Reducing the data: Analysis of the role of vascular geometry on the features of blood flow in curved vessels

Jordi Alastruey[1,2], Jennifer Siggers[1], Véronique Peiffer[2], Luca Antiga[3], Denis Doorly[2], Spencer Sherwin[2]

Departments of Bioengineering[1] and Aeronautics[2], Imperial College London, UK
[3]Medical Image Unit, Mario Negri Institute, Bergamo, Italy

Motivation

- Vascular disease location is associated with haemodynamic factors (*e.g.* WSS)

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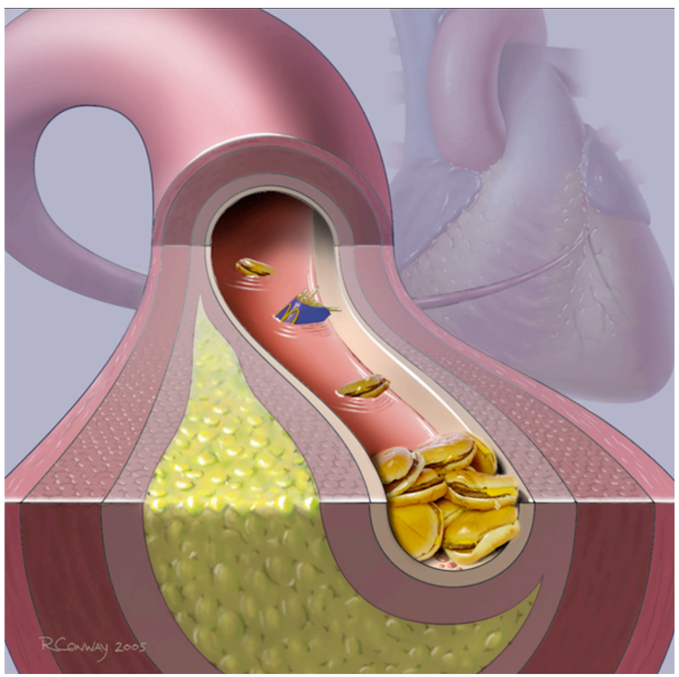
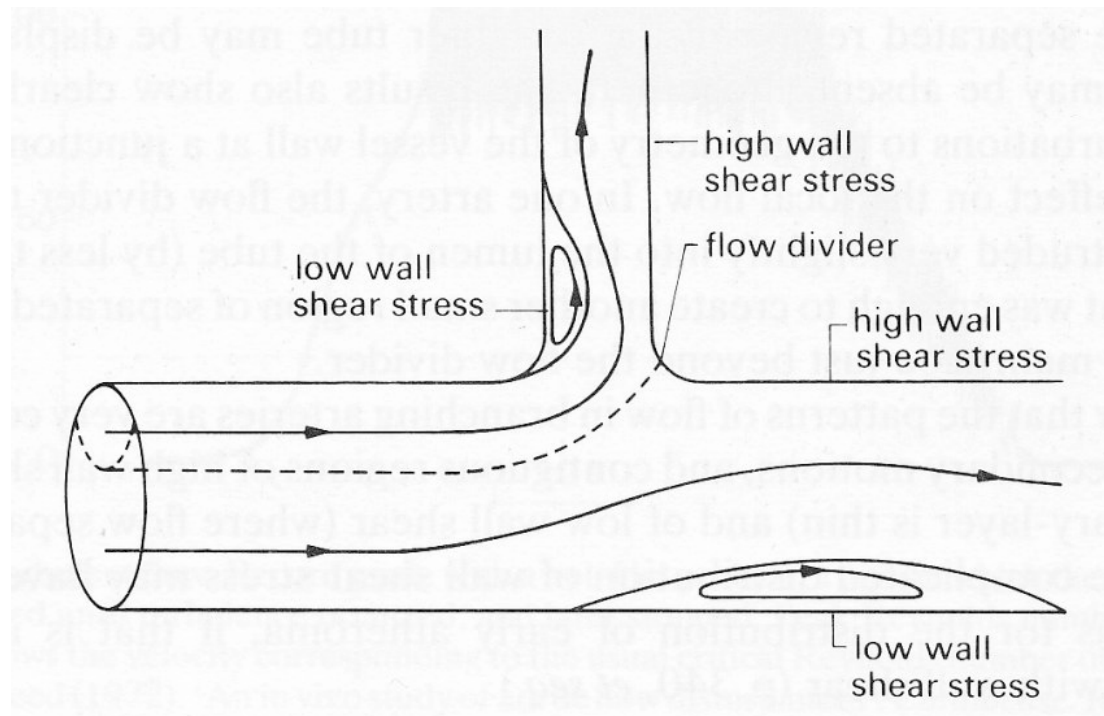
- Vascular geometry strongly affects these factors

Atherosclerosis

- Chronic inflammatory disease of the arterial lumen
- Good correlation with low or oscillatory WSS
- Particularly prevalent on the inner wall of curved arteries and the outer wall of bifurcations

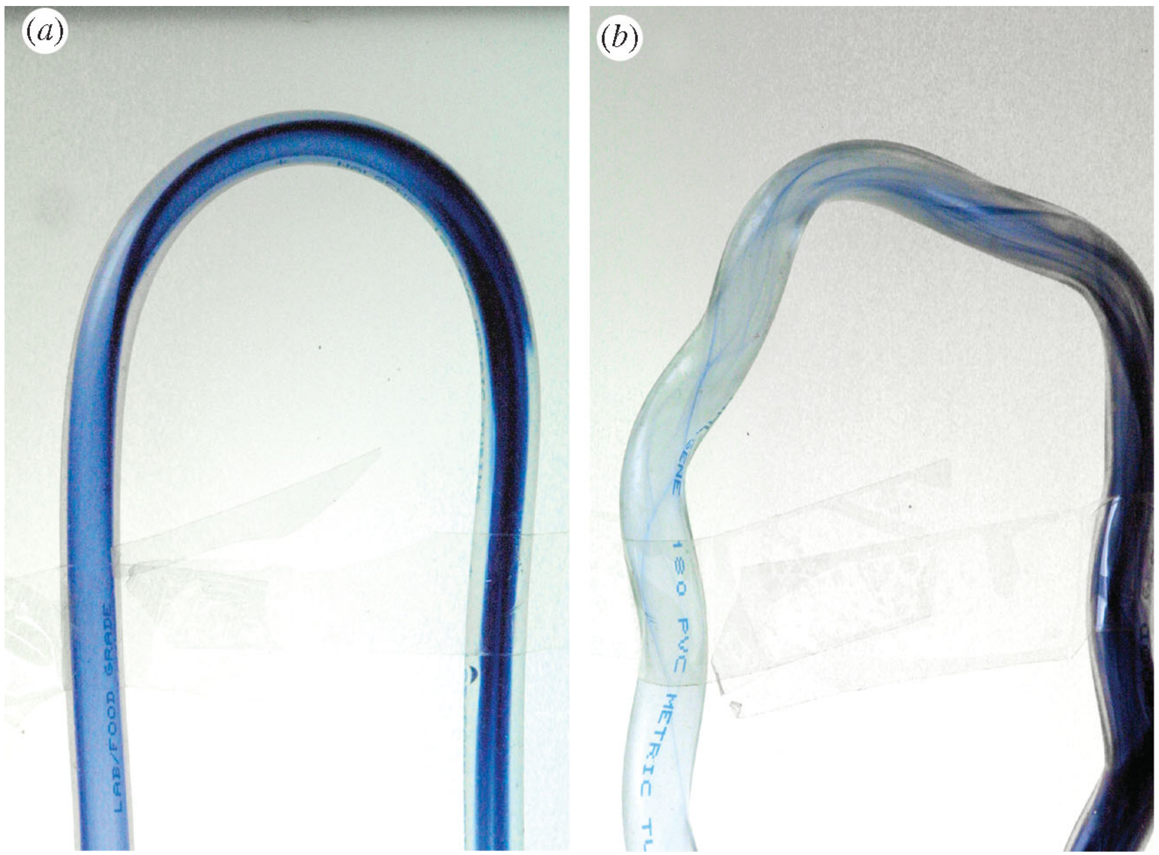




Caro *et al.* 1978. The Mechanics of the Circulation

Helical geometries

- Vascular geometry affects the risk of occlusion of prothesis (e.g. bypass grafts, stents and arterio-venous shunts) and surgical vascular reconstructions



(a)

(b)

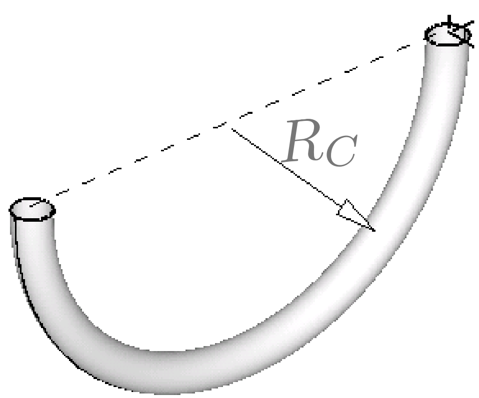
Caro *et al.* 2005. *J. R. Soc. Interface*

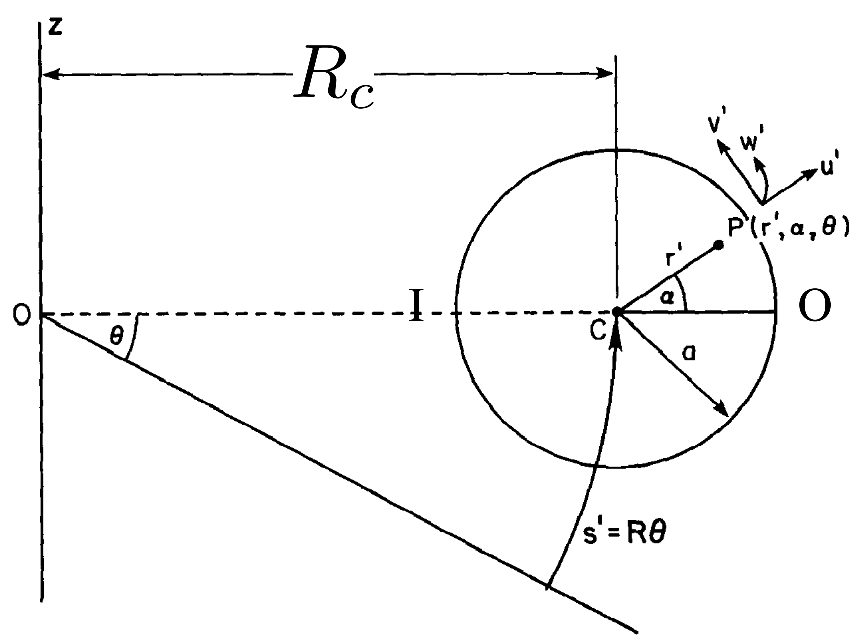
Asymptotic solutions

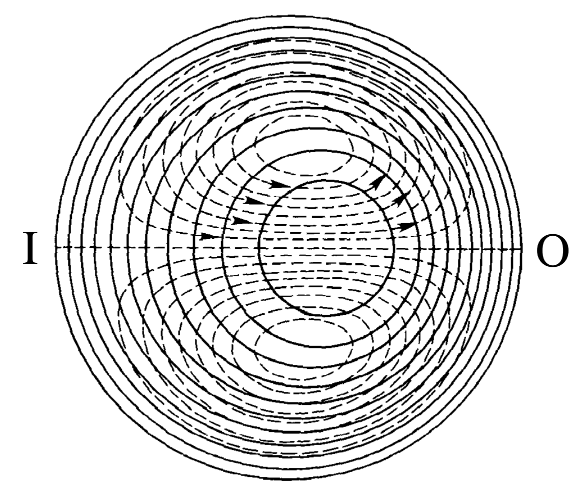
- Data reduction to a few parameters by constructing asymptotic solutions
- *E.g.* Dean flow: fully developed steady and laminar flow in a bend of constant curvature

$$\frac{W(r,\alpha)}{2\overline{W}} = 1 - \left(\frac{r}{a}\right)^2 + \left(\frac{\mathrm{De}}{96}\right)^2 \left[\frac{19}{40}\left(\frac{r}{a}\right) - \left(\frac{r}{a}\right)^3 + \frac{3}{4}\left(\frac{r}{a}\right)^5 - \frac{1}{4}\left(\frac{r}{a}\right)^7 + \frac{1}{40}\left(\frac{r}{a}\right)^9\right]\sin(\alpha)$$

