HADLEY CELL EXPANSION IN TODAY'S CLIMATE AND PALEOCLIMATES

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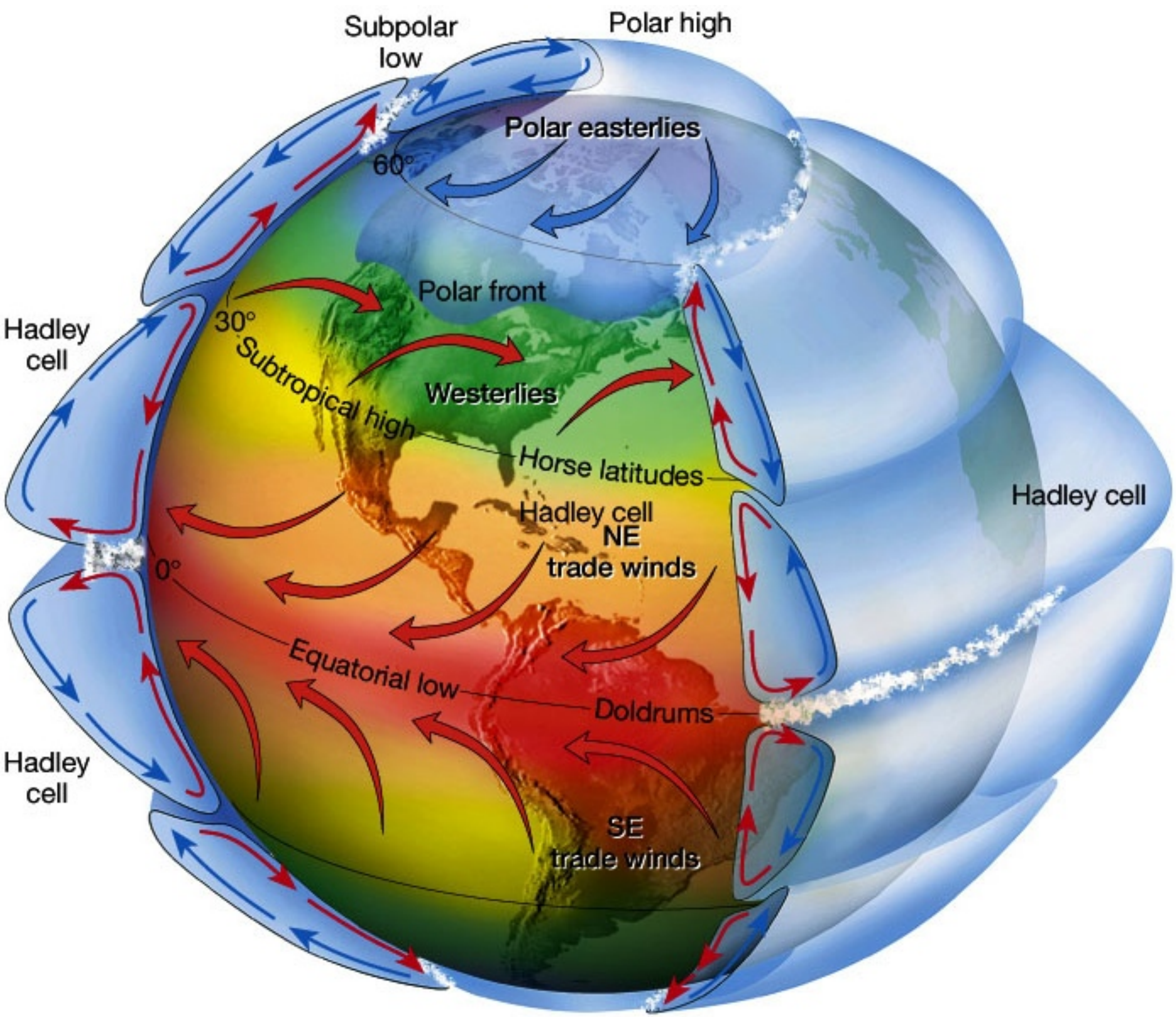
OUTLINE

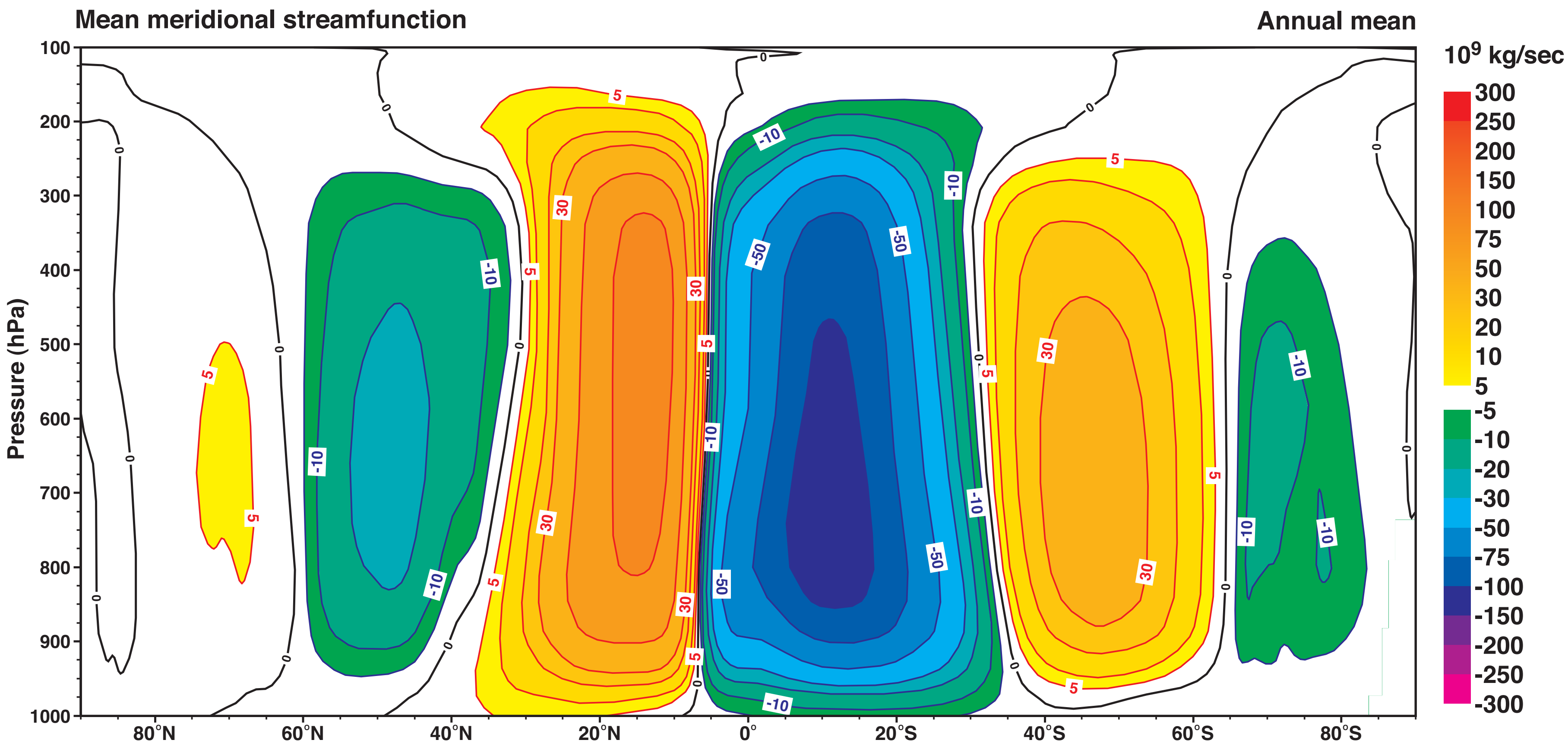
- Present-day climate changes
- Greenhouse and icehouse climate modes
- Questions from paleoclimatology
- Mathematical model of Hadley convection
- Numerical analysis by Greg Lewis
- Implications of the model
- The Pliocene Paradox and future work

