))$

Feynman-Kac implies $f(s,t,\lambda) := E[e^{-\int_s^t \lambda_u du} | \mathcal{G}_s]$

solves
$$\begin{cases} \partial_s f + Af - \lambda f = 0 & s<t \\ f(t,t,\lambda) = 1 \end{cases}$$

Put $f = \exp[A(s,t) + B(s,t)\lambda]$ and plug in:

$$(\partial_s A + \partial_s B\lambda)f + (k(\theta-\lambda) - Jc)Bf + cf[e^{BJ} - 1] - \lambda f = 0$$

$$\Rightarrow \begin{cases} \partial_s A = (Jc - k\theta)B + c[1-e^{BJ}] & , A(t,t) = 0 \\ \partial_s B = kB - 1 & , B(t,t) = 0 \end{cases}$$

[8] Note $P[\tau > t | \mathcal{F}_s] = H_s^c f(s,t,\lambda_s)$. (by Lando formula).

2. Ex 37 (5.31): apply $E[\cdot]$ to get

$$d\mu(t) = k(\theta - \mu(t))dt \quad , \quad \mu(t) = E[\lambda(t)]$$

[2] As $t \to \infty$ $\mu(t) \to \theta$ if $k>0$.

MATLAB = [4]

4. Ex 44 Price at time $t<T := H_t^c \tilde{V}_t$

$$= E^Q[H_T^c e^{-\int_t^T r_s ds} V_T | \mathcal{F}_t]$$ Risk Neutral Pricing

$$= H_t^c E^Q[e^{-\int_t^T r_s ds} \cdot V_T \cdot e^{-\int_t^T \lambda_s ds} | \mathcal{G}_t]$$ by Prop'n 5.3.1 #3 with $Y = e^{-\int\lambda} \cdot V$.

[3]

* NUMBERING GOT SCREWED UP * similar soln for #43 (2)

4. Ex 45 If $\tau \le T$, contract pays you $(\tilde{V}_\tau)^-$

which is of course negative.

Here $\tilde{V}_t$ is the predefault value of swap (to you).

If $\tau > T$, contract pays V_T at time T.

RN valuation for $t < T$ gives

$$H_t^c \cdot \tilde{V}_t = E^Q\left[e^{-\int_t^T r_u du} V_T H_T^c + e^{-\int_t^\tau r_u du} (\tilde{V}_\tau)^- (H_t^c - H_T^c) \,\Big|\, \mathcal{F}_t \right]$$

$$= H_t^c \, E^Q\left[e^{-\int_t^T (r_u + \lambda_u) du} V_T + \int_t^T (\tilde{V}_u)^- \lambda_u e^{-\int_t^u (r_v + \lambda_v) dv} \,\Big|\, \mathcal{G}_t \right]$$

[5] by Prop'n 5.3.1 #3 and #4. (note we need $\tilde{V}$ $\mathcal{G}$-meas).

5. Value at $t = 0 = 1 = \left(\frac{c^*}{2}\right) \sum_{i=1}^{40} E^Q\left[e^{-\int_0^{i/2} r_u du} \cdot 1_{\{\tau > i/2\}} \,\Big|\, \mathcal{F}_0 \right]$

$$+ E^Q\left[e^{-\int_0^{20} r_u du} 1_{\{\tau > 20\}} \,\Big|\, \mathcal{F}_0 \right]$$

[3] $$+ R \int_0^{20} E^Q\left[e^{-\int_0^t r_u du} (H_0^c - H_T^c) \,\Big|\, \mathcal{F}_0 \right]$$

where $\left(\frac{c}{2}\right)$ is the twice yearly coupon.

c^* obtained by solving.

Default-free case

[2] $$1 = \left(\frac{c}{2}\right) \sum_{i=1}^{40} E^Q\left[e^{-\int_0^{i/2} r_u du} \right] + E^Q\left[e^{-\int_0^{20} r_u du} \right]$$

Expect $c^* - c \sim (1-R) \times$ "average λ".