$a/R_c \ll 1$ $\qquad$ $De < 96$







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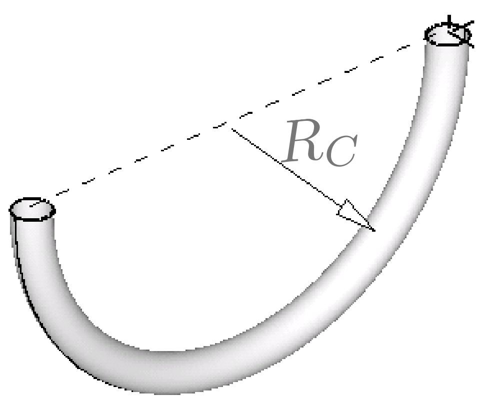
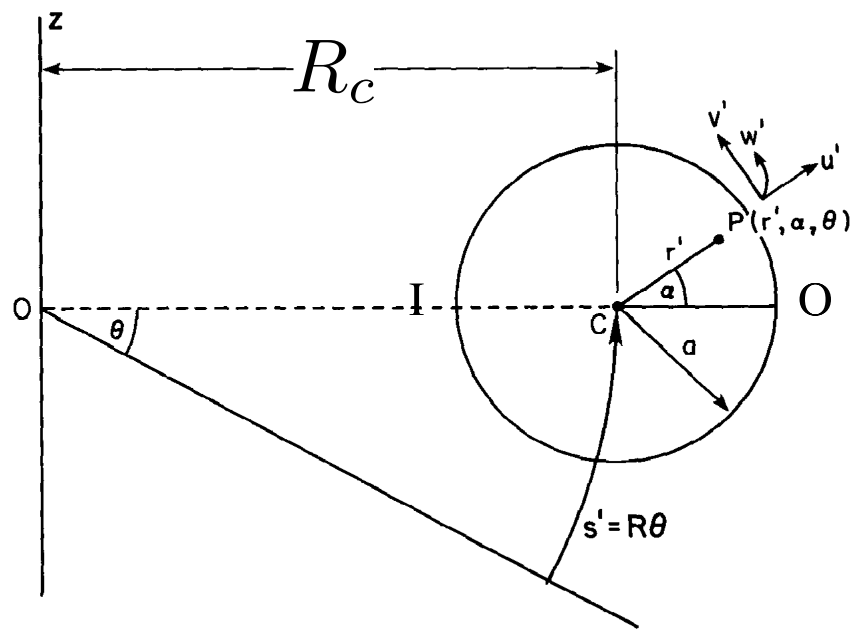
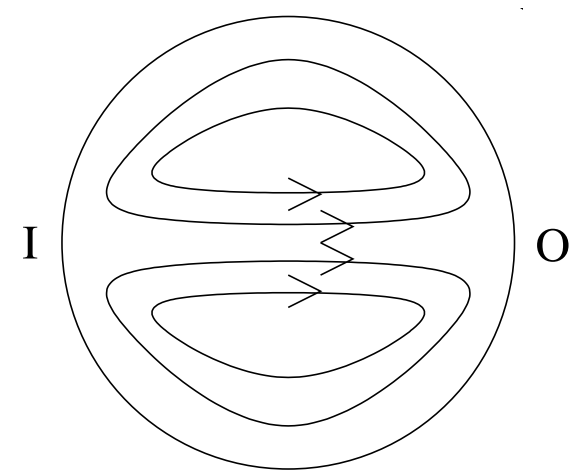
$a/R_c << 1$ $\qquad De < 96$





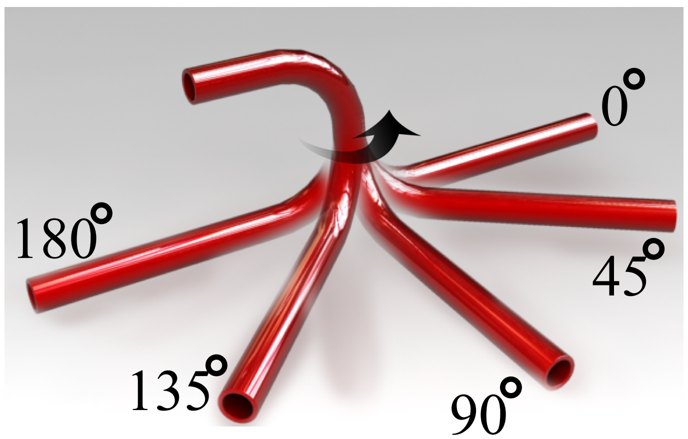


Asymptotic solutions

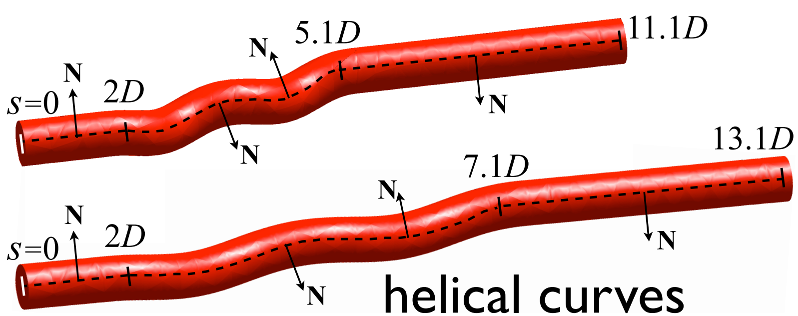
- Data reduction to a few parameters by constructing asymptotic solutions
- *E.g.* Dean flow: fully developed steady and laminar flow in a bend of constant curvature
- The degree of validity of these solutions in blood vessels is unknown
- In the human vasculature we have sequences of non-planar bends with changing curvature and torsion

Our goal

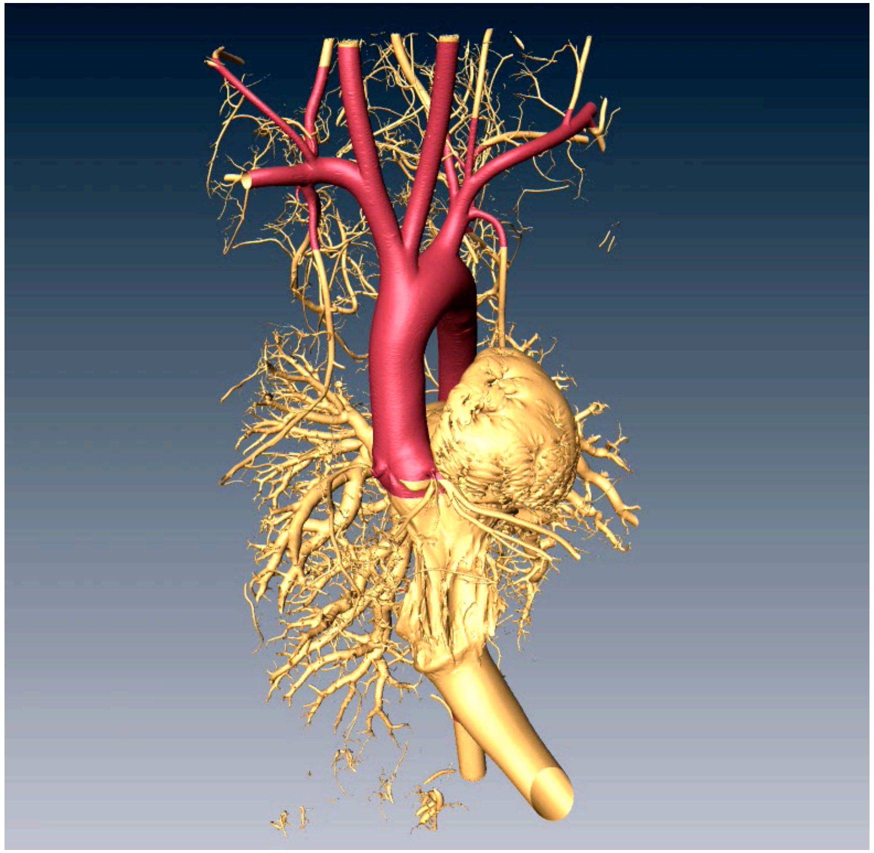
- Reduce the amount of data whilst retaining the clinically relevant mechanisms
- Quantify the effect of vascular geometry on primary and secondary flows in curved vessels and their association with velocity profiles, vortical structures and wall stresses



planar and non-planar double bends



helical curves

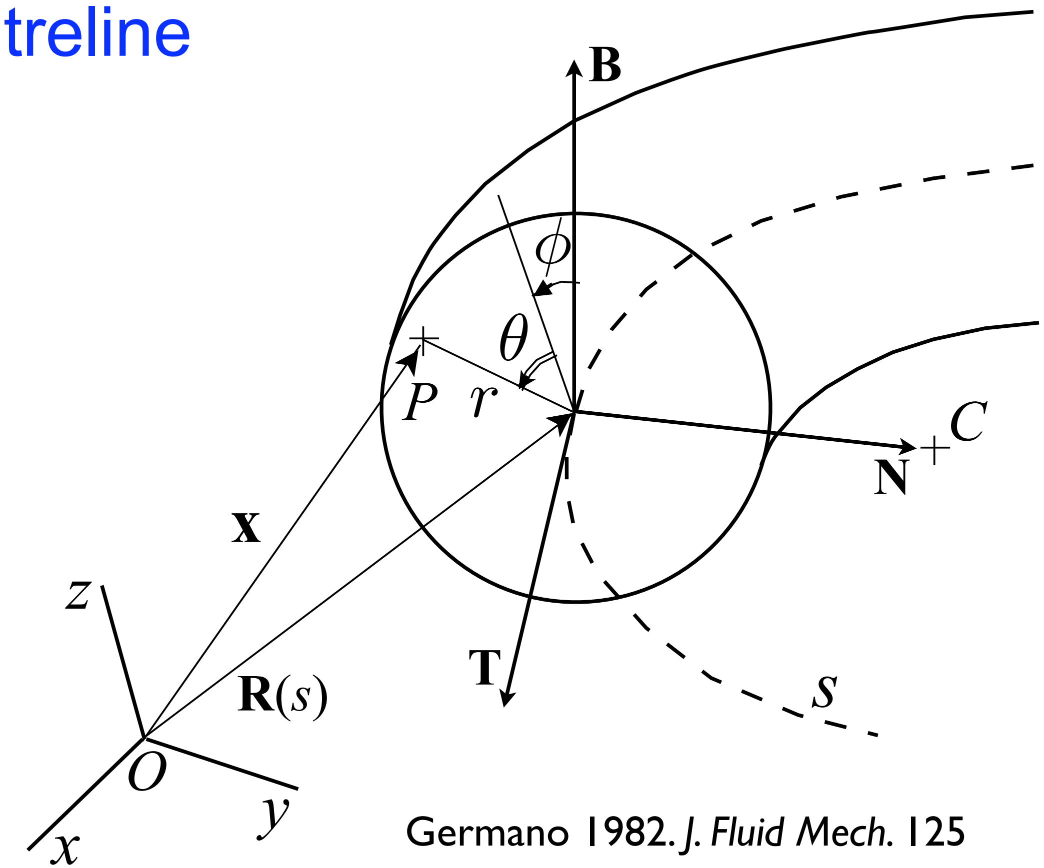


Vincent *et al.* 2011. *J. R. Soc. Interface*

Centreline analysis

1. Solve N-S equations in Cartesian coordinates (incompressible flow, Newtonian fluid and fixed geometry)

2. Transform the Cartesian velocity and pressure fields (and their derivatives) into an orthogonal (local) coordinate system following the vessel centreline



Germano 1982. *J. Fluid Mech.* 125

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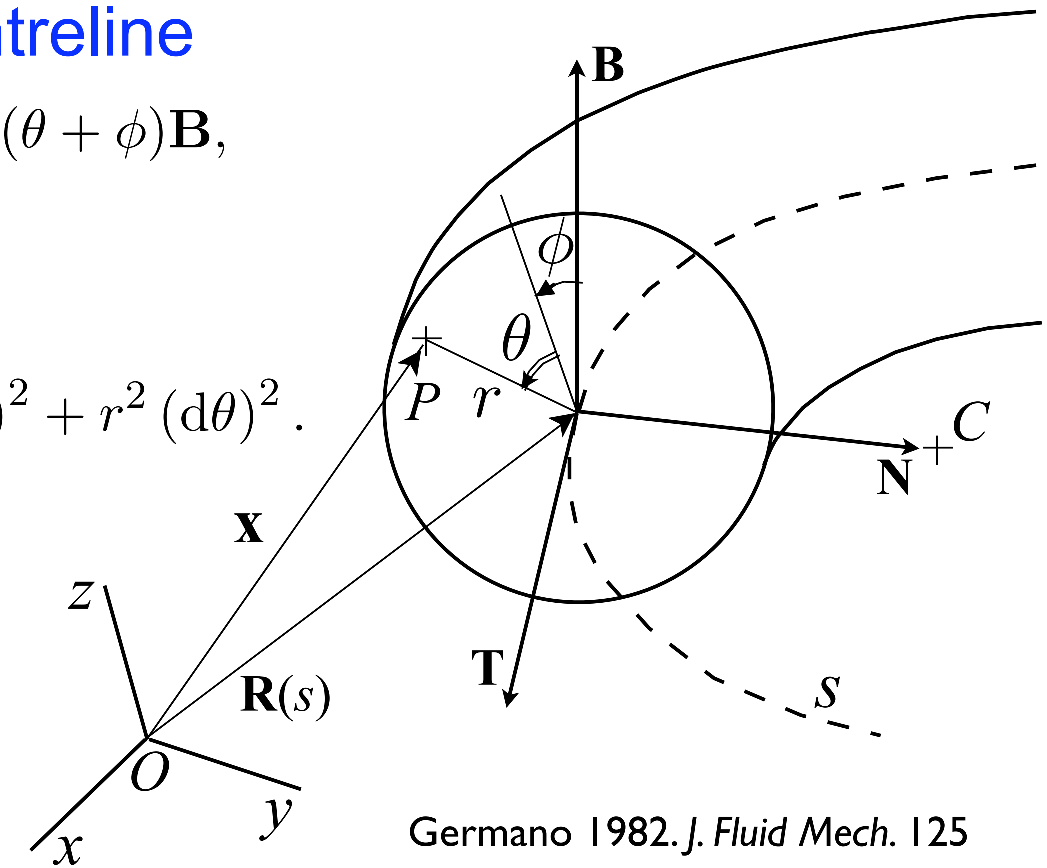
$$\mathbf{x} = P - O = \mathbf{R} - r\sin(\theta+\phi)\mathbf{N} + r\cos(\theta+\phi)\mathbf{B},$$

$$\phi(s) = -\int_{s_0}^{s} \tau(s')ds',$$

$$\mathrm{d}\mathbf{x}\cdot\mathrm{d}\mathbf{x} = [1+\kappa r\sin(\theta+\phi)]^2(\mathrm{d}s)^2 + (\mathrm{d}r)^2 + r^2(\mathrm{d}\theta)^2.$$

$\tau(s)$: torsion

$\kappa(s)$: curvature



Germano 1982. *J. Fluid Mech.* 125

Centreline analysis

1. Solve N-S equations in Cartesian coordinates (incompressible flow, Newtonian fluid and fixed geometry)

2. Transform the Cartesian velocity and pressure fields (and their derivatives) into an orthogonal (local) coordinate system following the vessel centreline