Mean Circulation of Today's Atmosphere



Meridional Streamfunction 1979-2001: Annually and Longitudinally Averaged

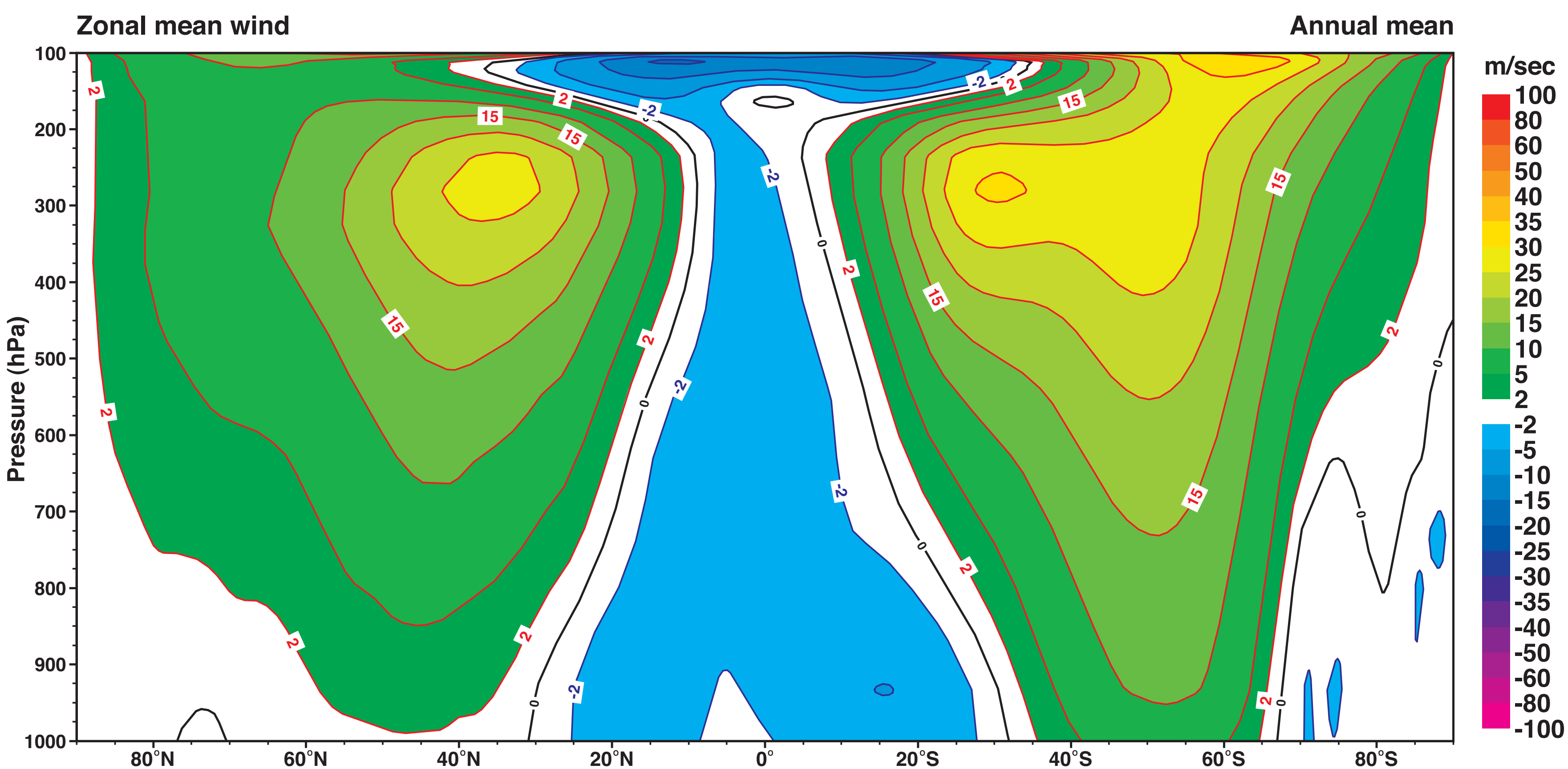
[European Centre for Medium Range Weather Forecasts]



Note: Hadley, Ferrel and Polar Cells.

Zonal (Longitudinal) Wind 1979-2001: Annually and Longitudinally Averaged

[European Centre for Medium Range Weather Forecasts]



Note: Trade Winds and Jet Streams.

PRESENT-DAY CLIMATE CHANGES

Intergovernmental Panel on Climate Change
[IPCC Report 2007 and beyond]

- The mean global temperature is rising.
- The poles are warming faster than the tropics.
- More rain in equatorial regions, less in subtropics.
- The Hadley cells are expanding poleward.
- The Hadley circulation is slowing.
- The jet streams are moving poleward.

Understanding the Causes of Today's Climate Change

- Mean global warming is believed to be driven by greenhouse gas buildup (anthropogenic).
- Enhanced polar warming is caused by positive feedbacks, such as a decrease in albedo due to the melting of ice caps.
- There is no consensus on the causes of the changes in Hadley cells and jet streams.

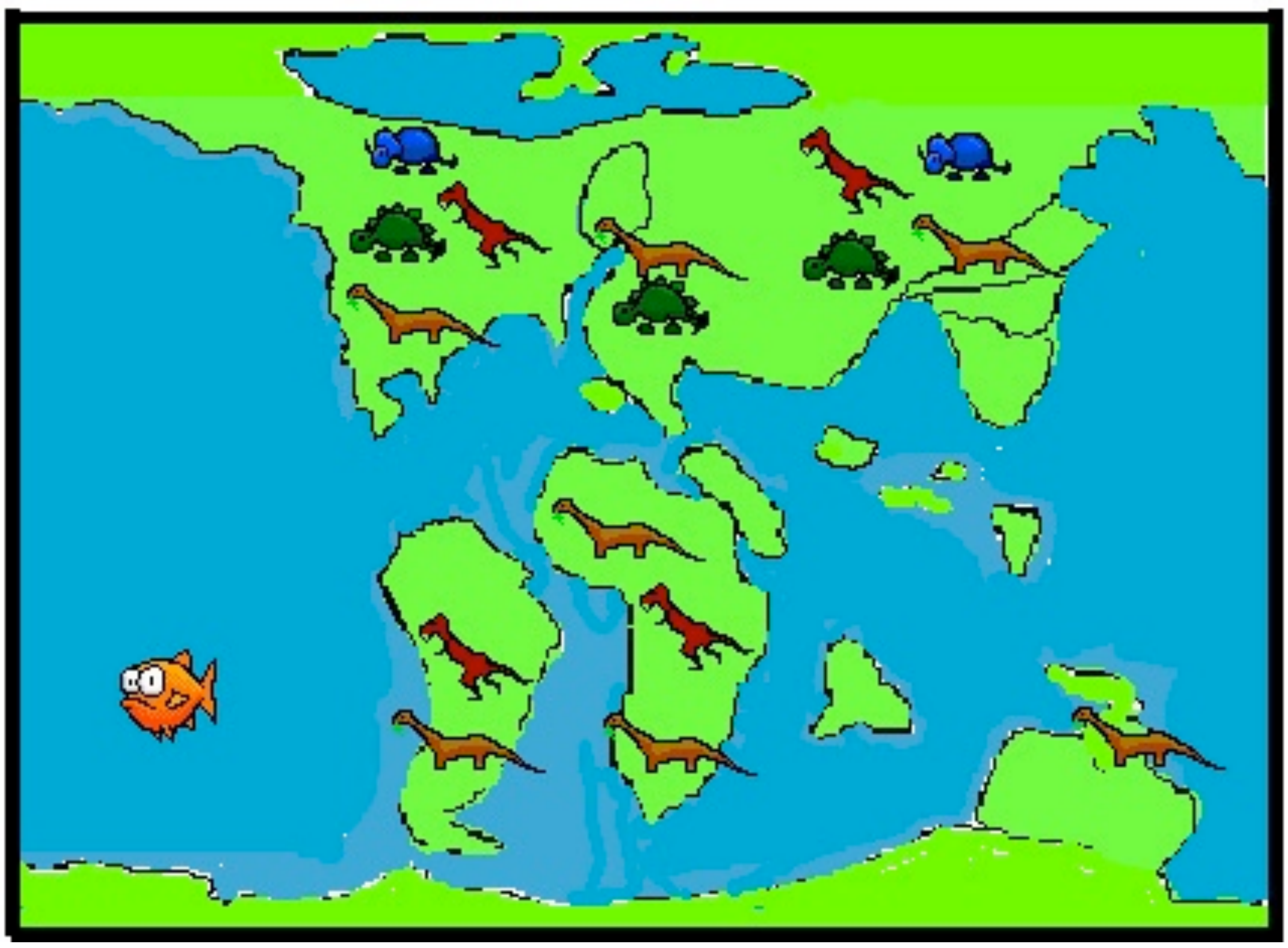
PALEOCLIMATES

- The Earth has experienced dramatically different climate "modes" in its geological history.
- The two dominant modes of the past half-billion years are often called "Greenhouse Climate" and "Icehouse Climate".
- Better knowledge of *paleoclimate changes* will help us to understand modern day climate changes, and vice-versa.

