FIELDS FOLLOWUP WORKSHOP
MAY 30–JUNE 4
PROGRAM AND ABSTRACTS

1. Schedule

time	M	T	W	Th	F
9:30	Coffee	Coffee	Coffee	Coffee	Lacey*
10:30	Sawyer	Iosevich	Guth	Lyall	Uriate-Tuero
11:30-2pm	Lunch	Lunch	Łaba	Lunch	Bond*
2	Pramanik	Hamel		Costea	
3	Ou	Zhai		Shen	
3:30	Break	Break		Break	
4	Wick	Koh		Bilyk	

Michael Lacey will lecture at 9 am on Friday. Break will be from 10 to 10:30
Matthew Bond will give a one hour talk.

2. Abstracts

Dmitriy Bilyk, University of South Carolina

Title: Directional discrepancy

It is well-known in the theory of irregularities of distribution that the geometry of the underlying sets plays an important role. In particular, if one considers the discrepancy with respect to axis-parallel rectangles, it depends on the number of points logarithmically, while for arbitrarily rotated rectangles these estimates are polynomial. We shall present an attempt to understand the delicate correlation between the asymptotic behavior of the discrepancy and the structure of the set of rotations. This is joint work with Xiaomin Ma, Jill Pipher, and Craig Spencer.

Matthew Bond, Michigan State University

Title: How likely is Buffon's needle to land near a 1-dimensional Sierpinski gasket? A power estimate via Fourier analysis.

Abstract: It is well known that a needle thrown at random has zero probability of intersecting any given irregular planar set of finite 1-dimensional Hausdorff measure. Sharp quantitative estimates for small open neighborhoods of such sets are still not known, even for such sets as the Sierpinski gasket and the 4-corner Cantor set (with self-similarities $1/4$ and $1/3$). In 2008, Nazarov, Peres, and Volberg provided the sharpest known upper bound for the 4-corner Cantor set. Volberg and I have recently used the same ideas to get a similar estimate for the Sierpinski gasket. Namely, the probability that Buffon's needle will land in a 3^{-n}-neighborhood of the Sierpinski gasket is no more than C_p/n^p, where p is any small enough positive number.

Serban Costea, McMaster University

Title:Corona Theorems for Multiplier Algebras on $\mathbb{B}_n$

Carleson's Corona Theorem from the 1960's has served as a major motivation for many results in complex function theory, operator theory and harmonic analysis. In a simple form, the result states that for $N \geq 2$ bounded analytic functions, $f_1, \dots, f_N$ on the unit disc with no common zeros in a quantitative sense, it is possible to find N other bounded analytic functions, $g_1, \dots, g_N$ such that $f_1 g_1 \dots f_N g_N = 1$. Moreover, the functions $g_1, \dots, g_N$ can be chosen with some norm control. In this talk we will discuss some new generalizations of this result to certain function spaces on the unit ball in several complex variables. In particular, we will highlight the Corona Theorem for the Drury-Arveson space and for the space of BMO analytic functions. This is joint work with Eric T. Sawyer and Brett D. Wick.

Larry Guth, University of Toronto

Multilinear Kakeya estimates and Dvir's polynomial method

Abstract: In 2007, Dvir proved the Kakeya conjecture over finite fields. The proof is surprisingly short. The key idea is the polynomial method: finding a polynomial that vanishes on a hypothetical bad set. This proof has not so far led to a proof of the Kakeya conjecture in Euclidean space, and it looks very difficult to make the transition. The best progress to date is a new proof of the Bennett-Carbery-Tao multilinear Kakeya inequality in Euclidean space, based on Dvir's method.

Alex Iosevich, University of Rochester

Title: A sharpness example for the Falconer estimate and connections with geometric incidence theory

Abstract: In 1985 Falconer proved that if the Hausdorff dimension of E is greater than $\frac{d+1}{2}$, then the Lebesgue measure of the distance set is positive. He did that by proving that

$$(1) \qquad \mu \times \mu\{(x, y) : 1 \leq |x - y| \leq 1 + \epsilon\} \lesssim \epsilon$$

for any Borel measure μ supported on E with

$$I_{\frac{d+1}{2}}(\mu) = \int\int |x - y|^{-\frac{d+1}{2} - \epsilon} d\mu(x) d\mu(y) < \infty .$$

Mattila proved that in two dimensions for no $s < \frac{3}{2}$ does $I_s(\mu) < \infty$ imply (1). His construction can be extended to three dimensions but appears to break down in higher dimensions. We are going to construct an Ahlfors-David regular example in all dimensions showing that for no $s < \frac{d+1}{2}$ does $I_s(\mu) < \infty$ imply (1). Our example is based on a combinatorial construction due to Pavel Valtr and naturally leads us into consideration of certain geometric incidence problems.

Doo Won Koh, Michigan State University

Title: Harmonic analysis related to homogeneous varieties in three dimensional vector spaces over finite fields.

Abstract: we study the boundedness of the extension operators and the averaging operators associated with arbitrary homogeneous varieties in three dimensional vector spaces over finite fields. In the case when homogeneous varieties in three dimension do not contain any plane, we obtain the generally best possible results on aforementioned two problems. In particular, our results on extension problems recover and generalize the work due to Mockenhaupt and Tao who studied the conical extension problems in three dimension and provided us of the complete answer. Investigating the Fourier decay on homogeneous varieties, we give the complete mapping properties of averaging operators on homogeneous varieties in three dimension. Joint work with Chun-Yen Shen.