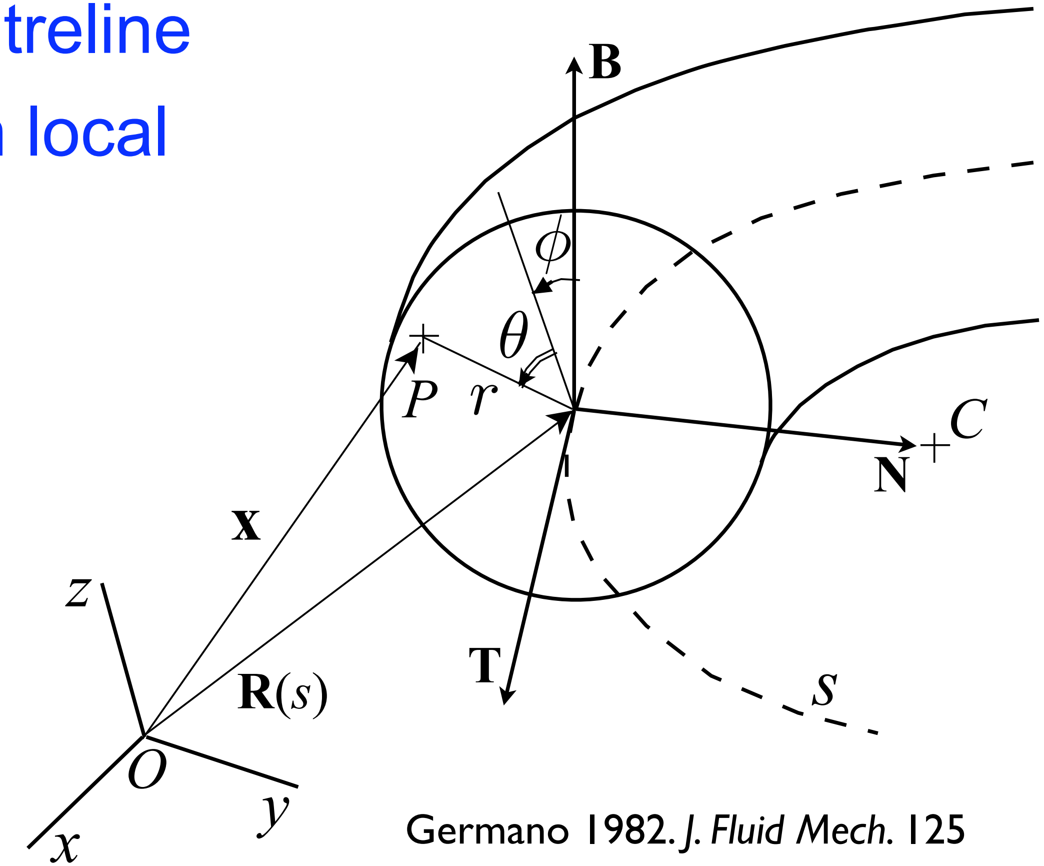
3. Express the N-S equations in local coordinates,

x,y,z: $$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\boldsymbol{\nabla} p + \nu \nabla^2 \mathbf{u}$$

x: $CA_x = PG_x + VF_x$

y: $CA_y = PG_y + VF_y$

z: $CA_z = PG_z + VF_z$



Germano 1982. *J. Fluid Mech.* 125

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3. Express the N-S equations in local coordinates,

T: $CA_T = Co + PG_T + VF_T$

N: $CA_N = CF_N + TF_N + PG_N + VF_N$

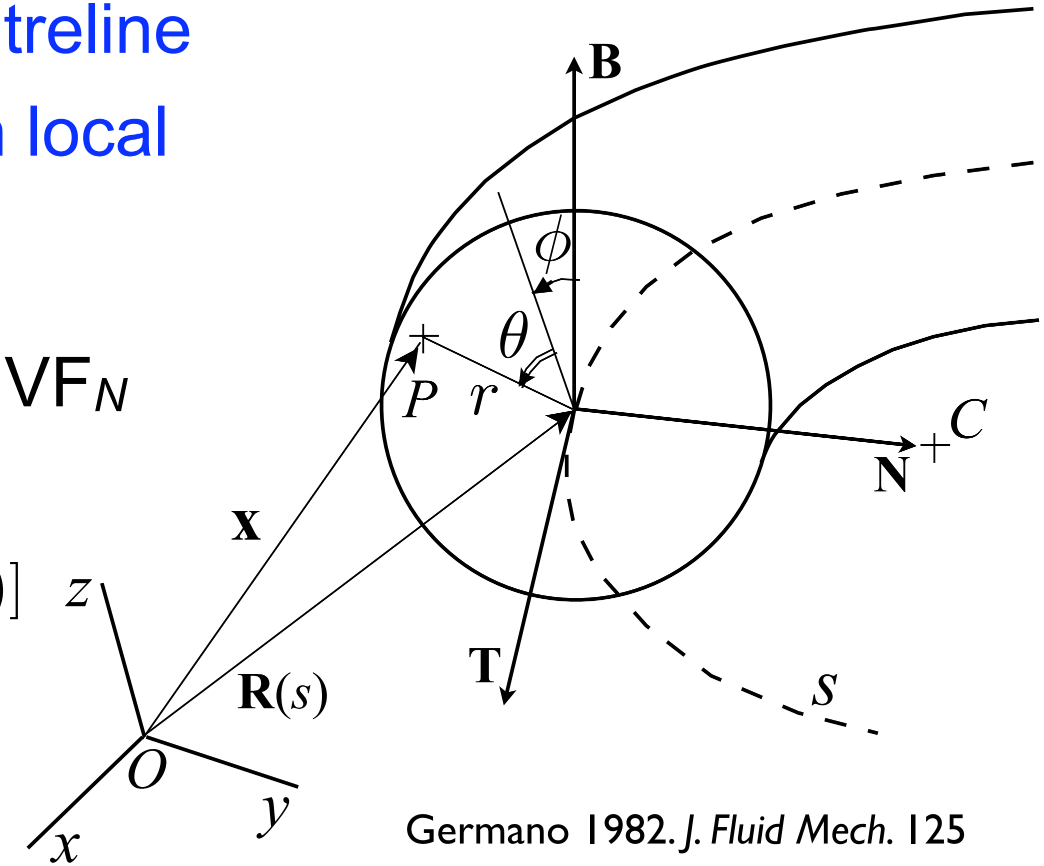
B: $CA_B = TF_B + PG_B + VF_B$

$$\mathrm{Co} = -\frac{\kappa u}{h}\left[v\sin(\theta+\phi) + w\cos(\theta+\phi)\right]$$

$$\mathrm{CF}_N = -\frac{\kappa u^2}{h}$$

$$\mathrm{TF}_N = \frac{\tau u}{h}V_B, \quad \mathrm{TF}_B = -\frac{\tau u}{h}V_N$$

$$h = 1 + \kappa r\sin(\theta+\phi)$$



Germano 1982. *J. Fluid Mech.* 125

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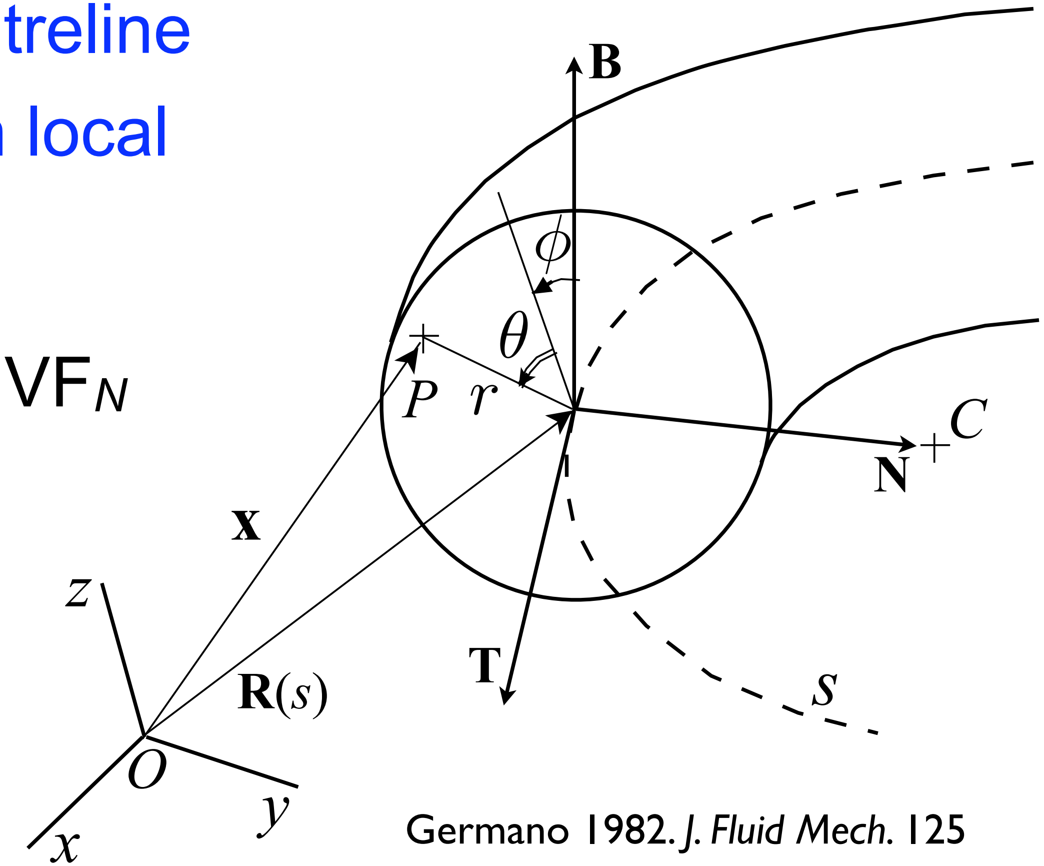
$$T:\ CA_T = Co + PG_T + VF_T$$

$$N:\ CA_N = CF_N + TF_N + PG_N + VF_N$$

$$B:\ CA_B = TF_B + PG_B + VF_B$$

4. Take cross-sectional averages of the local quantities to reduce the terms onto the centreline



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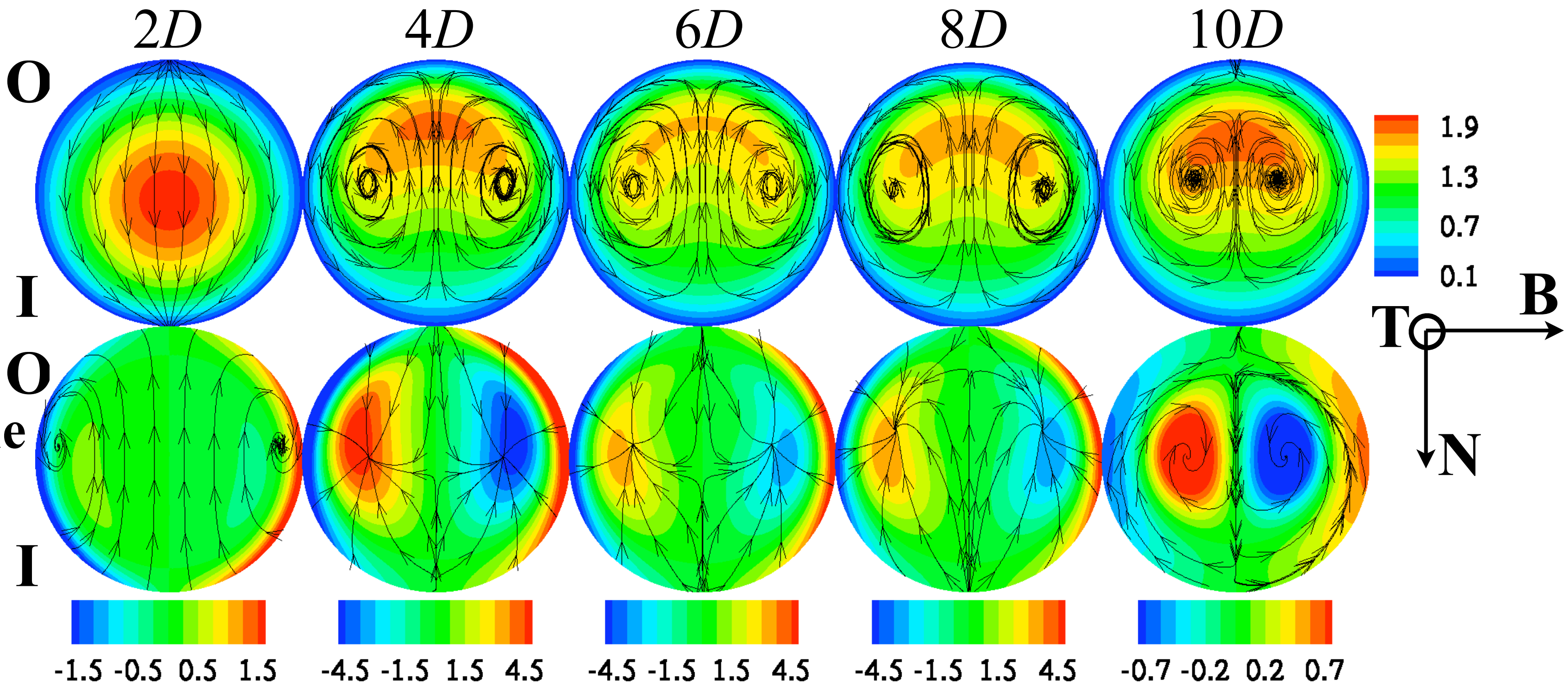
Single bend