Greenhouse Climate Mode

- Mean annual temperature (MAT) was a few degrees warmer than today. (But *means* can mislead.)
- The global climate was more EQUABLE than today. *Equable climate* means:
    1. Warmer winters without much warmer summers; i.e. low seasonality.
    2. Low temperature gradient, pole-to-equator.
- Most of the higher MAT is due to the warmer winters and warmer polar regions.

Greenhouse Climate dominated the Mesozoic "*Age of Dinosaurs*" 240-65 Mya



Icehouse Climate Mode

- Permanent polar icecaps (all year).
- Large pole-to-equator temperature gradient.
- Cold winters and warm/hot summers for mid-latitudes of Earth; i.e. high seasonality.
- Equatorial region has climate similar to that in the greenhouse climate mode.
- This "icehouse climate mode" has been dominant only in the past 30 million years.

Outstanding Questions of Paleoclimatology

- How can such different but stable climate modes both exist on the same Earth?
- Why has the Earth "preferred" greenhouse to icehouse climate for most of 250 My?
- What has caused abrupt changes between greenhouse and icehouse climate modes?

Our Mathematical Model

Question

Can the mean behaviour of the Hadley cells and atmospheric flow be replicated in a simple model based on convection, rotation and spherical geometry ?

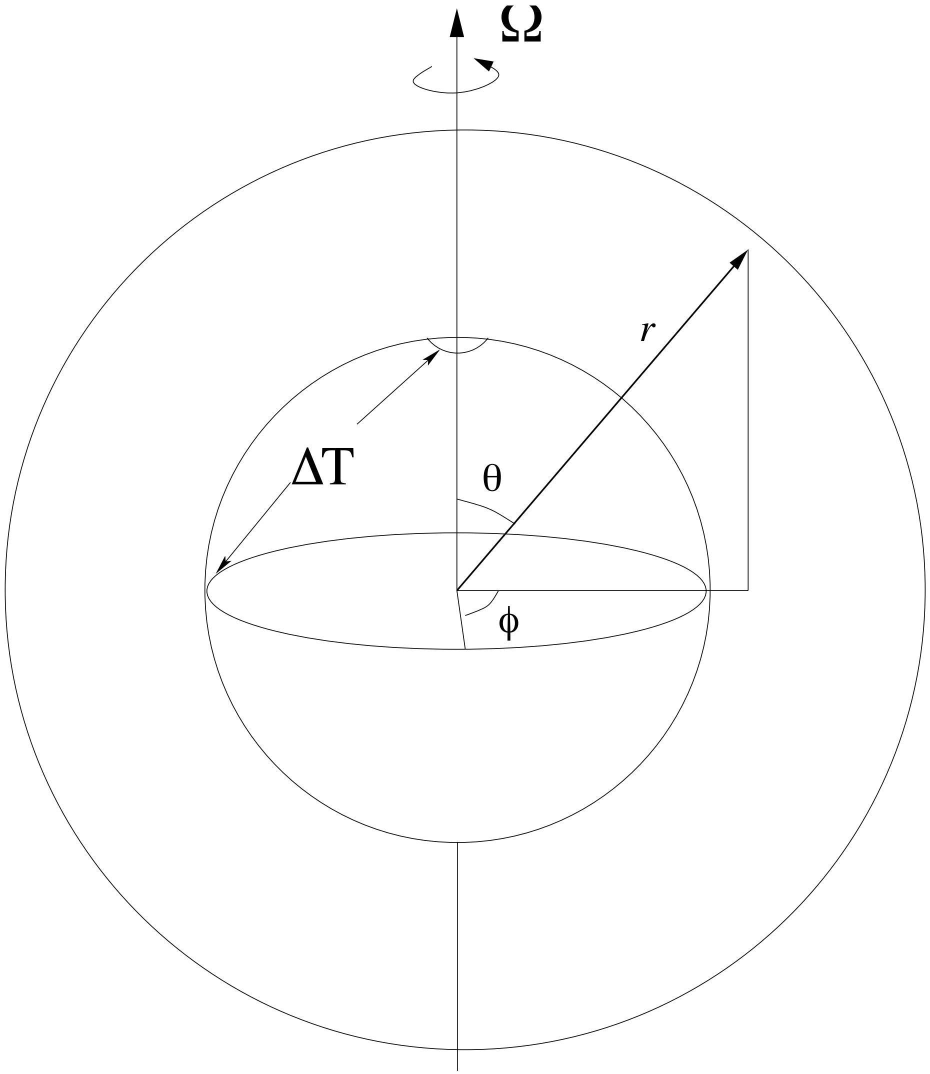
Basic Components of the Mathematical Model

- Navier-Stokes PDE in rotating spherical shell.
- Boussinesq fluid [density varies linearly with temperature].
- Incompressible fluid for convenience.
- Convection is driven by the latitudinal temperature gradient on the inner boundary.

Rotating spherical shell of fluid

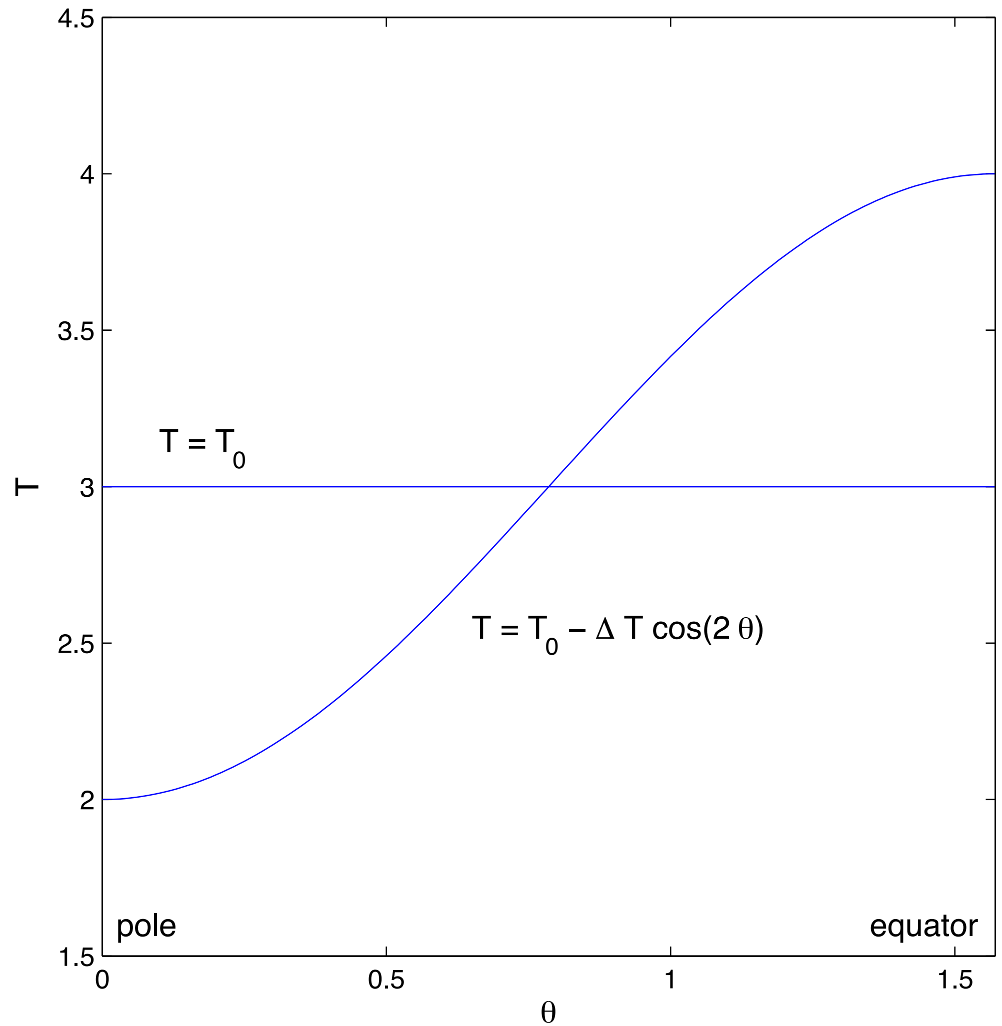
differentially heated on the interior boundary