Mariah Hamel, University of Georgia

Title: Sumsets of dense subsets of primes

Let A be a subset of the primes with positive relative density δ. In this talk we will show that the sumset $A + A$ must have positive upper density $C_1 \delta e^{-C_2 (\log(1/\delta))^{2/3} (\log\log(1/\delta))^{1/3}}$ in the integers. Our argument uses modifications of the Fourier analytic techniques developed by Green and

Green-Tao to find arithmetic progression in the primes, together with a result on sums of subsets of the multiplicative subgroup of the integers modulo M. Joint work with Karsten Chipeniuk.

Izabella Łaba, University of British Columbia

Title: Maximal operators and differentiation theorems in sparse sets.

Abstract: We study maximal averages associated with singular measures on $\mathbf{R}$. Our main result is a construction of singular Cantor-type measures in one dimension for which the corresponding maximal operator are bounded on L^p for all $p > 1$. As a consequence, we answer a question of Aversa and Preiss on density and differentiation theorems in one dimension. Our proof combines probabilistic techniques with the methods developed in multidimensional harmonic analysis. (Joint work with Malabika Pramanik.)

Michael T. Lacey, Georgia Institute of Technology

Title: Recent results on two weight inequalities for singular integrals

Abstract: We will survey recent results on two weight inequalities for singular integrals, namely inequalities of the form

$$\|T(uf)\|_{L^2(v)} \leq C\|f\|_{L^2(u)}$$

for two weights u, v, and singular integral T. We will discuss (1) characterizations of the weak-type inequalities for the maximal truncations (2) L^p characterizations, when one weight is doubling (3) and new sufficient conditions for the L^2 inequality. Joint work with Eric Sawyer and Ignacio Uriate-Tuero, among others.

Neil Lyall, University of Georgia

Title: On the density of sets with no square differences

Abstract: We propose to give an exposition of the best-known bound (established by Pintz, Steiger and Szemerédi) on the density of sets whose difference set contains no squares.

Winston Ou, Scripps College Title: Structure results on A_∞ Abstract:

We present some results on how knowledge of the structure of the limiting Muckenhoupt weight classes (A_1 and RH_∞) simplifies Jones (-Cruz-Uribe–Neugebauer) factorization; also some work-in-progress (with Slavin and Wall) on using the Bellman technique to get sharp control of the Hardy-Littlewood maximal operator's behavior on A_∞.

Malabika Pramanik, University of British Columbia (May 31 or June 1)

Title: A multi-dimensional resolution of singularities and applications

Abstract: We discuss an algorithm for analyzing the zero sets of real-analytic functions in all dimensions. Applications include the computation of the critical integrability index of real-analytic functions, sublevel set estimates and oscillation index of scalar oscillatory integrals with real-analytic phases. This is joint work with Tristan Collins and Allan Greenleaf.

Eric T Sawyer, McMaster University
Two weight inequalities for singular integrals: L^p results

Chun-Yen Shen, Indiana University

Title : Expanding phenomena in fields

Abstract : We investigate the sum-product phenomena in fields, in which in the finite field setting we use Fourier analytic methods to characterize the polynomials so that when a set behaves like an arithmetic progression, then the size of its image under the polynomials is large. In the real setting, we use algebraic methods to derive the same result. If time permits, we will also discuss the problems of two variables expanding maps.

Ignacio Uriate-Tuero, Michigan State University

Title: Astala's conjecture on Hausdorff measure distortion under planar quasiconformal mappings and related removability problems

Abstract: In his celebrated paper on area distortion under planar quasiconformal mappings (Acta 1994), Astala proved that if E is a compact set of Hausdorff dimension d and f is K-quasiconformal, then fE has Hausdorff dimension at most $d' = \frac{2Kd}{2+(K-1)d}$, and that this result is sharp. He conjectured (Question 4.4) that if the Hausdorff measure $\mathcal{H}^d(E) = 0$, then $\mathcal{H}^{d'}(fE) = 0$. This conjecture was known to be true if $d' = 0$ (obvious), $d' = 2$ (Ahlfors), and $d' = 1$ (Astala, Clop, Mateu, Orobitg and UT, Duke 2008.) The approach in the last mentioned paper does not generalize to other dimensions.
UT showed that Astala's conjecture is sharp in the class of all Hausdorff gauge functions (IMRN, 2008).

Lacey, Sawyer and UT jointly proved completely Astala's conjecture in all dimensions (Acta, 2009?) The proof uses Astala's 1994 approach, geometric measure theory, and new weighted norm inequalities for Calderón-Zygmund singular integral operators which cannot be deduced from the classical Muckenhoupt A_p theory.
These results are intimately related to removability problems for various classes of quasiregular maps. I will particularly mention sharp removability results for bounded K-quasiregular maps (i.e. the quasiconformal analogue of the classical Painleve problem) recently obtained jointly by Tolsa and UT.

Brett Wick, Georgia Institute of Technology

Title: Bergman-Type Singular Integral Operators and Non-Homogeneous Harmonic Analysis

Abstract: In this talk we will discuss how the method of non-homogeneous harmonic analysis of Nazarov, Treil and Volberg can be extended to handle "Bergman–type" singular integral operators. As an application of these techniques, we illustrate how these methods can be used to study the Carleson measures for the Besov–Sobolev space of analytic functions B_2^σ on the complex ball of $\mathbb{C}^d$. In particular, we demonstrate that the Carleson measures for the space are characterized by a "T1 Condition" arising in the problem. This talk is based on joint work with A. Volberg.

Kelan Zhai, University of British Columbia

Title: Favarad Length of Cantor Type Sets Abstract For a two dimensional

set E, the Favard length is defined as the integral over all directions of the its projection onto lines. Nazarov, Peres and Volberg proved that the Favard length of the n-th iteration of the four-corner Cantor set is bounded from above by n^c for an appropriate c. In a joint work with Izabella Łaba, we generalize this result to all product Cantor sets whose projection in some direction has positive 1-dimensional measure.