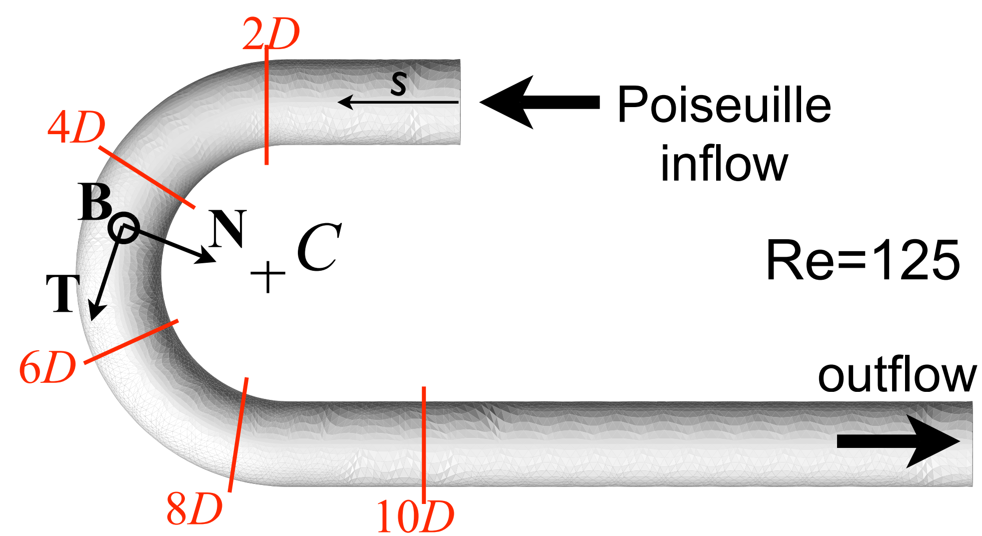
Axial & in-plane velocities

ω_s, in-plane CA



Axial vorticity

$$\omega_s = \frac{1}{r}\frac{\partial(rw)}{\partial r} - \frac{1}{r}\frac{\partial v}{\partial \theta}$$



Single bend

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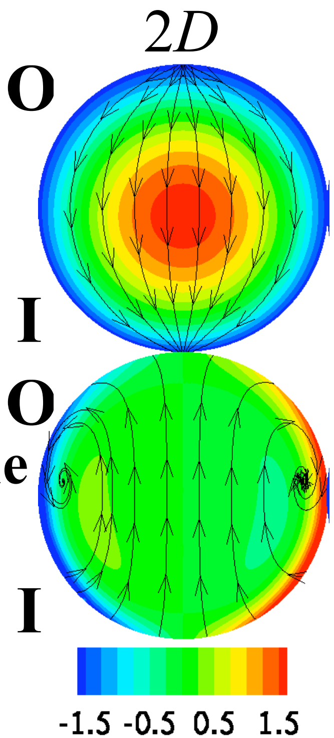

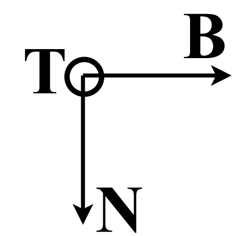

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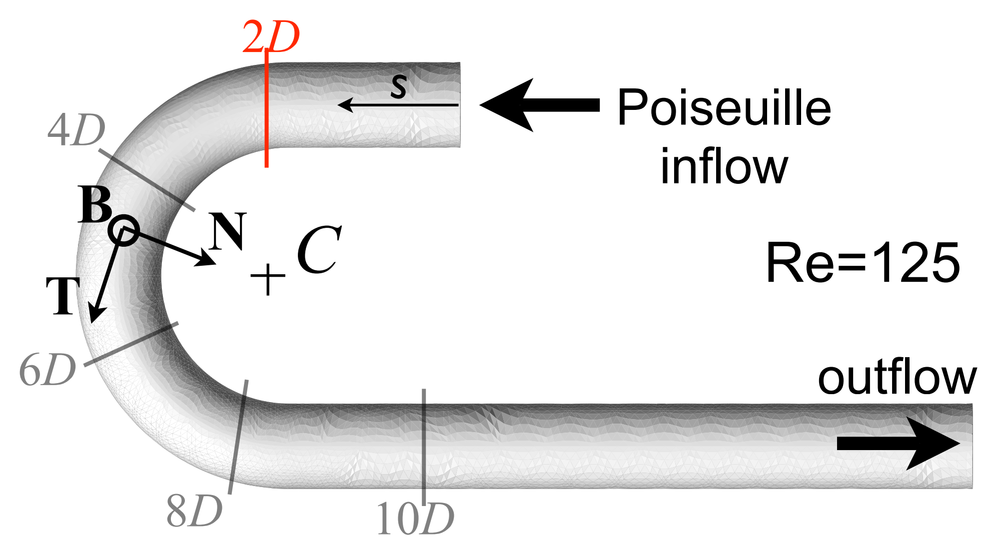
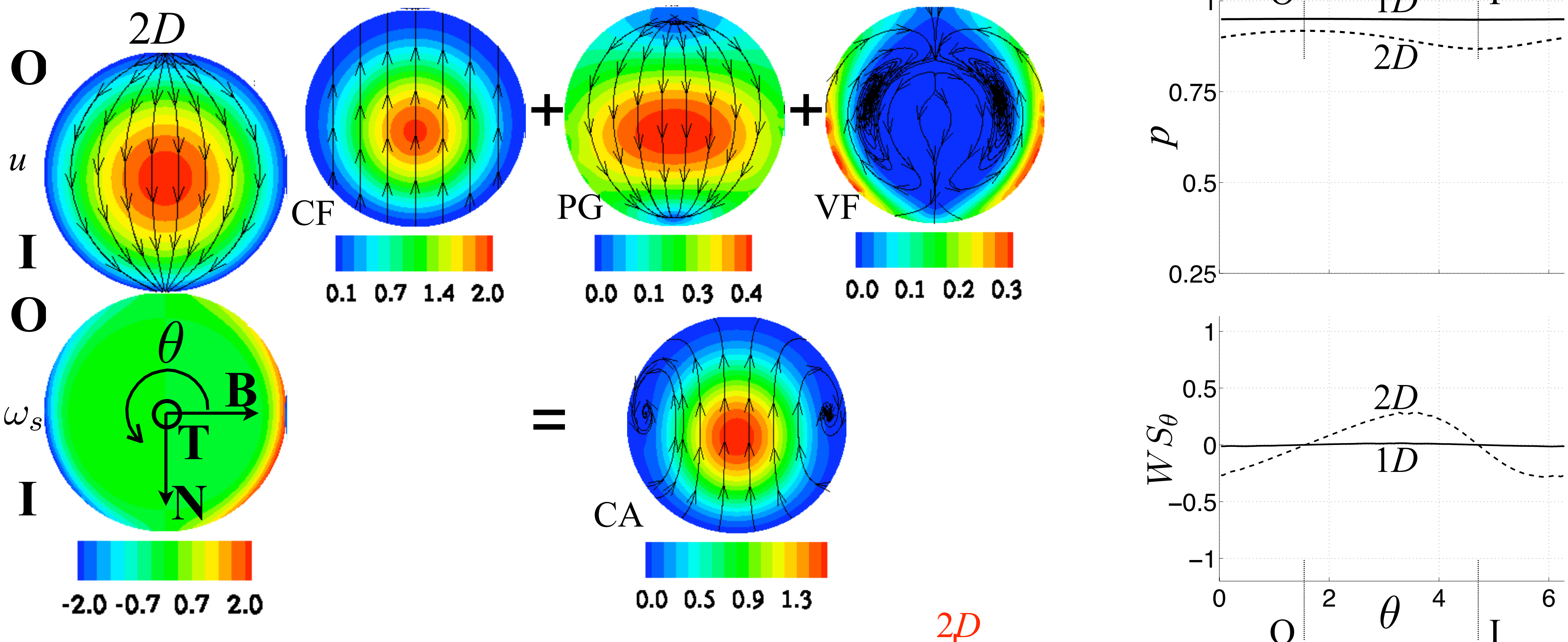
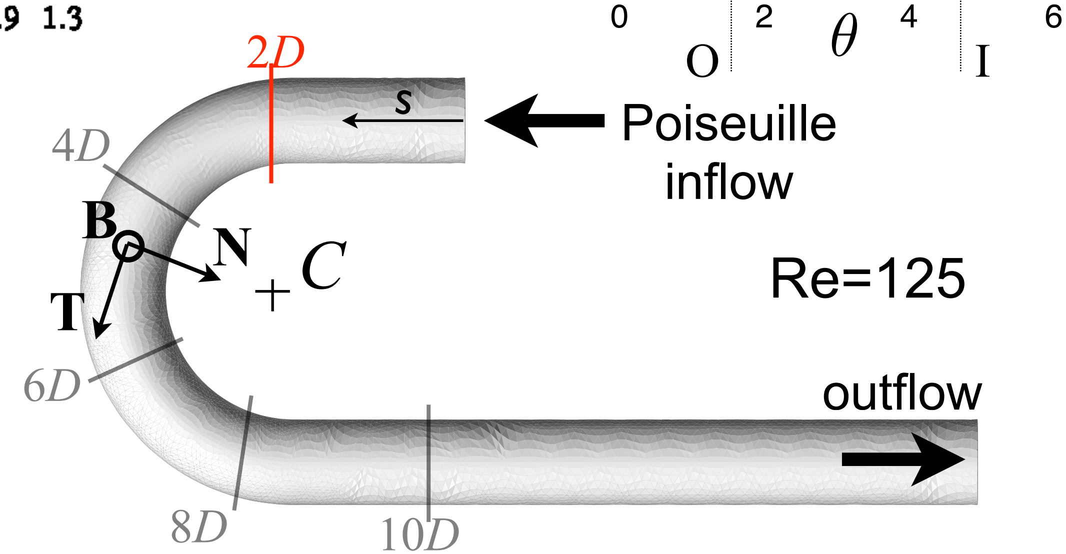


Entry region (s=2D)