Assume rotational symmetry and north-south reflectional symmetry.



Averaged solar heating
of the rotating tilted earth

Fix this as the boundary temperature on the surface of the Earth.



Navier-Stokes Boussinesq Equations

$$
\begin{aligned}
\frac{\partial \mathbf{u}}{\partial t} &= \nu \nabla^2 \mathbf{u} - 2\boldsymbol{\Omega} \times \mathbf{u} + \left[g \mathbf{e}_r + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})\right] \alpha \left(T - T_r\right) - \frac{1}{\rho_0} \nabla p - \left(\mathbf{u} \cdot \nabla\right) \mathbf{u}, \\
\frac{\partial T}{\partial t} &= \kappa \nabla^2 T - \left(\mathbf{u} \cdot \nabla\right) T, \\
& \nabla \cdot \mathbf{u} = 0,
\end{aligned}
$$

where $\mathbf{u}$ is the velocity vector, T is temperature, $\boldsymbol{\Omega}$ is the rotation vector, p is pressure deviation, ν is kinematic viscosity, κ is thermal diffusivity, g is gravitational acceleration and ∇ is the gradient operator.

Here $\left[g \mathbf{e}_r + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})\right] \alpha \left(T - T_r\right)$ is the buoyancy force.

Boundary Conditions

- Inner Boundary
  - T varies with $-\Delta T \cos(2\theta)$
  - Non-slip BC for velocity u
- Outer Boundary
  - T is insulated
  - Stress-free BC for velocity u

Numerical Analysis

- Discretize on NxN grid.
- Get $3N^2 + 5N$ sparse nonlinear equations.
- Solve nonlinear system by Newton iteration.
- Use Keller continuation from the trivial solution, which is known exactly at $\Delta T = 0$.

Bifurcation Analysis

- Use the temperature difference ΔT as both bifurcation parameter and Keller continuation parameter.
- Monitor eigenvalues of the linearized system to determine bifurcation and stability as ΔT varies.
- Identify both symmetry-preserving and symmetry-breaking bifurcations.

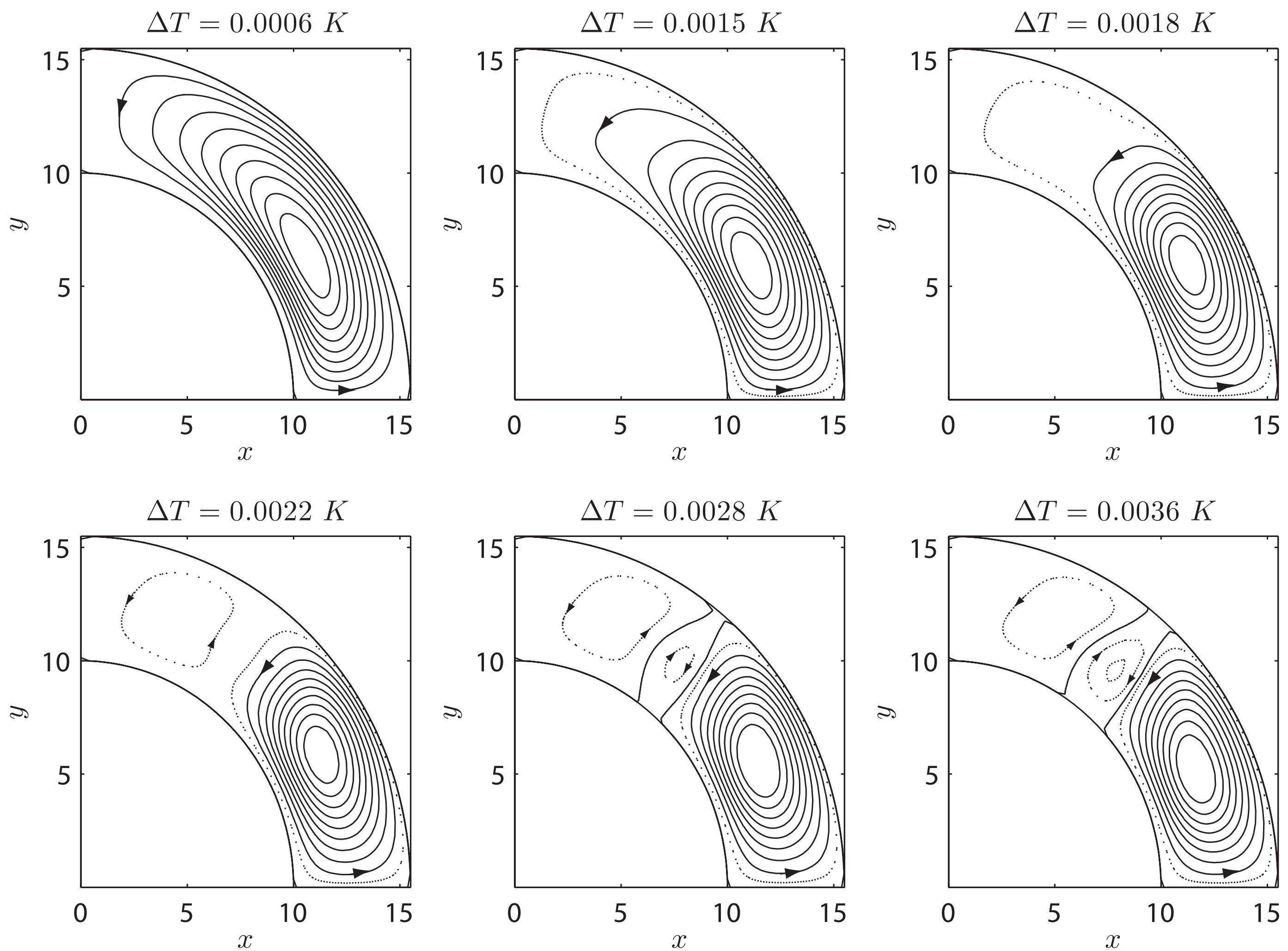
Stability Analysis

- Calculate the leading eigenvalues of the linearized system.
- Use implicitly restarted Arnoldi method.
- Find critical (zero) eigenvalues.
- Over 3000 lines of code.

