

Workshop on Groups and Group Actions in  
Operator Algebra Theory

University of Ottawa  
Ottawa, Canada

July 12–16, 2010

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WORKSHOP SCHEDULE

<b>Monday</b>	12.30pm - 1.25	<i>Registration</i> <sup>†</sup> + <i>Coffee</i> <sup>*</sup>
	1.30 - 2.30	Vadim Kaimanovich I
<b>July 12</b>	2.40- 3.40	Ilijas Farah
	4.00- 5.00	Zhuang Niu
	6.00- 8.00	<i>Reception</i> <sup>*</sup>
<b>Tuesday</b>	9.00-10.00	Ruy Exel I
	10.00-10.30	<i>Coffee</i> <sup>*</sup>
	10.30-11.30	Volodymyr Nekrashevych
	11.35-12.15	Catalin Rada
<b>July 13</b>	2.00-3.00	Jean Renault
	3.00- 3.30	<i>Coffee</i> <sup>*</sup>
	3.30-4.30	George Elliott
	7.00-	<i>Banquet</i> <sup>‡</sup>
<b>Wednesday</b>	9.00-10.00	Vadim Kaimanovich II
	10.00-10.30	<i>Coffee</i> <sup>*</sup>
	10.30-11.30	David Kerr
	11.30-12.30	Alcides Buss
<b>July 14</b>	2.00-3.00	Ruy Exel II
	3.00- 3.30	<i>Coffee</i> <sup>*</sup>
	3.30-4.30	Benjamin Steinberg
<b>Thursday</b>	9.00-10.00	Matthias Neufang
	10.00-10.30	<i>Coffee</i> <sup>*</sup>
	10.30-11.30	Benoît Collins
	11.35-12.15	Ion Nechita
<b>July 15</b>	2.00-3.00	Daniel Gonçalves
	3.00- 3.30	<i>Coffee</i> <sup>*</sup>
	3.30-4.30	Ping Wong Ng
<b>Friday</b>	9.00-10.00	Vadim Kaimanovich III
	10.00-10.30	<i>Coffee</i> <sup>*</sup>
	10.30-11.30	Ruy Exel III

*Lectures* are taking place in the room A0150 of the SITE building (code STE on the campus map). Wireless access will be provided to registered participants.

<sup>†</sup>*Registration*, as well as <sup>\*</sup>*Coffee breaks/discussions*, will be held in the room STE G0103.

<sup>\*</sup>*Reception* and <sup>‡</sup>*Banquet*: locations to be revealed on Monday.

\* \* \*

*Workshop organizers*: Thierry Giordano and Vladimir Pestov

**Cartan subalgebras, Fell bundles and twisted actions of inverse semi-groups**

Ruy Exel

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Coauthors: The first part will be based on work by the speaker and the second part will be based on joint work with Alcides Buss

The minicourse will focus on the rich interplay between  $C^*$ -algebras and dynamical systems. We will begin by briefly discussing Jean Renault's description of Cartan subalgebras in terms of twisted tale groupoids, as well as a non-commutative generalization found by the speaker. As we shall see, the concept of Fell bundles over inverse-semigroups underlies this circle of ideas in a fundamental way and we will conduct a careful study of these. The goal is twofold: we will completely characterize the semi-abelian Fell bundles over inverse-semigroups in terms of twisted tale groupoids, and then we will characterize a large class of the non-abelian ones as resulting from twisted actions of inverse-semigroups on  $C^*$ -algebras.

**Markov chains and groupoids**

Vadim Kaimanovich

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The algebraic structure of groupoids is ideally suitable for describing various probabilistic models generalizing random walks on groups (random walks in random environment and with internal degrees of freedom, along classes of equivalence relations etc.). On the other hand, the notion of an invariant Markov operator is very natural from the groupoid point of view as well, as, for instance, multiplication of these operators precisely corresponds to the groupoid convolution operation. We shall concentrate on the problem of triviality of the Poisson boundary for invariant Markov operators on groupoids and its application to amenability of groupoids.