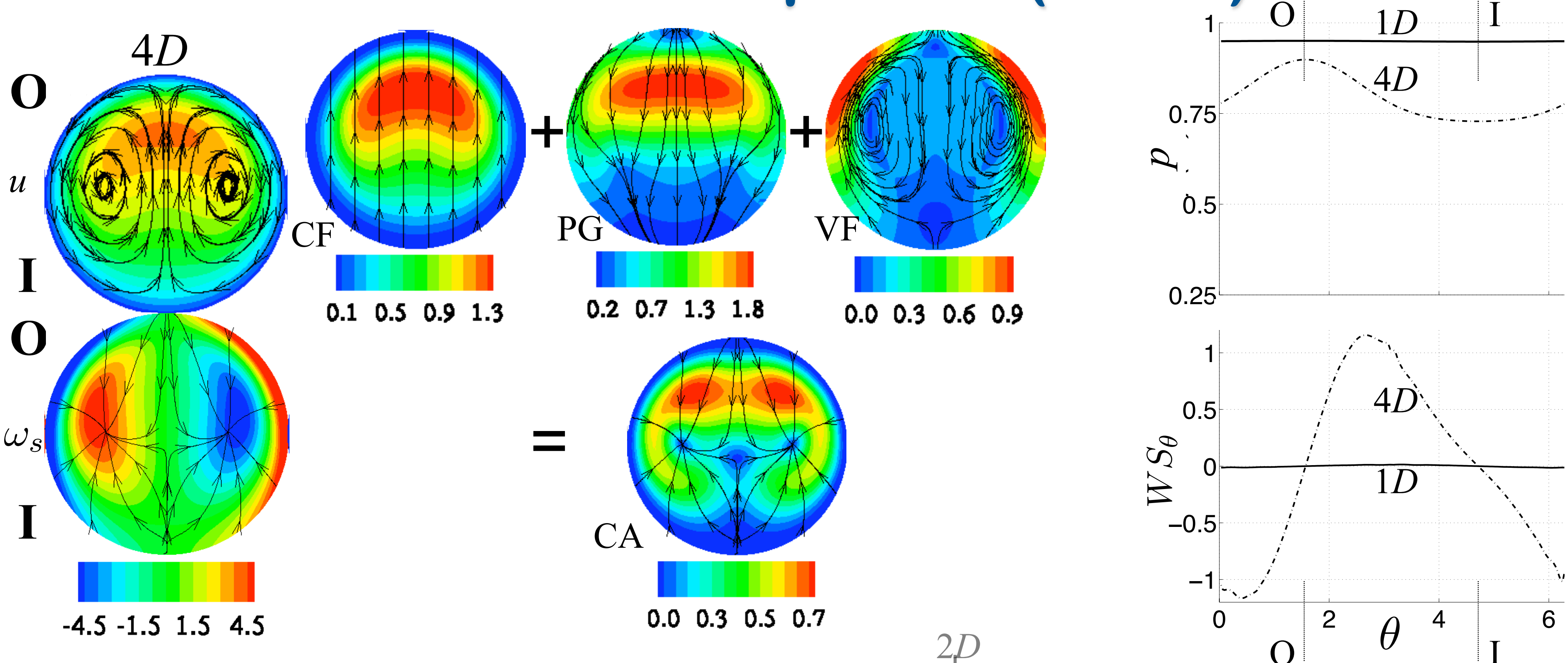


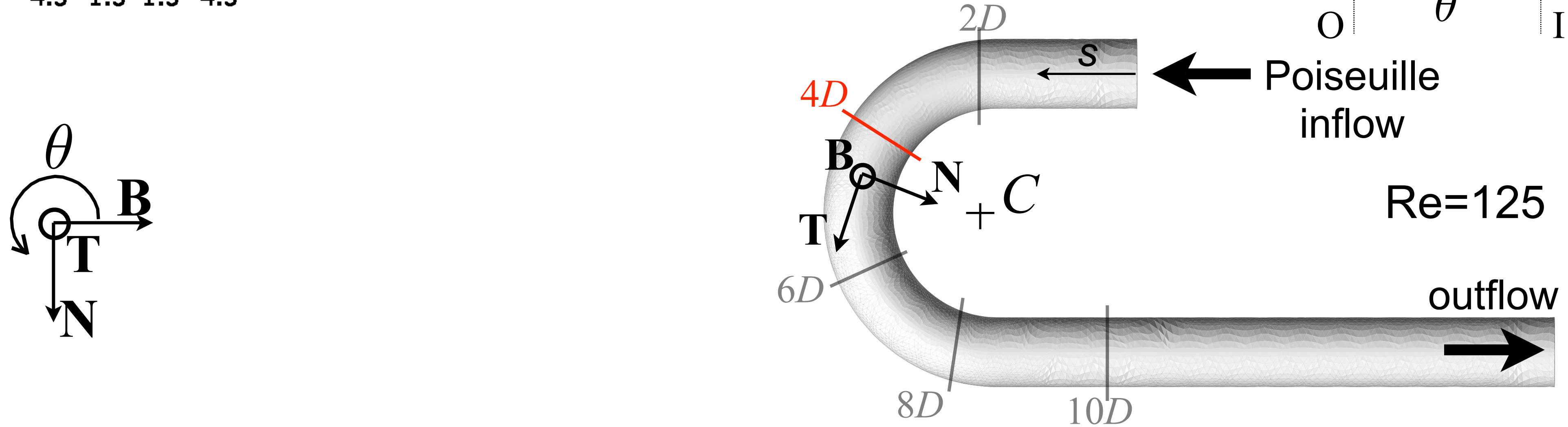


$$\mathrm{CF}_N = -\frac{\kappa u^2}{h}$$

$$WS_\theta = -\nu \rho \omega_s|_{r=D/2}$$

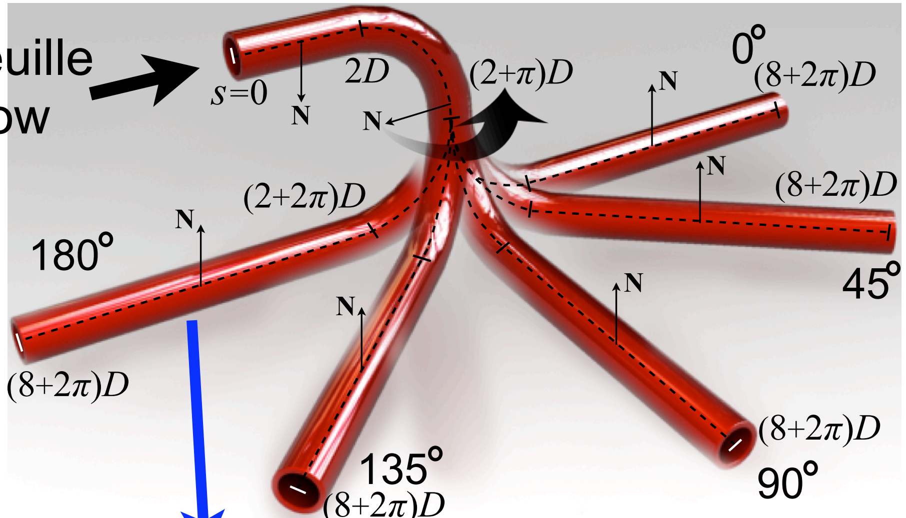
Flow development (s=4D)



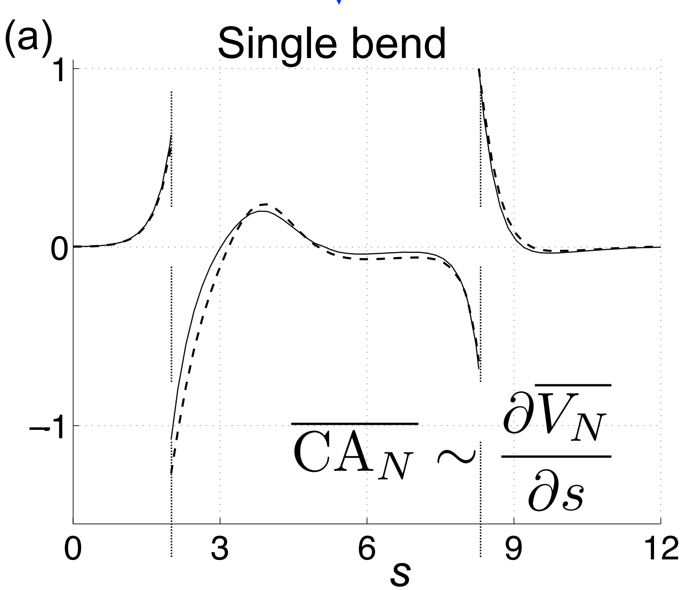


Average in-plane velocities and CAs

Poiseuille inflow

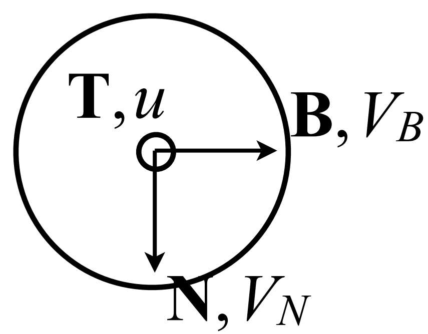


Re=125



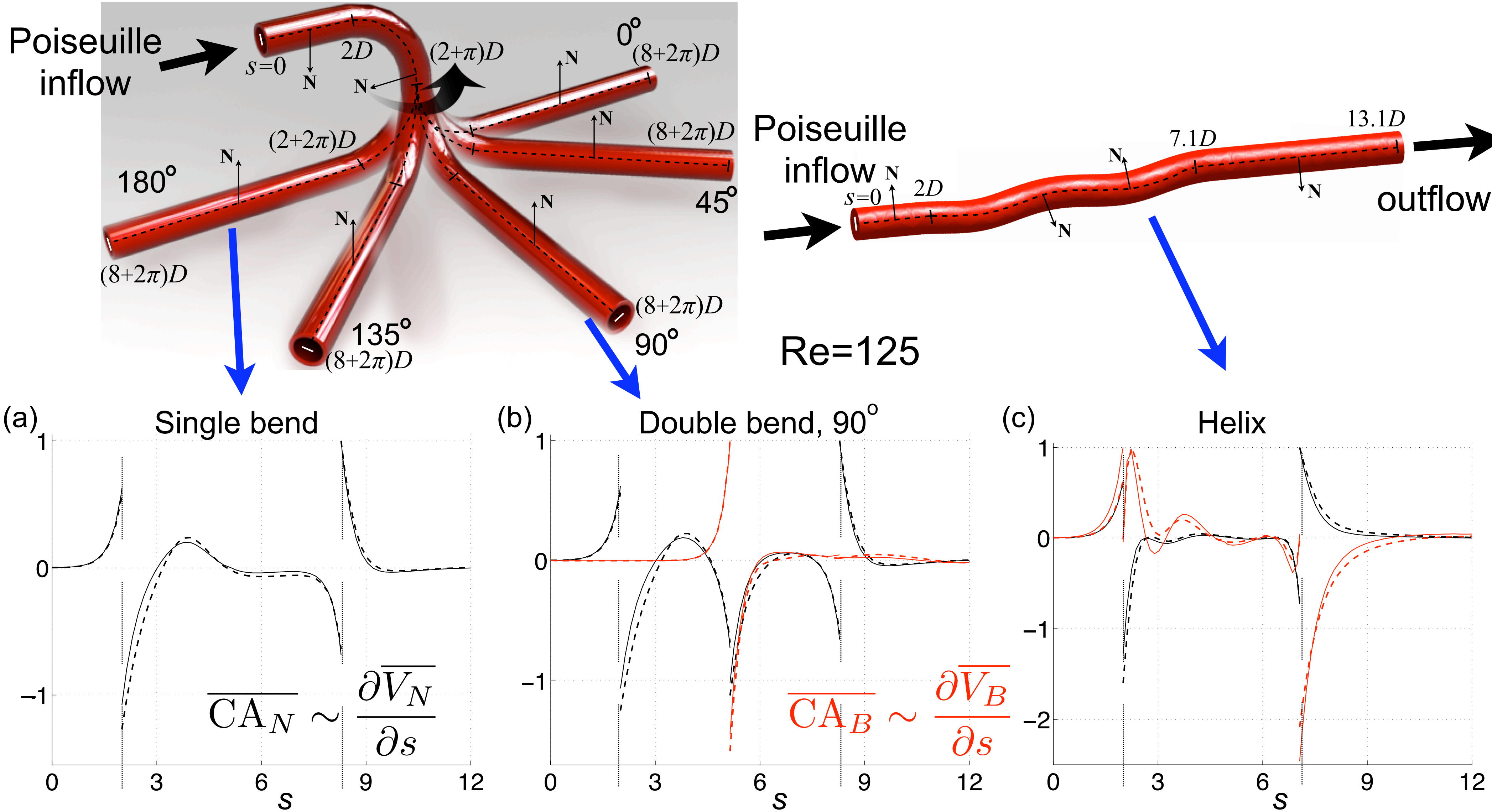
$$\overline{\mathrm{CA}_N} \sim \frac{\partial \overline{V_N}}{\partial s}$$

$$\mathrm{CA}_N = \frac{u}{h}\frac{\partial V_N}{\partial s} + V_N \frac{\partial V_N}{\partial n} + V_B \frac{\partial V_N}{\partial b},$$



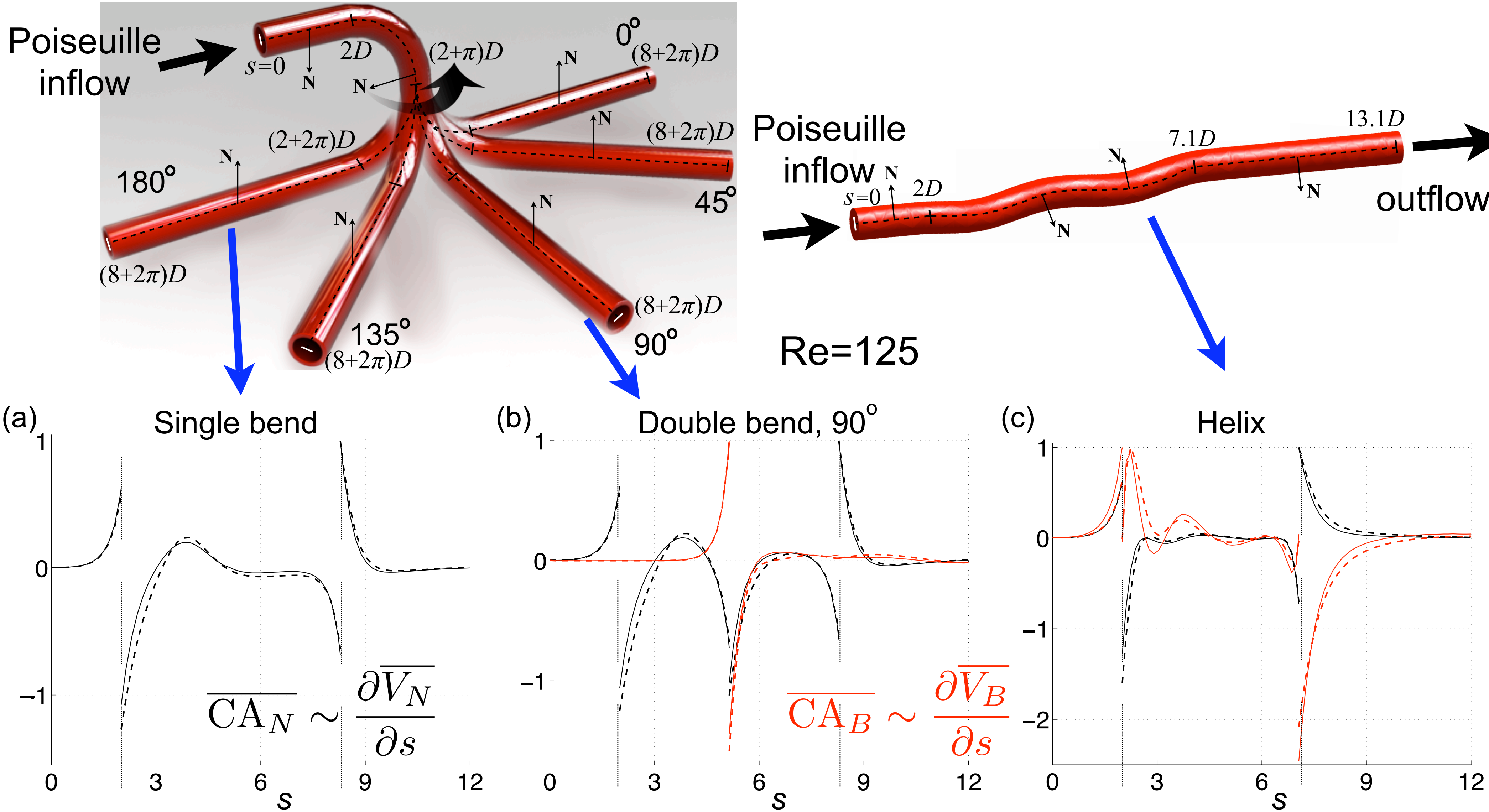
Given a field $\xi(s, r, \theta)$, we define its cross-sectional average $\bar{\xi}$ at $\mathbf{R}(s)$ as $\bar{\xi} = \frac{1}{S}\int_S \xi \mathrm{d}A$.

Average in-plane velocities and CAs



Cross-sectional averages in the direction of the normal (**N**) and binormal (**B**).