OUR MODEL RESULTS

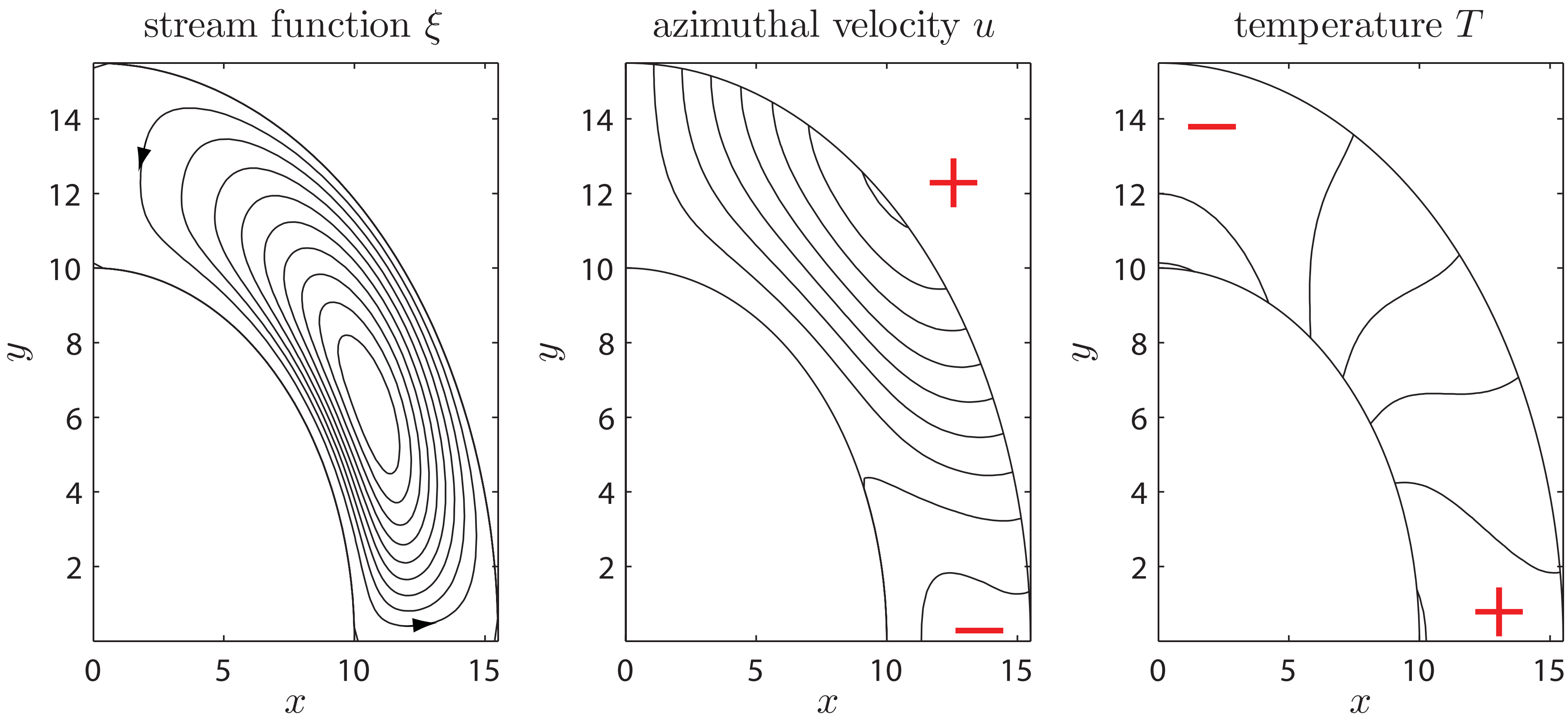
Ref. Gregory M. Lewis and William F. Langford (2008). Hysteresis in a rotating differentially heated spherical shell of Boussinesq fluid. SIAM J. Applied Dynamical Systems, V. 7, pp. 1421-1444.

HADLEY CELL CHANGES



As ΔT increases, the Hadley cell shrinks toward the equator and the Ferrel and polar cells appear.

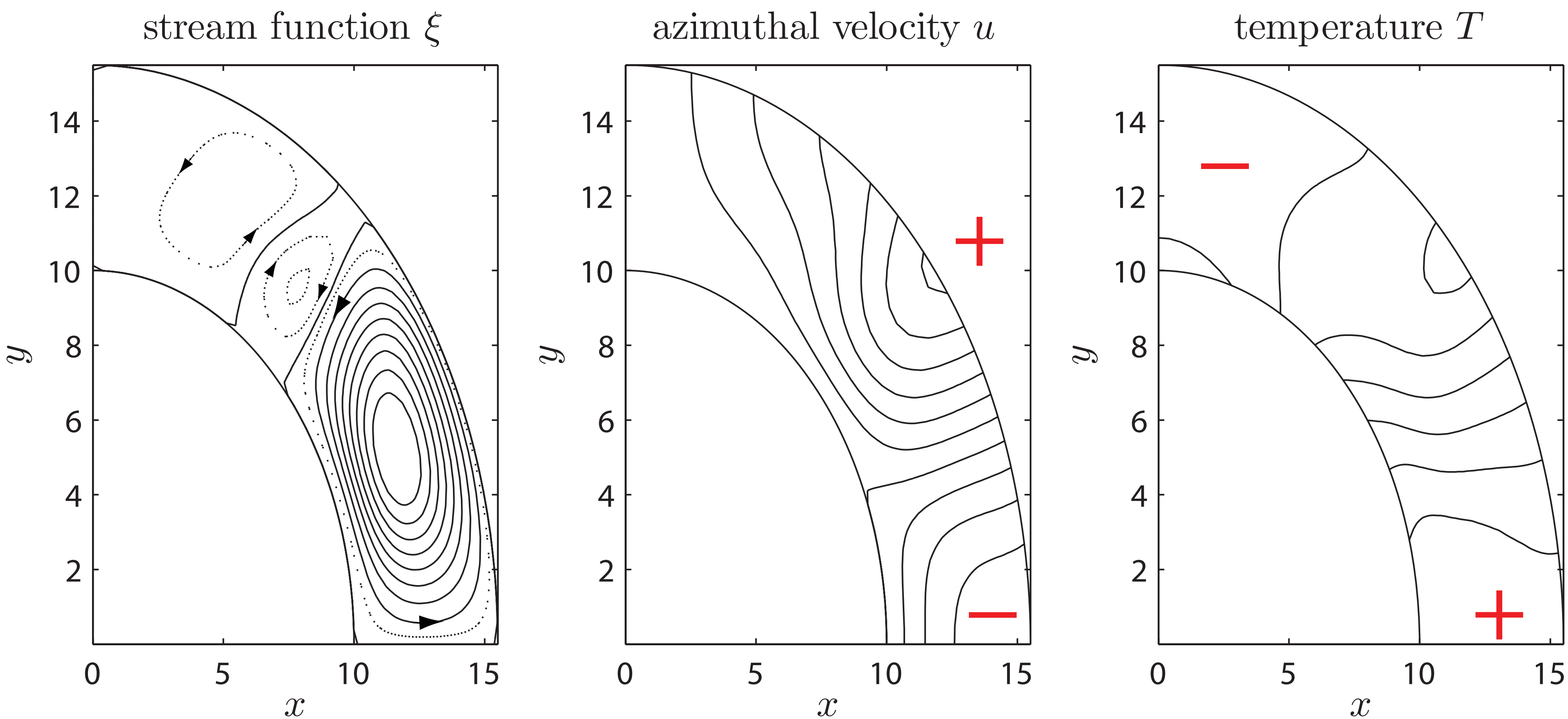
One-Cell Pattern for Small ΔT



A single large Hadley cell extends from equator to pole.

Note the jet stream at high altitude and trade winds in the tropics.