**Fell bundle structures for  $C^*$ -algebras**

Alcides Buss

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A Fell bundle over a locally compact group  $G$  is a (continuous) Banach bundle  $A$  over  $G$  together with a multiplication and an involution satisfying some natural conditions. Associated to  $A$  is the cross-sectional  $C^*$ -algebra  $C^*(A)$ , which generalizes the crossed product construction for a  $C^*$ -dynamical system. The  $C^*$ -algebra  $C^*(A)$  has a rich structure, including a canonical dual coaction of  $G$ .

Given a  $C^*$ -algebra  $B$  we consider the problem of finding and classifying the Fell bundle structures over  $G$  for  $B$ , that is, Fell bundles  $A$  over  $G$  together with an isomorphism  $C^*(A) = B$ . We are going to explain how to solve this problem using the theory of integrable coactions.

**Random permutation matrices and sofic groups**

Benoît Collins

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Coauthors: Ken Dykema

In this talk we will explain why the free product of sofic groups amalgamated over amenable groups is again sofic. This requires estimates on the moments of products of random permutation matrices.

**A brief survey of classification theory for amenable  $C^*$ -algebras**

George Elliott

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Over the last twenty years, the classification theory of Glimm and Bratteli has been extended to broad classes of amenable (= nuclear)  $C^*$ -algebras, including many of those arising from actions of a group on a given  $C^*$ -algebra. Some aspects of this work will be reviewed.

**Classifiable  $C^*$ -algebras are not classifiable...**

Ilijas Farah

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Coauthors: Asger Törnquist (Kurt Gödel Research Center for Mathematical Logic, Vienna), Andrew Toms (York University)

...by countable structures. On the other hand, the isomorphism of nuclear separable simple unital  $C^*$ -algebras is reducible to an orbit equivalence relation of a Polish group action.

**Representations of Graph and Exel-Laca algebras and the Perron-Frobenius operator.**

Daniel Gonçalves

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Coauthors: Danilo Royer

In this talk we show how to obtain representations of certain universal  $C^*$ -algebras, given by generators and relations, on the bounded operators on  $L^2(X)$ . In the specific case of graph and Exel-Laca  $C^*$ -algebras, we show how the representations obtained relate to a Perron-Frobenius operator defined on  $L^1(X)$ .

**Topological entropy and a variational principle for actions of sofic groups**

David Kerr

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Coauthors: Hanfeng Li (SUNY at Buffalo)

Recently Lewis Bowen introduced a notion of entropy for measure-preserving actions of a countable sofic group on a standard probability space admitting a generating partition with finite entropy. Using an operator algebra perspective we develop a more general approach to sofic entropy which produces both measure and topological dynamical invariants. We establish the variational principle in this context, and use it to compute the topological entropy of certain algebraic actions of residually finite groups in terms of the Fuglede-Kadison determinant.

## **A random matrix / free probability approach to additivity problems for quantum channels**

Ion Nechita

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Coauthors: Benoit Collins (University of Ottawa / CNRS), Serban Belinschi (University of Saskatchewan / Institute of Mathematics “Simion Stoilov” of the Romanian Academy)

We shall present an approach to additivity questions in Quantum Information Theory involving random quantum channels. We study spectral properties of such random channels with free probability techniques, obtaining a precise description of the set of eigenvalues of the output matrices.

## **Hyperbolic groupoids**

Volodymyr Nekrashevych

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I will define a notion of hyperbolicity of étale groupoids, consisting of two conditions: one topological, and one large-scale. Using the action of a hyperbolic groupoid on its boundary, I will show that every hyperbolic groupoid has a naturally defined dual groupoid, which is also hyperbolic.

An example of a hyperbolic groupoid is the action of a hyperbolic group on its boundary, which is self-dual. Other examples (some of them coming from group actions) will be presented. We will also discuss applications of the duality theory, as well as properties of the  $C^*$ -algebras of hyperbolic groupoids.