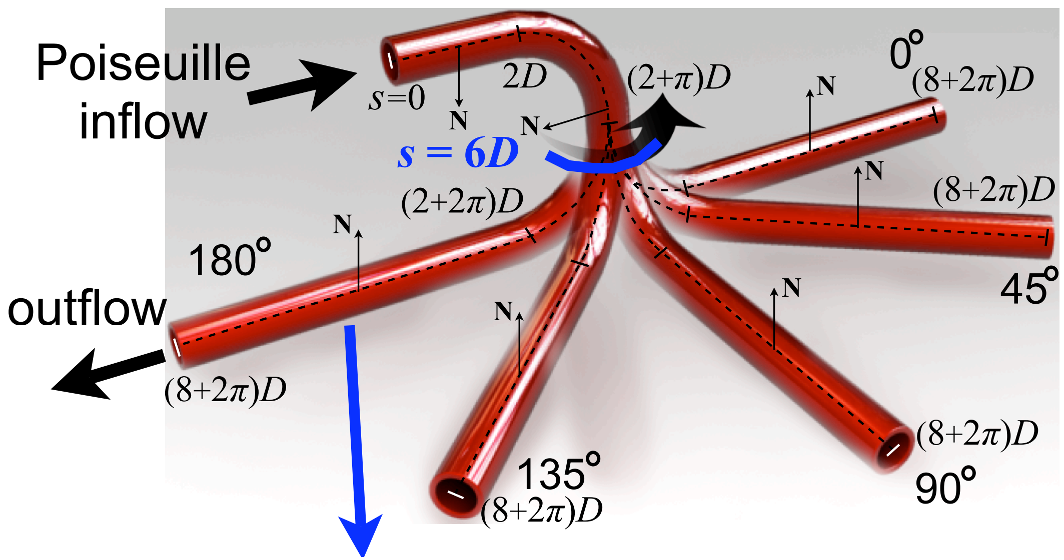
Average in-plane velocities and CAs

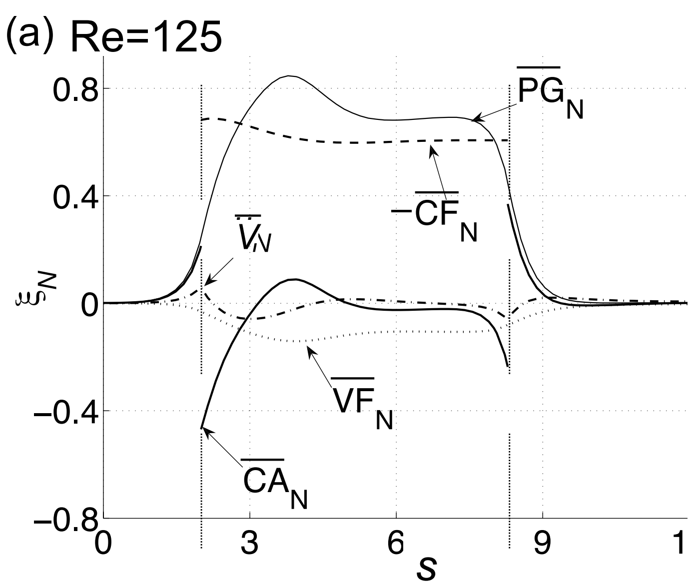


$\overline{V_N}$ mainly governed by $\overline{\mathrm{CA}_N} = \overline{\mathrm{CF}_N} + \overline{\mathrm{TF}_N} + \overline{\mathrm{PG}_N} + \overline{\mathrm{VF}_N}$

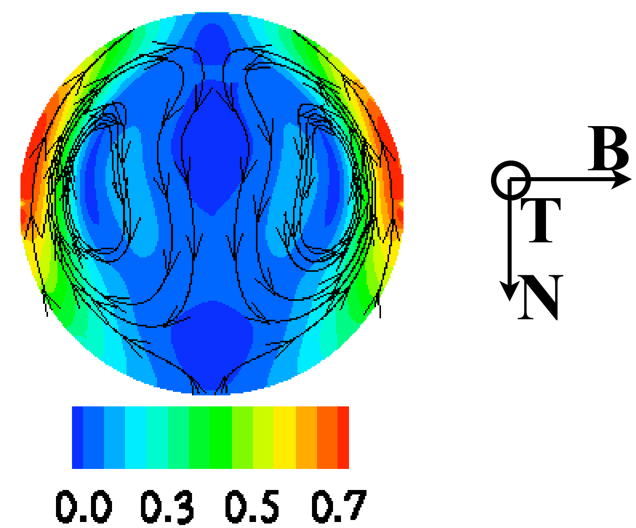
$\overline{V_B}$ mainly governed by $\overline{\mathrm{CA}_B} = \overline{\mathrm{TF}_B} + \overline{\mathrm{PG}_B} + \overline{\mathrm{VF}_B}$

Average in-plane forces - Single bend





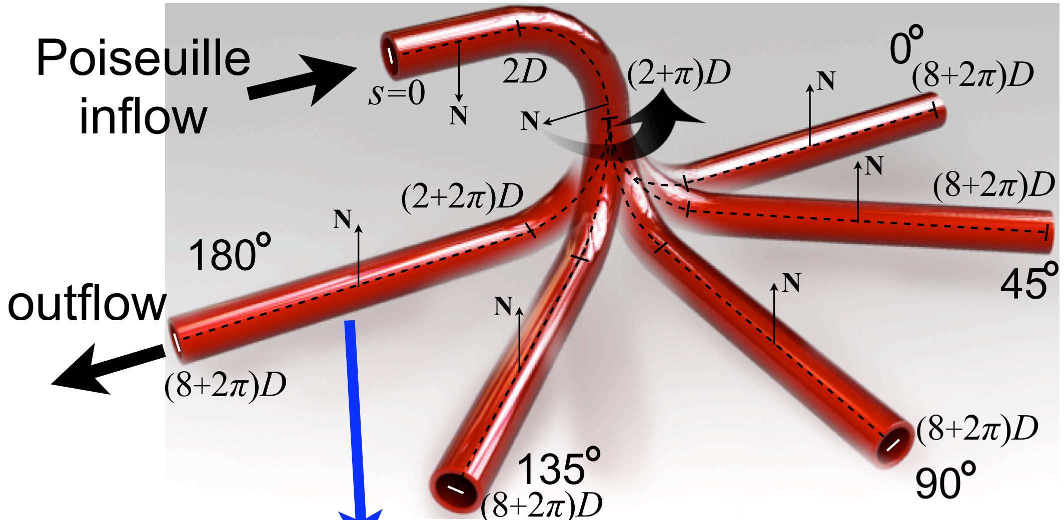
In-plane VF at $s = 6D$

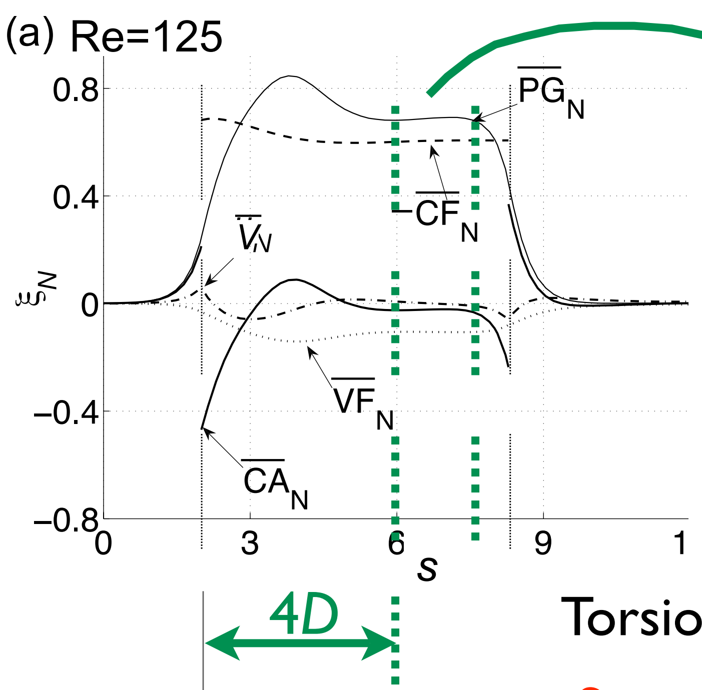


Torsion = 0: $\overline{\mathrm{CA}_N} = \overline{\mathrm{CF}_N} + \cancel{\overline{\mathrm{TF}_N}} + \overline{\mathrm{PG}_N} + \overline{\mathrm{VF}_N}$

Symmetry: $\overline{\mathrm{CA}_B} = \overline{\mathrm{TF}_B} = \overline{\mathrm{PG}_B} = \overline{\mathrm{VF}_B} = 0$

Close to full flow development





- Close to fully developed flow
- For fully developed flow we have:

$$\overline{V_N} = \overline{\mathrm{CA}_N} = 0$$
$$\overline{\mathrm{PG}_N} = -\overline{\mathrm{CF}_N} - \overline{\mathrm{VF}_N}$$

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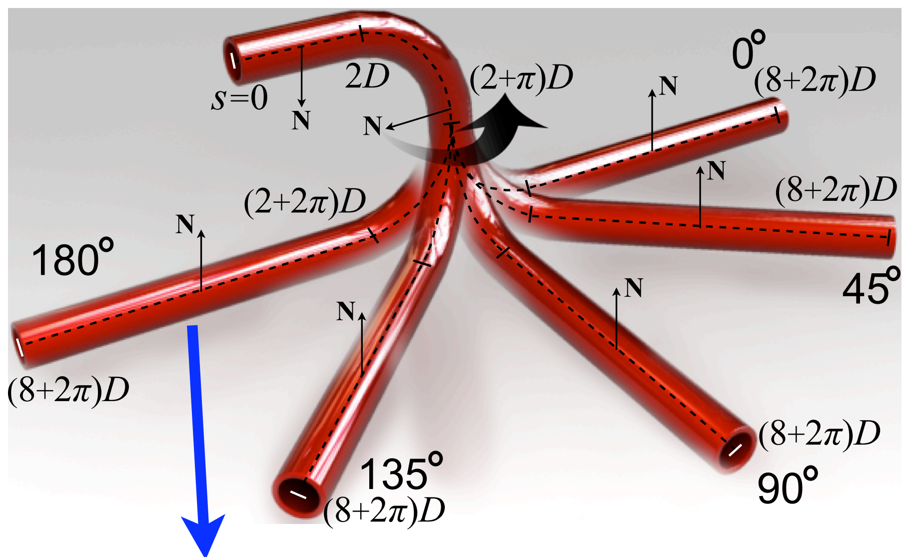
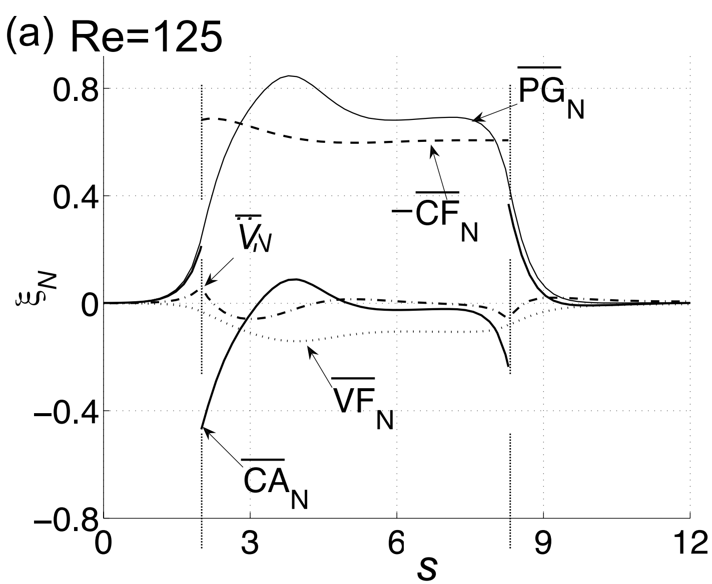
Analogy with an underdamped oscillator





$\overline{V_N}$ and $\overline{\mathrm{CA}_N}$ play the role of the velocity and acceleration of an underdamped oscillator around the fully-developed state, with

$\overline{\mathrm{CF}_N}$: driving force

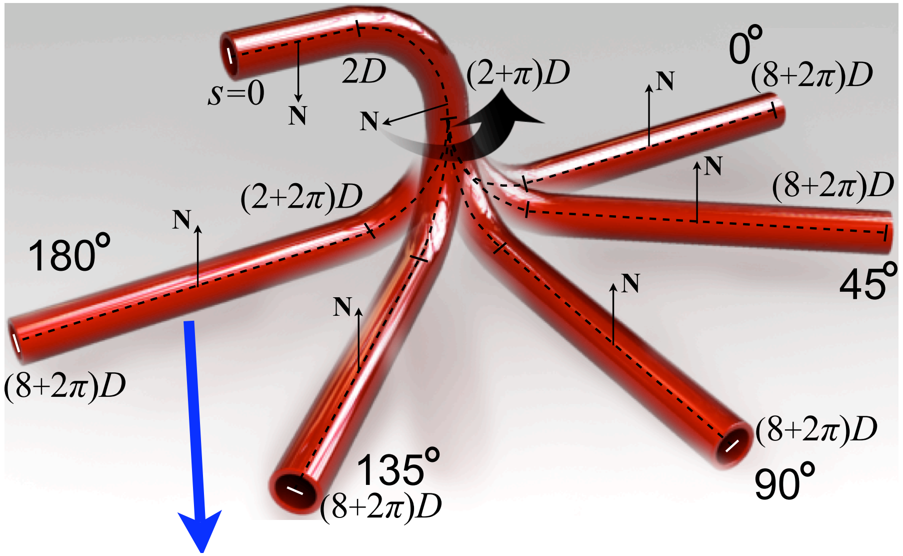
$\overline{\mathrm{PG}_N}$: restoring force