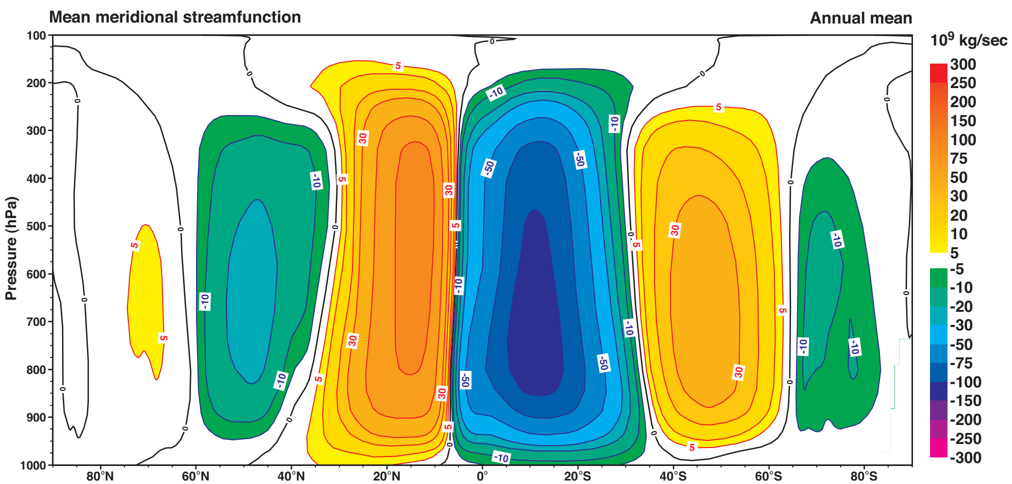
Three-Cell Pattern for Large ΔT

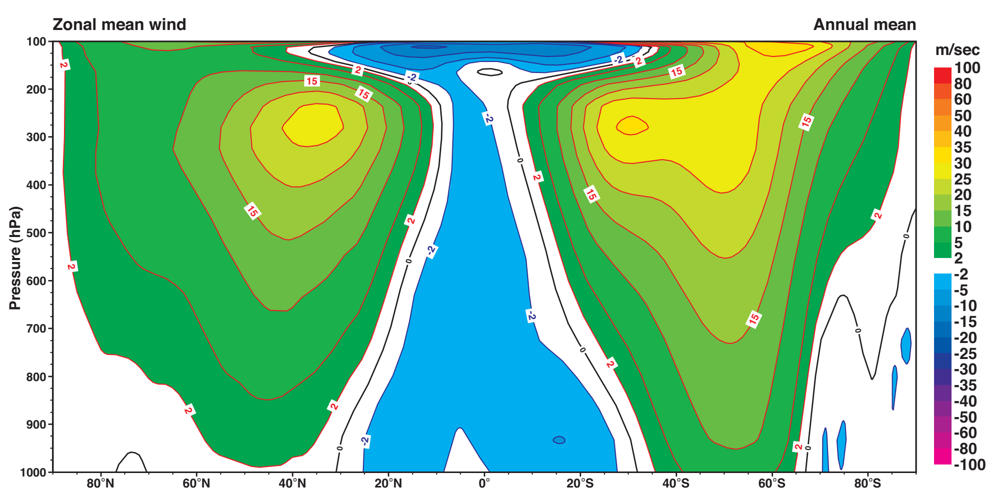


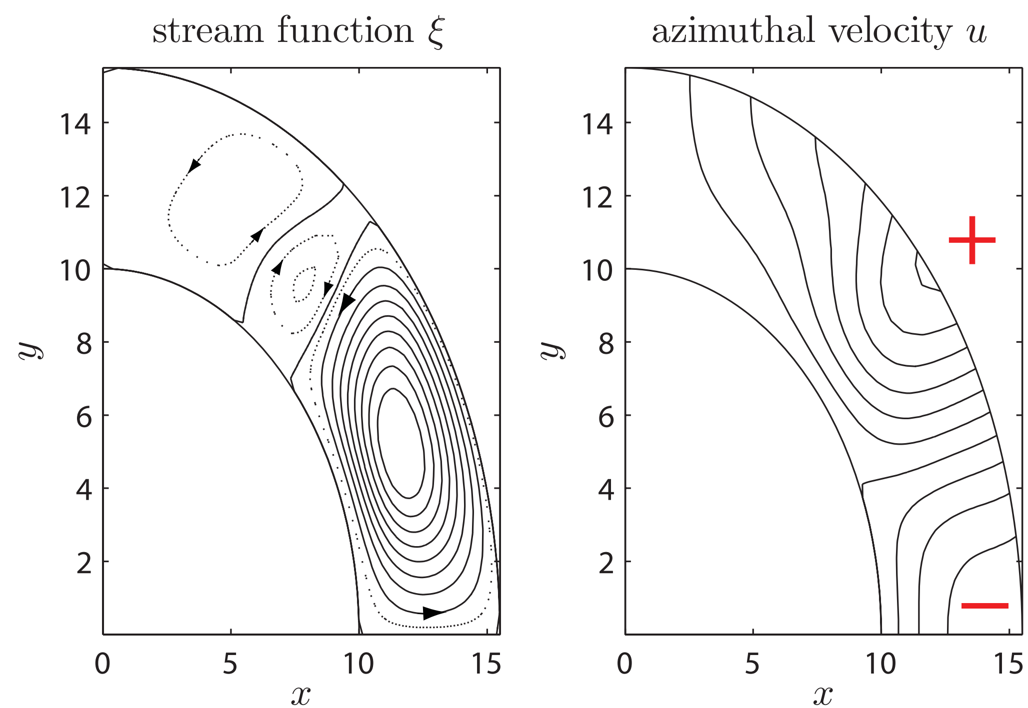
Hadley, Ferrel and Polar cells all exist.

The jet stream has moved toward the equator.

Compare the Model with Real Data







Implications for Today's Climate Change

A decrease in pole-to-equator temperature gradient can cause:

1. Poleward expansion of the Hadley cells.
2. Slowing of the Hadley circulation.
3. Poleward movement of jet streams.

All of these changes are occurring today.

Furthermore:

Changes in other parameters (rotation rate, radii, ...) do not alter the conclusions. This behaviour of Hadley cells is robust in the model.

The changes in Hadley circulation depend strongly on small changes in the temperature gradient.