## **Harmonic analysis and quantized functional analysis**

Matthias Neufang

*Carleton University and Fields Institute*

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Coauthors: Zhiguo Hu, Wojciech Jaworski, Marius Junge, Mehrdad Kalantar, Zhong-Jin Ruan, Nico Spronk

We discuss a representation theoretical framework, based on operator space theory, for a large class of Banach algebras arising in abstract harmonic analysis, such as the measure algebra and the completely bounded multipliers of the Fourier algebra over a locally compact group. We show that this approach, initiated independently by F. Ghahramani, U. Haagerup and E. Størmer, can be generalized to the setting of locally compact quantum groups; this is based on joint work with M. Junge and Z.-J. Ruan. At the level of our representation, we see that Pontryagin type quantum group duality can be expressed through

a commutation relation, and the latter leads to a quantum group version of Heisenberg's commutation relation.

We also present links to Banach space geometry, e.g., concerning Godefroy-Talagrand's property  $(X)$ , and to quantum information theory, such as the calculation of the completely bounded minimal entropy of certain quantum channels arising from quantum groups. Moreover, we obtain a crossed product formula for the Poisson boundary of certain random walks on quantum groups; this result generalizes our earlier solution, obtained with W. Jaworski, of a problem raised by M. Izumi (2004) on the corresponding random walks on groups.

The talk is mainly based on collaborations - involving, along with the above-mentioned, Z. Hu, (my Ph.D. student) M. Kalantar, and N. Spronk.

### **On the structure of certain topological groups associated with operator algebras**

Ping Wong Ng

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Coauthors: Parts are joint work with Giordano, Kucerovsky and Ruiz.

We look at certain topological groups coming from  $C^*$ -algebras with an emphasis on those from the classification theory and extension theory of  $C^*$ -algebras. (E.g. unitary groups, automorphism groups.) We ask some basic questions – focussing on structure theory – of which many are still open.

### ***AH*-algebras with diagonal maps**

Zhuang Niu

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Simple unital *AH*-algebras with diagonal maps are considered. It is shown that if the projections of such an *AH*-algebra separate traces, then this *AH*-algebra is isomorphic to an *AH*-algebra without dimension growth.



## Regular AF subalgebras of certain crossed products by $\mathbf{Z}$

Catalin Rada

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Let  $X$  be the Cantor set and let  $T : X \rightarrow X$  be a minimal homeomorphism. If  $C(X) \rtimes_T \mathbf{Z}$  is the associated crossed product, we characterize the regular AF subalgebras of  $C(X) \rtimes_T \mathbf{Z}$  that can arise as the algebra  $A_Y = \langle C(X), uC_0(X \setminus Y) \rangle$  for some closed subset  $Y$  of  $X$ . We also characterize the minimal homeomorphisms in  $A_Y$  terms. Other properties on traces and  $K$ -theory of certain  $A_Y$  algebras are discussed.

## KMS states on groupoid $C^*$ -algebras

Jean Renault

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I shall first describe a general framework where the determination of KMS states can be reduced to finding quasi-invariant measures with a prescribed Radon-Nikodym derivative. Then I shall present some examples coming from statistical mechanics, hyperbolic geometry and number theory.

## Etale groupoids and inverse semigroups

Benjamin Steinberg

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Coauthors: David Milan (UT Tyler)

In this talk we are interested in the question: When is an etale groupoid Morita equivalent to the groupoid of germs of an inverse semigroup action? More precisely, we define the notion of a cocycle from an etale groupoid to an inverse semigroup and give conditions that guarantee it leads to such a Morita equivalence.

Haefliger, Khoshkam, Skandalis, Exel, Abadie and other have considered this question for the case of a discrete group.

As an application, we give conditions on an inverse semigroup homomorphism  $f : S \rightarrow T$  so that  $C^*(S)$  is Morita equivalent to a cross product  $C_0(X) \rtimes T$ . This generalizes the results of Khoshkam and Skandalis, who considered the case when  $T$  is a group. We also show that the  $C^*$ -algebra of a strongly 0- $E$ -unitary inverse semigroup is a partial group action cross product and give conditions for the enveloping action to be Hausdorff.