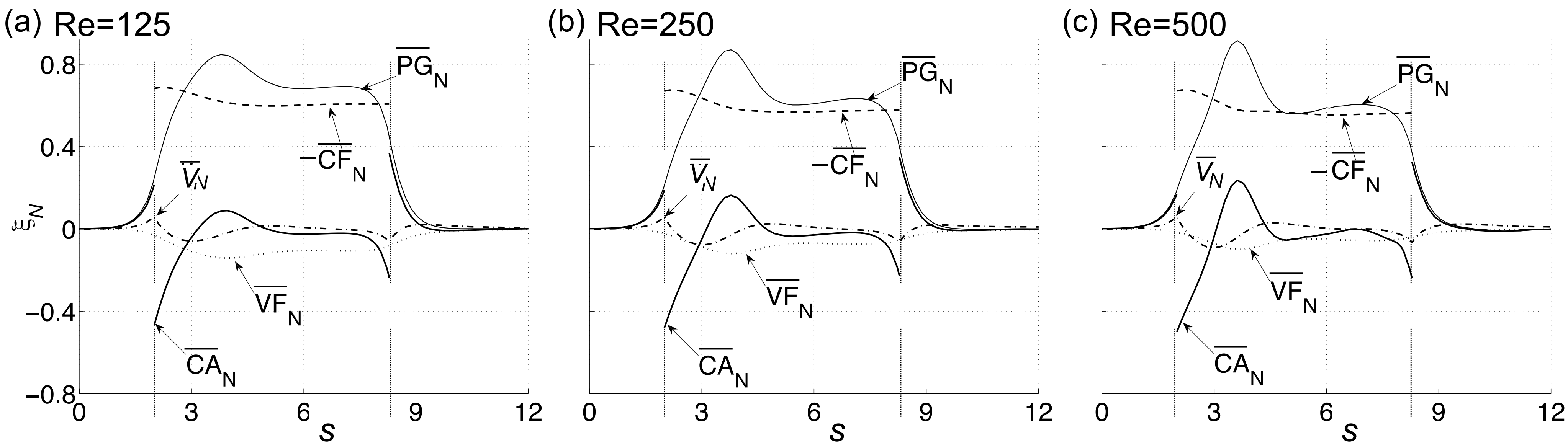
$\overline{\mathrm{VF}_N}$: frictional force

Torsion = 0: $\mathrm{CA}_N = \mathrm{CF}_N + \cancel{\mathrm{TF}_N} + \mathrm{PG}_N + \mathrm{VF}_N$

Symmetry: $\overline{\mathrm{CA}_B} = \overline{\mathrm{TF}_B} = \overline{\mathrm{PG}_B} = \overline{\mathrm{VF}_B} = 0$

Higher Reynolds numbers





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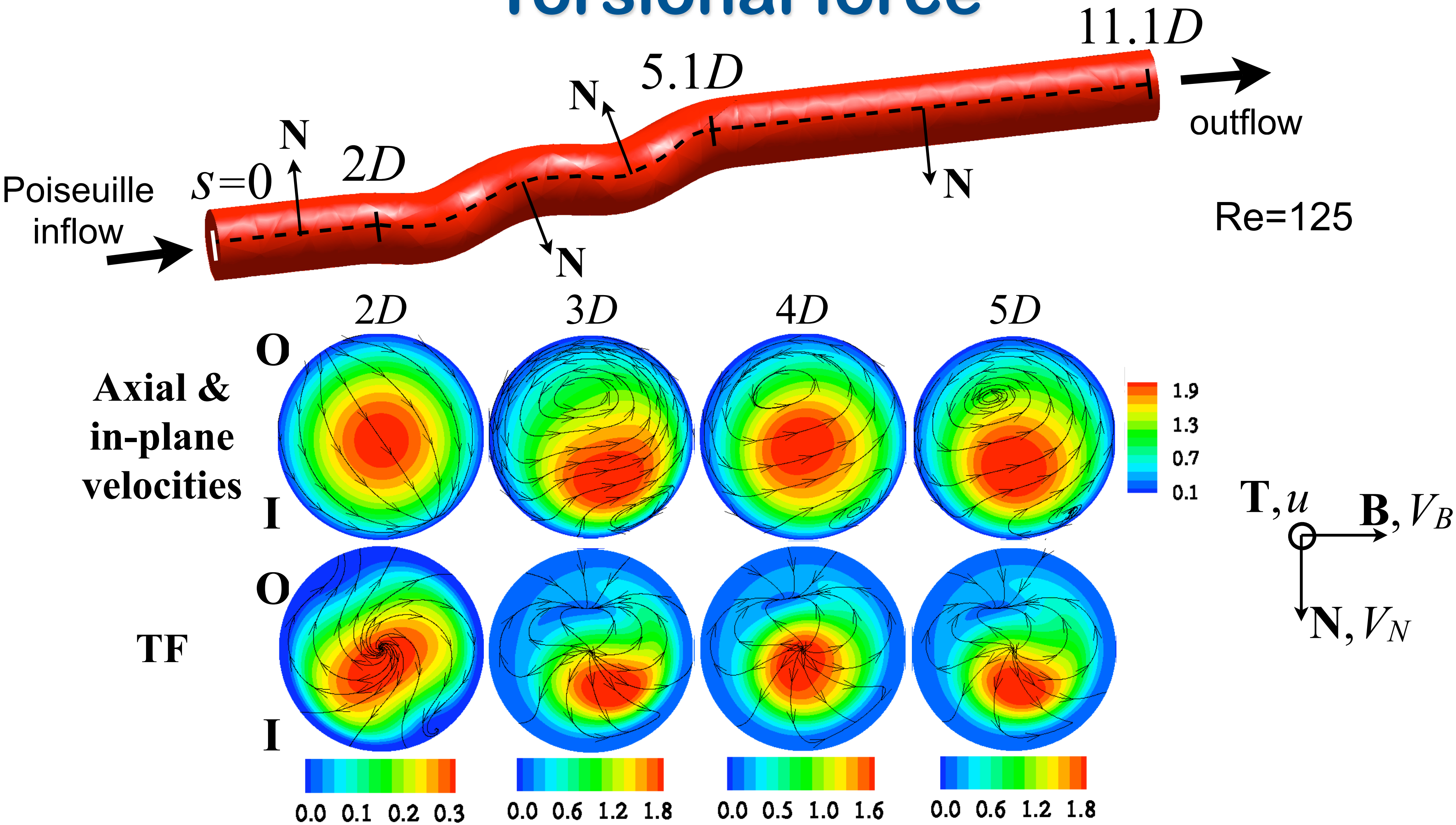
$\overline{V_B}$ mainly governed by $\overline{\mathrm{CA}_B} = \overline{\mathrm{TF}_B} + \overline{\mathrm{PG}_B} + \overline{\mathrm{VF}_B}$

Torsional force



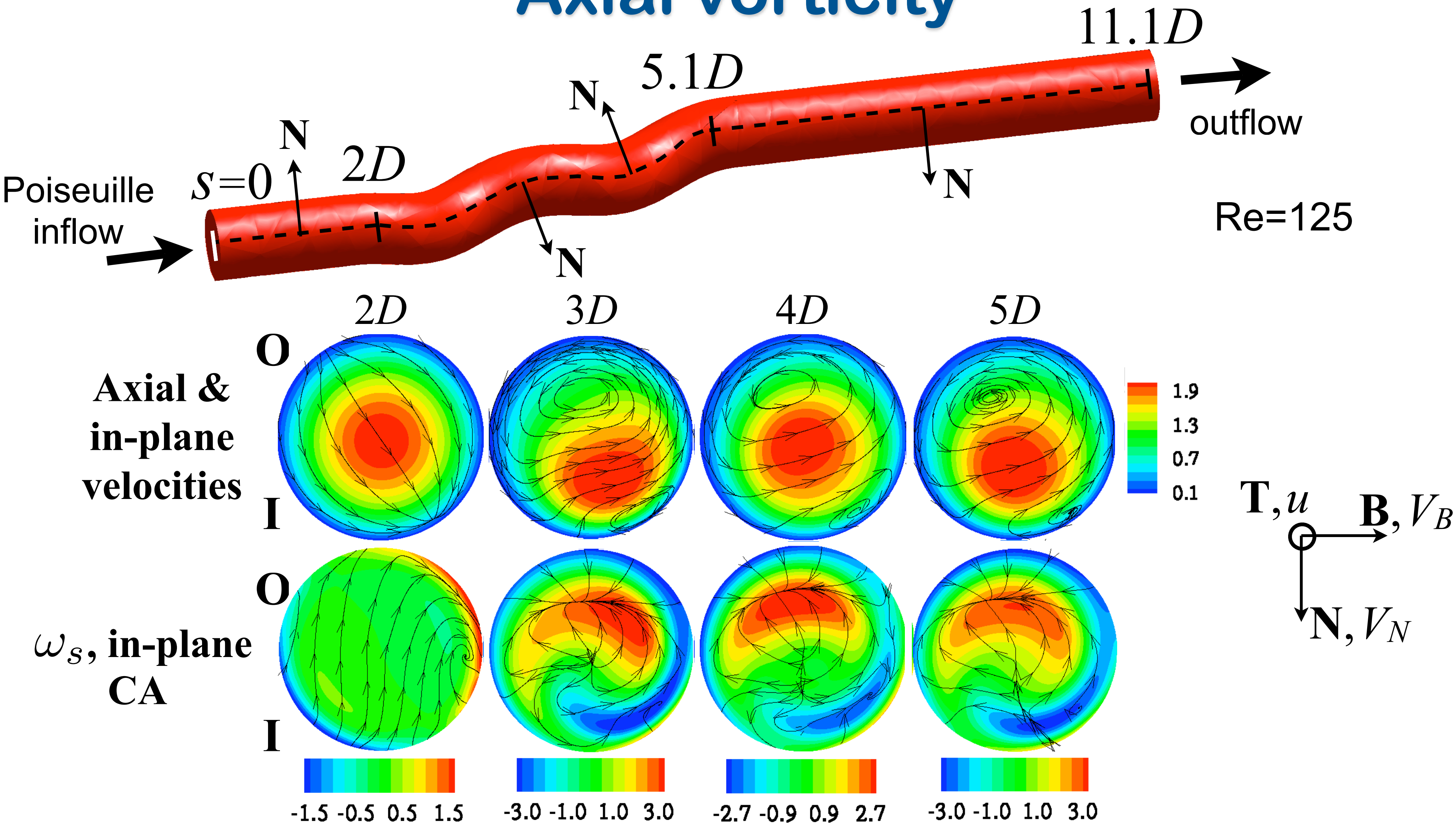
$$\mathbf{TF} = (0, \mathrm{TF}_N, \mathrm{TF}_B)$$

$$\mathrm{TF}_N = \frac{\tau u}{h} V_B, \quad \mathrm{TF}_B = -\frac{\tau u}{h} V_N$$

$$\mathrm{CA}_N = \mathrm{CF}_N + \mathrm{TF}_N + \mathrm{PG}_N + \mathrm{VF}_N$$

$$\mathrm{CA}_B = \mathrm{TF}_B + \mathrm{PG}_B + \mathrm{VF}_B$$

Axial vorticity

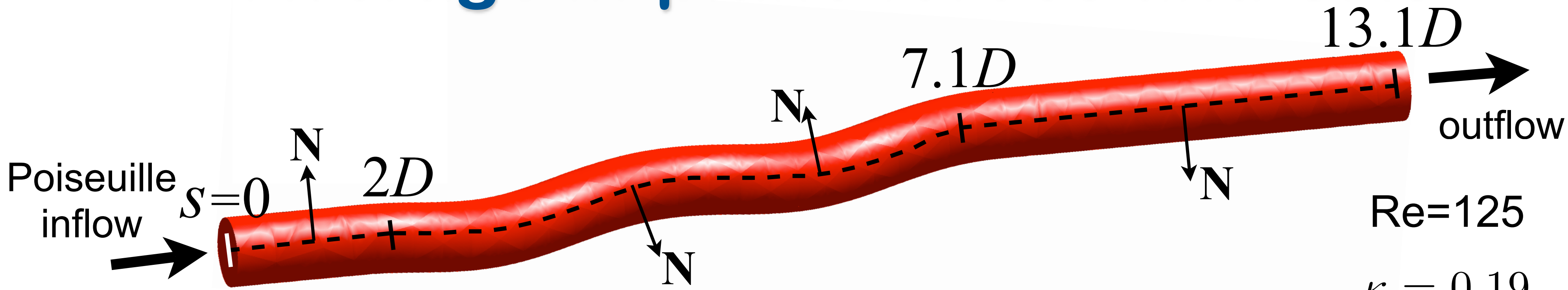


$$\omega_s = \frac{1}{r}\frac{\partial(rw)}{\partial r} - \frac{1}{r}\frac{\partial v}{\partial \theta}$$

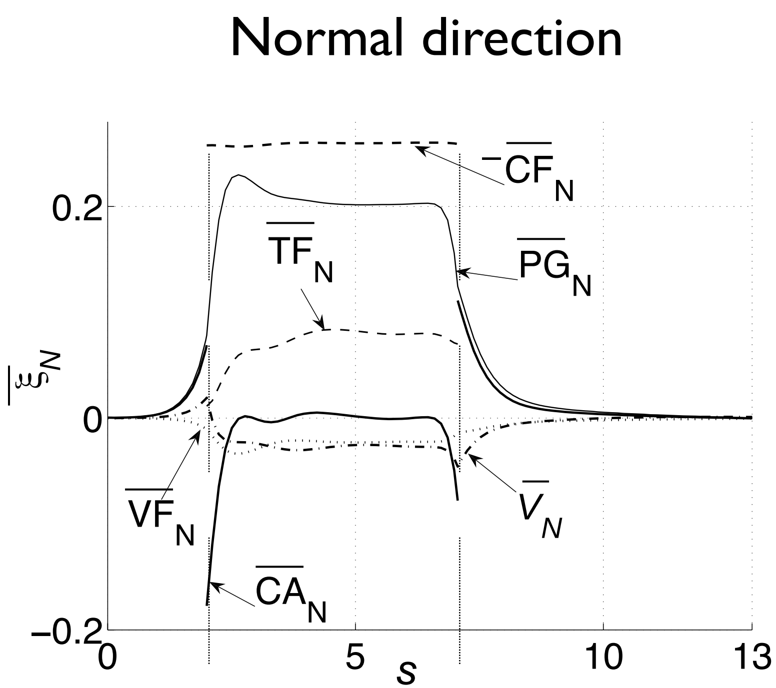
$$\mathrm{CA}_N = \mathrm{CF}_N + \mathrm{TF}_N + \mathrm{PG}_N + \mathrm{VF}_N$$