Computations are based on

$$\overline{P}_0(t) = E\left[e^{-\int_0^t (r+\lambda)} \right] = E\left[e^{-\int_0^t [(a+c)X_s + (b+d)Y_s]\,ds} \right]$$

(NOTE X, Y are independent!)

[2] $$= F^{VAS}(t; X_0; a+c, 0) \times F^{VAS}(t, Y_0; b+d, 0)$$

X parameters Y parameters

Similarly $P_0(t) = F^{VAS}(t, X_0; a, 0) \times F^{VAS}(t, Y_0; b, 0)$

[1]

Also: Recovery term

$$= R \int_0^{20} E\left[(cX_t + dY_t)\, e^{-\int_0^t ((a+c)X_s + (b+d)Y_s)\,ds} \right] dt.$$

Note: $E\left[X_t e^{-\int_0^t aX_s\,ds}\right] = -\partial_{c_2} F^{VAS}(t, X_0; a, c_2)\Big|_{c_2=0}$

$$= -\frac{1}{a}\partial_t F^{VAS}(t, X_0; a, 0).$$

Riemann sum approximation with $\Delta t = \frac{1}{2}$ should be OK.

[2] $$\sim \frac{R}{2} \sum_{i=1}^{40} \Big[c\left(-\partial_{c_2} F^{VAS}\right)(t_i, X_0; a+c, 0) \times F^{VAS}(t, Y_0; b+d, 0) + d\, F^{VAS}(t, X_0; a+c, 0) \times \left(-\partial_{c_2} F^{VAS}\right)(t, Y_0; b+d, 0) \Big]$$

Admittedly this quite a mess! USE MAPLE or Mathematica?

MORAL: It requires care + determination + MAPLE to correctly implement this kind of model!

[4]

6. Same as above, but replace $N=40$ by $N=1,2,\ldots,60$. Plot c^* vs $N/2$.

[6]

7. 1. • For $i \neq j$ $\Lambda_{ij}(s) = \lim_{t \downarrow s} \frac{\Pi_{ij}(s,t)}{t-s} \geq 0$

since $\Pi_{ij}(s,t) \geq 0$ all $i, j, s < t$.

• since $\Pi_{ii}(s,t) + \sum_{j \neq i} \Pi_{ij}(s,t) = 1$ for all i, $s<t$

For all i: $\Lambda_{ii}(s) = \lim_{t \downarrow s} \frac{\Pi_{ii}(s,t)-1}{t-s} = -\sum_{j \neq i} \lim_{t \downarrow s} \frac{\Pi_{ij}(s,t)}{t-s}$

$= -\sum_{j \neq i} \Lambda_{ij}(s)$ (from above).

$\therefore \Lambda_{ij}$ is stochastic matrix (according to eqns 6.26, 6.27)

[3]

2. Note that $\sum_{j=0}^{K} \Pi_{ij}(s,u)\Pi_{jk}(u,t) = \Pi_{ik}(s,t)$.

for all i k and $u \in (s,t)$.

Thus $\lim_{h \to 0^+} \frac{\Pi_{ik}(s,t+h) - \Pi_{ik}(s,t)}{h}$

$= \lim_{h \to 0^+} \frac{1}{h}\left(\sum_j \left(\Pi_{ij}(s,t)\Pi_{jk}(t,t+h) - \Pi_{ij}(s,t)\delta_{jk} \right)\right)$

$= \sum_j \Pi_{ij}(s,t) \lim_{h \downarrow 0} \frac{1}{h}\left(\Pi_{jk}(t,t+h) - \delta_{jk}\right)$

[3] $= \sum_j \Pi_{ij}(s,t)\Lambda_{jk}(t)$ as wanted

7. 3. [2]

$$\frac{\partial}{\partial t}\Pi(s,t) = B\, e^{\int_s^t \mu(u)\,du} \cdot \mu(t)\, B^{-1}$$

$$= \left(B\, e^{\int_s^t \mu(u)\,du} B^{-1}\right)\left(B \cdot \mu(t) \cdot B^{-1}\right)$$

$$= \Pi(s,t)\, \Lambda(t) \qquad \text{as wanted}$$

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