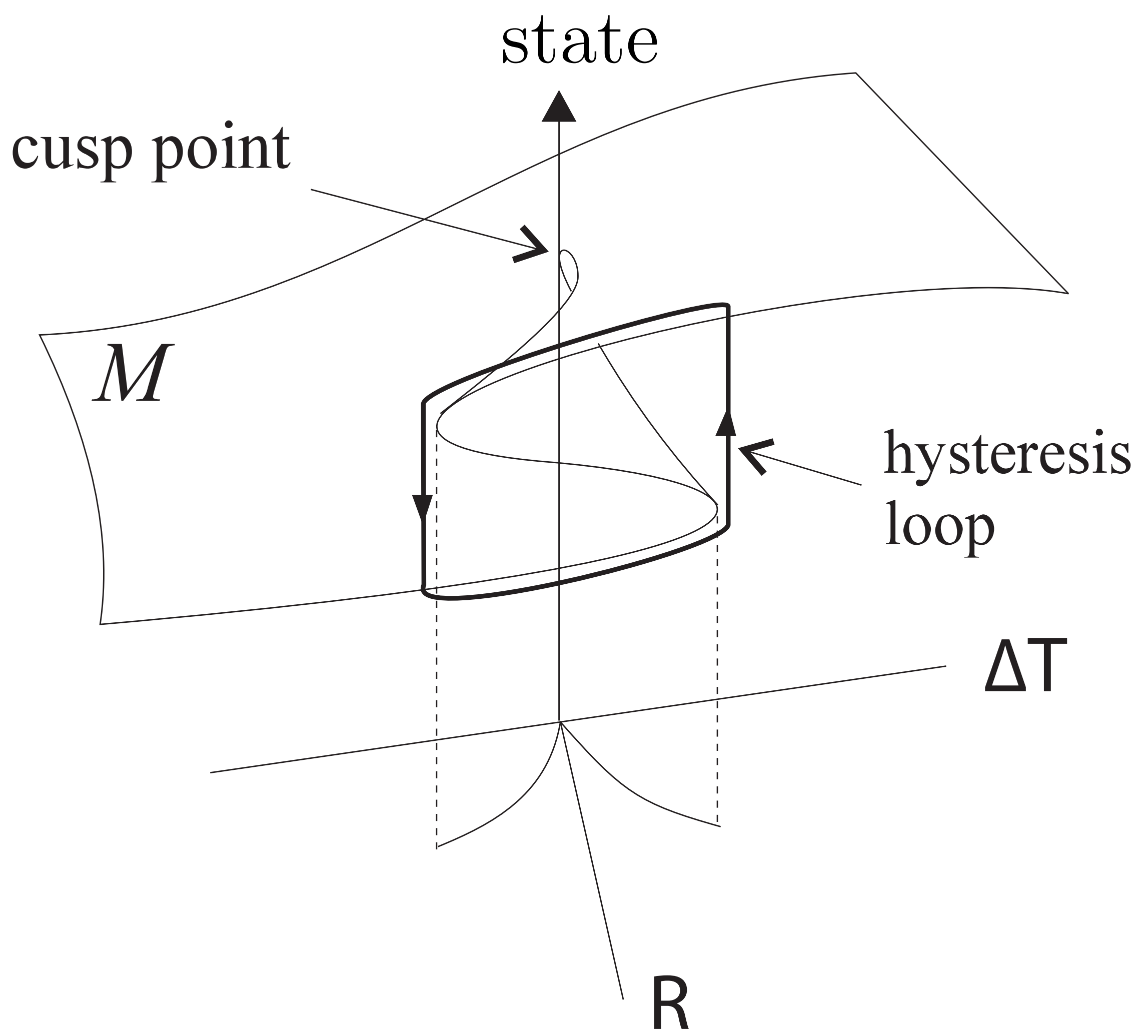
Implications for Paleoclimate Change

- Major changes in the Hadley cells could have caused dramatic changes in paleoclimate.
- For very small ΔT our model has a single large Hadley cell from equator to pole.
- We propose that a single large Hadley cell would yield an equable climate similar to that of the Mesozoic Era.

But there is more to our model!

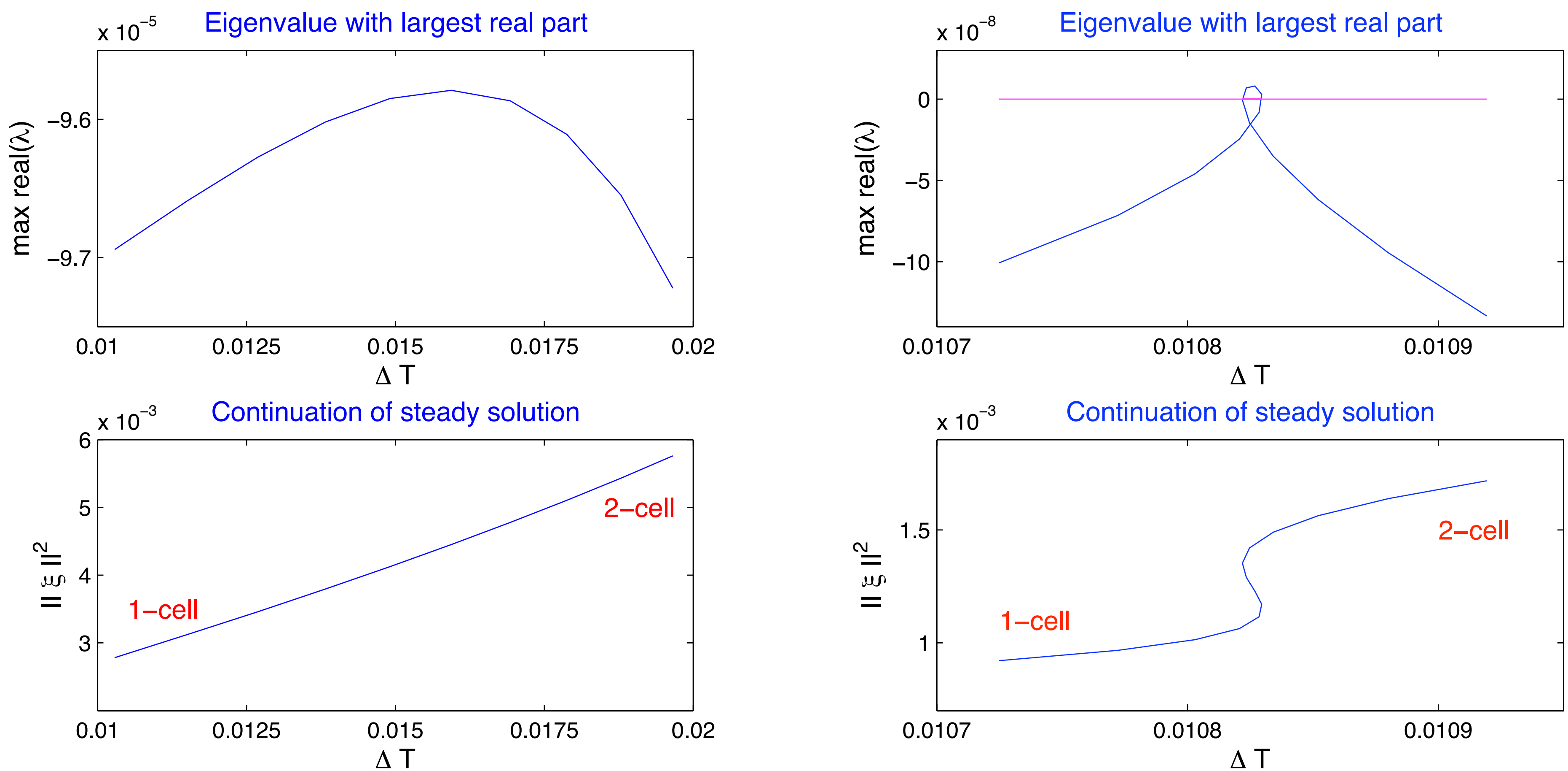
- The mathematical model exhibits *hysteresis bifurcation*.
- *Hysteresis* is a nonlinear phenomenon in which there is co-existence of two different stable states (or modes), with abrupt jumps from either state to the other state.
- In the model, the two states could represent greenhouse and icehouse climates.

HYSTERESIS BIFURCATION (Cusp)



A small change in ΔT can cause a jump in state.

CUSP BIFURCATION IN THE MODEL



$R = 0.5$ $R = 3.5$

Hysteresis Bifurcation Theorem

A hysteresis bifurcation point exists if:

1. There is an equilibrium point.
2. The linearization at the equilibrium point has a simple zero eigenvalue.
3. The coefficient of the second-order term of the normal form on the center manifold vanishes.
4. Certain other dominant terms in the normal form are nonzero.

Result of Greg Lewis

A hysteresis bifurcation point EXISTS in this model. It yields discontinuities in "climate", as the pole-to-equator temperature difference varies.

