# ARITHMETIC AND GEOMETRY OF ALGEBRAIC VARIETIES
# WITH SPECIAL EMPHASIS ON
# CALABI–YAU VARIETIES AND MIRROR SYMMETRY
# NOVEMBER 6–7, 2010

## ABSTRACTS

**Patrick Brosnan** (University of British Columbia)

### Split nilpotent orbits from the Tannakian viewpoint

It is a classic observation of Deligne that the category of split real mixed Hodge structures is equivalent to the category of representations of the Weil restriction $S$ of the multiplicative group of the complex numbers to the reals. The category of all real mixed Hodge structures has a slightly more complicated description, also due to Deligne: it is the category of finite dimensional representations of an affine group scheme which is an extension of the group $S$ by a prounipotent group $U$ whose associated Lie algebra is free on one one generator in each bidegree $(p, q)$ with $p, q < 0$. The talk will be about joint work with G. Pearlstein proving an analogue of Deligne's result to one-variable nilpotent orbits whose limits are split over the reals. Our original motivation for this was to understand another (unpublished) result of Deligne which is used in our work on the zero locus of admissible normal functions.

**Swantje Gahrs** (University of Hannover and the Fields Institute)

### Picard-Fuchs equations of invertible polynomials

In this talk I will deal with special one-parameter families of invertible polynomials. The goal is to compute the Picard-Fuchs equations for those families. For this purpose I will explain how to do this using the Griffiths-Dwork method. In the end I will state a conjecture how one can derive the Picard-Fuchs equation from the Bergland-Hubsch transpose of the polynomial.

**Eugene Ha** (The Fields Institute)

### C*-Algebras and the cohomology of Arakelov line bundles

Following the work of Roessler, Morishita, van der Geer-Schoof, and Borisov on the cohomological implications of Tate's Riemann-Roch theorem, we construct C*-algebras for the cohomology of an Arakelov line bundle. In this framework, the van der Geer-Schoof (real-valued) order of arithmetic cohomology is naturally recovered as the trace of a regular representation. There is natural connection between the heat equation and the C*-algebra for the first cohomology.

**Su-Jeong Kang** (Providence College)

### Kähler-de Rham cohomology and Chern classes

Any codimension $p$ subvariety of a complex smooth projective variety $X$ gives rise to an algebraic cycle in homology as well as in cohomology by Poincaré duality. Via the isomorphism $\mathrm{CH}^*(X) \otimes \mathbf{Q} \cong \mathbf{K^0(X)} \otimes \mathbf{Q}$, the space spanned by algebraic cycles coincides with the space spanned by Chern classes, and the classical Hodge conjecture tells us to view this space as the space of Hodge cycles. For a singular variety, by a natural Hodge conjecture, formulated by Lewis and Jennsen, we can also view the space of algebraic cycles in homology as the space of Hodge cycles. However, it is not clear how corresponding conjecture for Chern classes should be. In this talk, I would like to talk about constraints on Chern classes of vector bundles on a singular algebraic variety, which are stronger than obvious Hodge theoretical constraints. A part of this talk is based of recent work with Donu Arapura.

**James Lewis** (University of Alberta)

**Hodge Type Conjectures and the Bloch-Kato Theorem**

Let $U/\mathbf{C}$ be a smooth quasiprojective variety, $\mathrm{CH}^r(U,m)$ Bloch's higher Chow group, and $\mathrm{cl}_{r,m}$ : $\mathrm{CH}^r(U,m)\otimes\mathbf{Q}\to\hom_{\mathrm{MHS}}\big(\mathbf{Q}(0),H^{2r-m}(U,\mathbf{Q}(r))\big)$ the cycle class map. Beilinson once conjectured $\mathrm{cl}_{r,m}$ to be surjective; however Jannsen was the first to find a counterexample in the case $m=1$. In this talk I will explain the image of $\mathrm{cl}_{r,m}$ in more detail (as well as at the generic point) in terms of kernels of Abel-Jacobi mappings, and explain the impact of the Bloch-Kato theorem on the cycle class map at the generic point, in the case $r=m$. This talk is based on joint work with Rob de Jeu.

**Karol Palka** (UQAM)

**Open algebraic surfaces and embeddings of rational curves**

We discuss connections between embeddings of complex rational curves into the projective plane and the geometry of the complement. We present recent partial classification of embeddings of the punctured affine line into the affine plane and their connections with the Coolidge-Nagata Conjecture.

**Greg Pearlstein** (Michigan State Univeristy)

**The locus of Hodge classes in an admissible variation of mixed Hodge structure**

Some of the deepest evidence to date in favor of the Hodge conjecture is a result of Cattani, Deligne and Kaplan which gives an affirmative answer to the following question of A. Weil: "Does imposing a Hodge class upon the generic member of an algebraic family of polarized algebraic varieties amount to an algebraic condition on the parameters"? In this talk, I will discuss an extension of the work of Cattani, Deligne and Kaplan to the Hodge loci of admissible variations of mixed Hodge structure.

**Valdemar Tsanov** (Queen's University)

**Triangular groups, cusp forms, and torus knots**

This talk will be about the relation between several classical and well known objects: triangular Fuchsian groups, $\mathbf{C}^*$-equivariant singularities of plane curves, torus knot complements in the 3-sphere. Torus knots are the only nontrivial knots whose complements are homogeneous, i.e. admit a transitive Lie group action. In fact $\mathbf{S}^3\setminus K_{p,q}$ is diffeomorphic to a coset space $\widetilde{SL_2}(\mathbf{R})/G$, where $\widetilde{SL_2}(\mathbf{R})$ is the universal covering group of $PSL_2(\mathbf{R})$ and $G$ is a discrete subgroup contained in the preimage of a $(p,q,\infty)$-triangular Fuchsian group $\Gamma$. The existence of a diffeomorphism is known $\widetilde{SL_2}(\mathbf{R})/G$ and $\mathbf{S}^3\setminus K_{p,q}$ from a general topological classification of Seifert fibred 3-manifolds. Our goal is to construct an explicit diffeomorphism using automorphic forms. Such a construction is known for the trefoil knot $K_{2,3}$ and in fact we have $\mathbf{S}^3\setminus K_{2,3}\cong SL_2(\mathbf{R})/SL_2(\mathbf{Z})$. The connection between the two sides of the diffeomorphism comes via singularities of plane curves.

Most of the concepts involved in this construction are known. Our task is to bring all the elements together and be as explicit as possible about this beautiful example. It is interesting to observe the manifestation of various knot theoretic notions in the analytic setting, and vice versa.

**Noriko Yui** (Queen's University)

**The modularity of certain K3-fibered Calabi-Yau threefolds over Q**

We consider certain K3-fibered Calabi-Yau threefolds defined over $\mathbf{Q}$. These Calabi-Yau threefolds are constructed using the method of Voisin and Borcea, and are realized as smooth resolutions of quotients of $S\times E$ by some involution. (Here $S$ is an algebraic $K3$ surface and $E$ is an elliptic curve.)

First we will discuss the modularity of $K3$ surfaces $S$. We look into the famous 95 families of $K3$ surfaces found by Reid and Yonemura. Among them, we will pick $K3$ surfaces with involution. Our first result is to show that some of these $K3$ surfaces are of CM type.

Next, we will discuss the modularity of Calabi-Yau threefolds over $\mathbf{Q}$ obtained from products $S \times E$. We establish the modularity (automorphicity) of some of these Calabi-Yau threefolds and also thier mirror partners (if exist), in the sense of Arthur and Clozel. Several explicit examples are discussed.

This reports on a joint work in progress with Y. Goto (Hakodate) and R. Livne (Jerusalem).