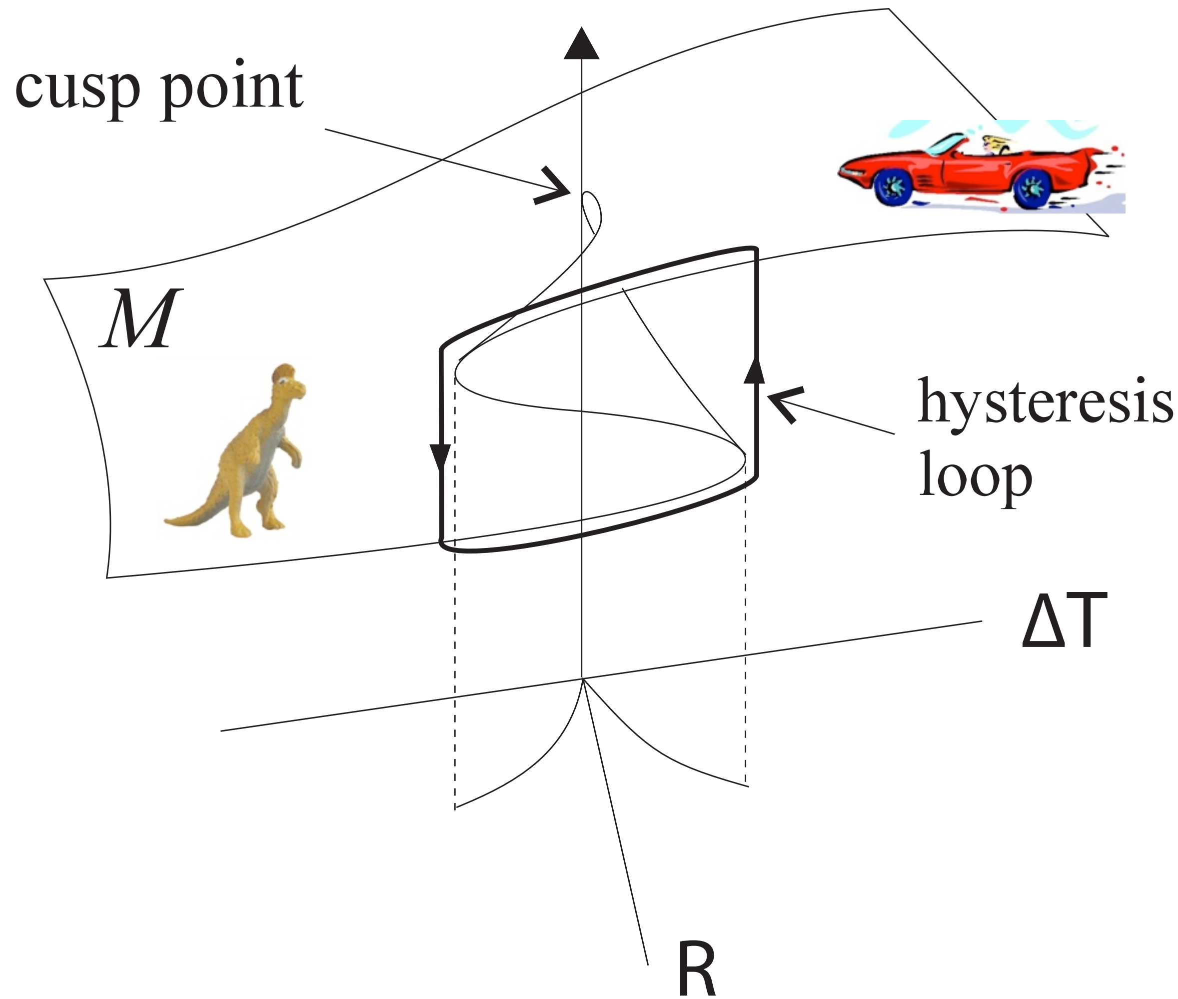
Proof: Lewis showed that the conditions of the Hysteresis Bifurcation Theorem are satisfied by the model equations.

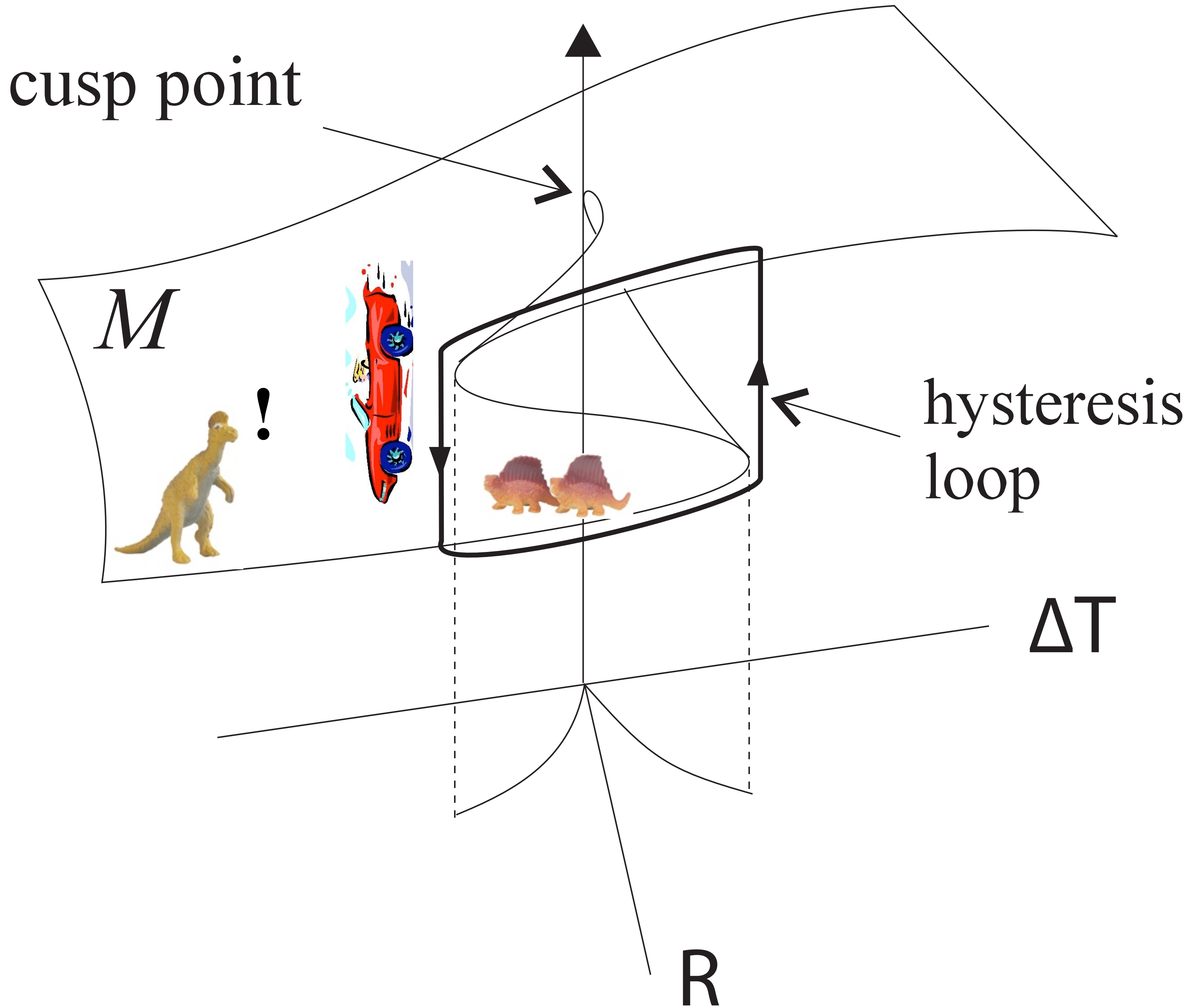
The model suggests:

Polar cooling may cause a global climate *bifurcation*, in which climate jumps abruptly from one climate mode to another.

Where are we today?

- In the model, we are on the upper branch, moving to the left (the poles are warming).
- Next question: How long before we fall over the edge?





Sudden transition from icehouse to greenhouse?

The Pliocene Paradox (3-5 million years ago)

- Greenland was ice-free, with palm trees on its southern coast [D. Greenwood].
- There were temperate rain forests on Canada's far northern arctic coastline [J. Basinger].
- Yet, all the "driving forces" were essentially the same as today's: CO_2 levels, solar radiation, Earth's axis tilt and orbit, continent positions, ocean currents, etc.

Pliocene Paradox 2.0

- The south pole switched from greenhouse climate to icehouse climate about 28 Mya (Oligocene).
- The north pole remained in a greenhouse climate until about 3 Mya (early Pliocene).
- Thus the north-south symmetry of the Earth's climate was *broken* for 25M years.
- Symmetry was *restored* when the north pole switched to icehouse climate about 3 Mya.

Work in Progress

- Drop the assumption of north-south symmetry in the mathematical model.
- Investigate the existence and stability of asymmetric modes.
- Study the relationship between asymmetric Hadley cells and asymmetric climates.

FUTURE WORK

- Add compressibility; assume an ideal gas law; break the rotational symmetry.
- Include in the model the "atmospheric conveyor belt" positive feedback. (Hadley cells carry heat from the equator to higher latitudes.)
- Add greenhouse gases and albedo to the model and compute Hopf bifurcations (ice age cycles).

Thank you!

References:

Gregory M. Lewis and William F. Langford (2008). *Hysteresis in a rotating differentially heated spherical shell of Boussinesq fluid.* SIAM J. Applied Dynamical Systems, V. 7, pp. 1421-1444.

William F. Langford and Gregory M. Lewis (2009). *Poleward expansion of Hadley cells.* Canadian Applied Mathematics Quarterly, V. 17, No.1, pp. 105-119.