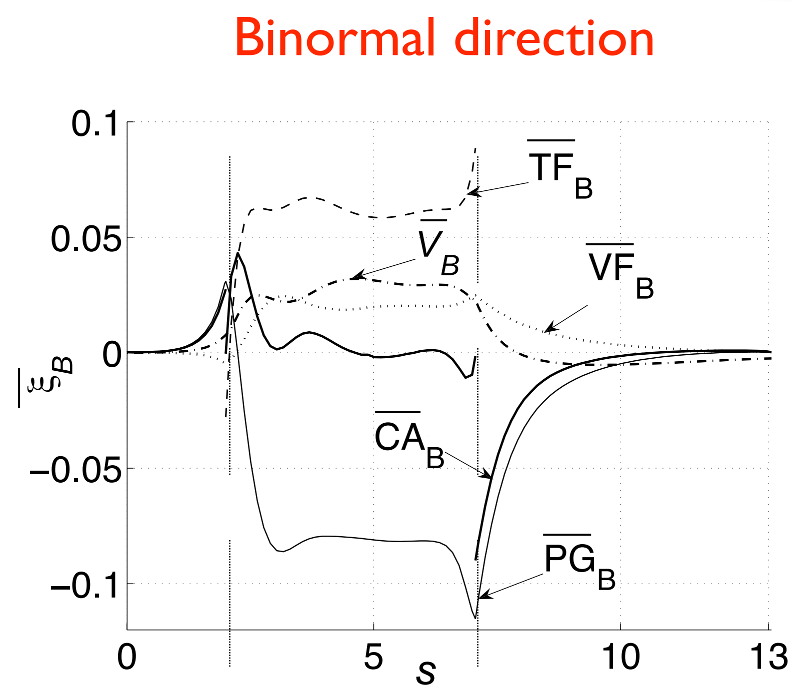
$$\mathrm{CA}_B = \mathrm{TF}_B + \mathrm{PG}_B + \mathrm{VF}_B$$

Average in-plane forces and CAs



$\kappa = 0.19$
$\tau = 1.23$





$$\mathrm{CF}_N = -\frac{\kappa u^2}{h}$$

$$\mathrm{TF}_N = \frac{\tau u}{h} V_B$$

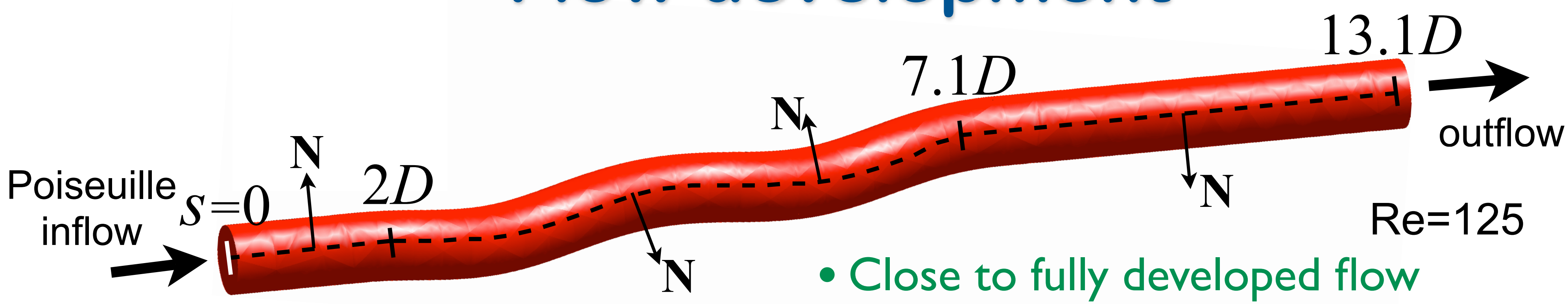
$$\mathrm{TF}_B = -\frac{\tau u}{h} V_N$$

$\overline{V_N}$ mainly governed by $\overline{\mathrm{CA}_N} = \overline{\mathrm{CF}_N} + \overline{\mathrm{TF}_N} + \overline{\mathrm{PG}_N} + \overline{\mathrm{VF}_N}$

$\overline{V_B}$ mainly governed by $\overline{\mathrm{CA}_B} = \overline{\mathrm{TF}_B} + \overline{\mathrm{PG}_B} + \overline{\mathrm{VF}_B}$

$$\bar{\xi} = \frac{1}{S}\int_S \xi \, \mathrm{d}A$$

Flow development



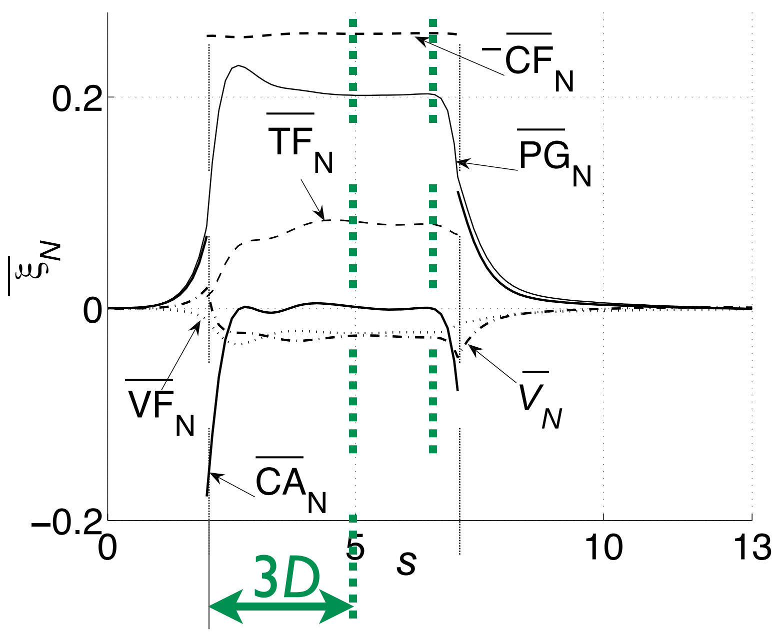
Re=125

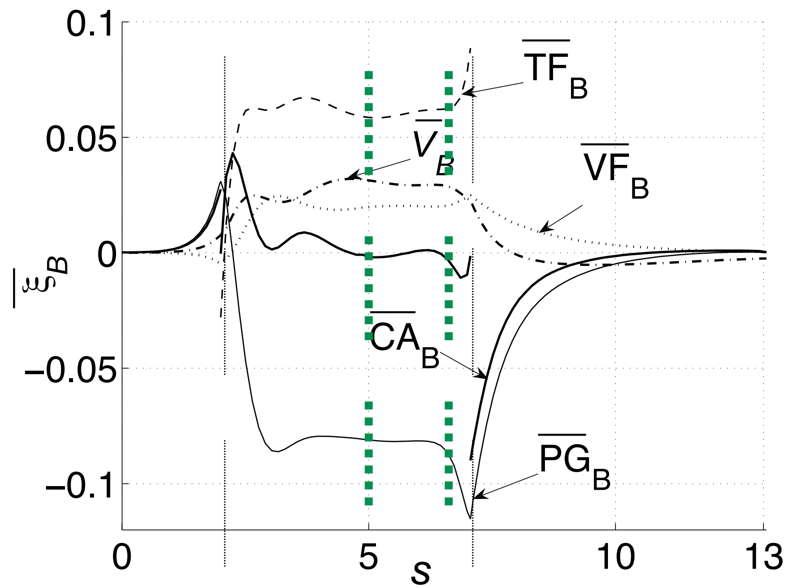
- Close to fully developed flow
- For fully developed flow we have:

$$\overline{\mathrm{CA}_N} = \overline{\mathrm{CA}_B} = 0$$

Normal direction

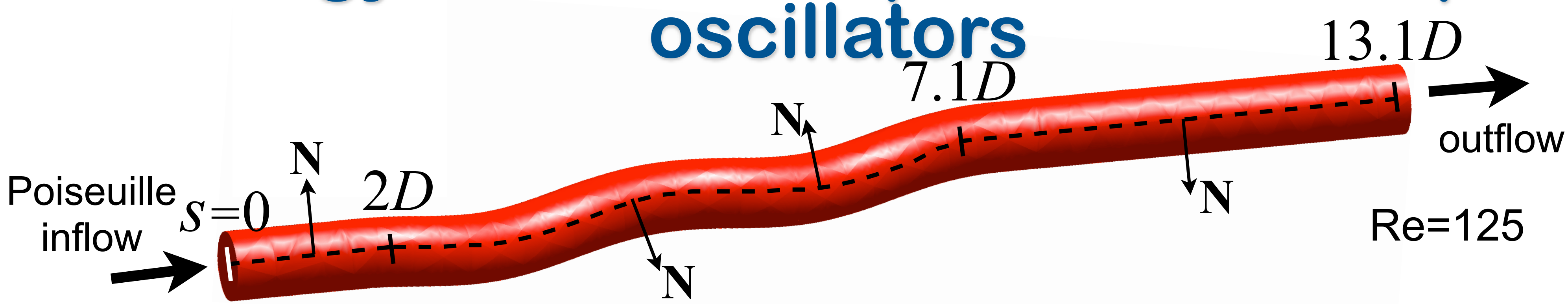
Binormal direction

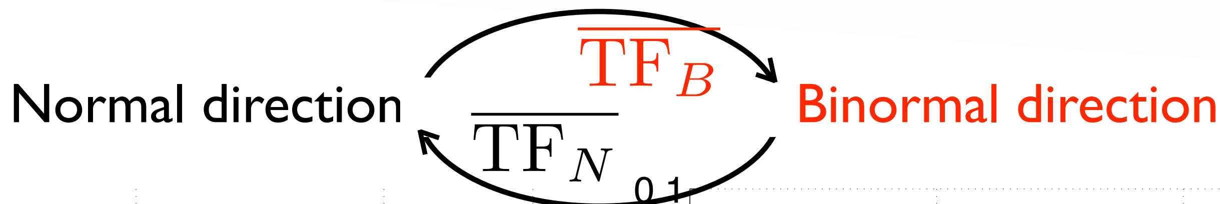




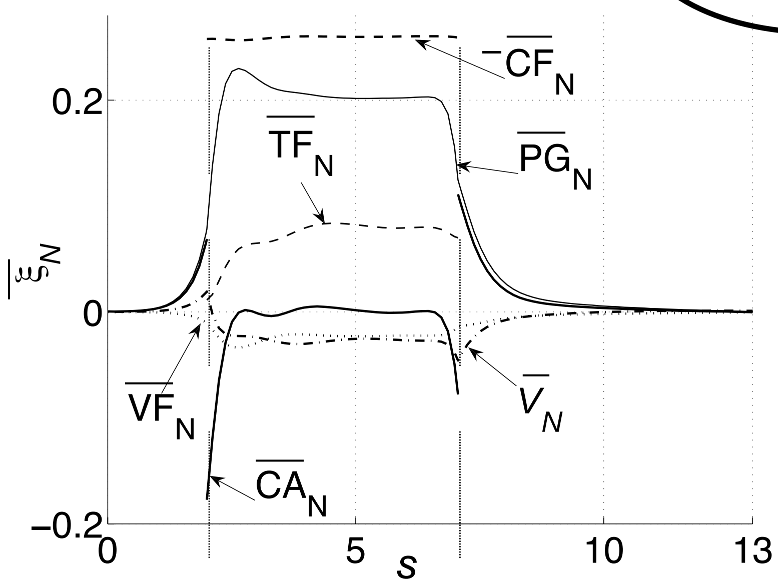
$$\overline{\mathrm{PG}_N} + \overline{\mathrm{TF}_N} = -\overline{\mathrm{CF}_N} - \overline{\mathrm{VF}_N} \qquad \overline{\mathrm{PG}_B} = -\overline{\mathrm{TF}_B} - \overline{\mathrm{VF}_B} \qquad \bar{\xi} = \frac{1}{S}\int_S \xi \,\mathrm{d}A$$

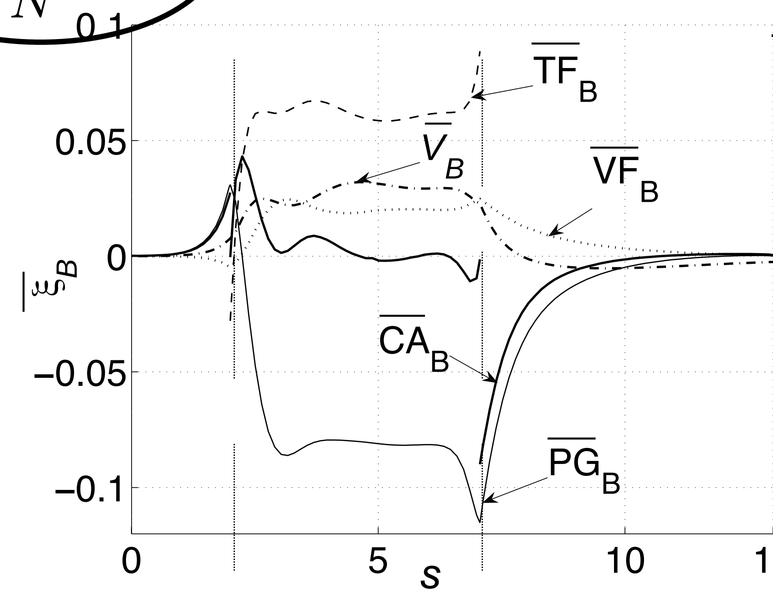
Analogy with two coupled underdamped oscillators





$\overline{\mathrm{CF}}_N$: driving force
$\overline{\mathrm{TF}}_N$: driving force
$\overline{\mathrm{PG}}_N$: restoring force
$\overline{\mathrm{VF}}_N$: frictional force





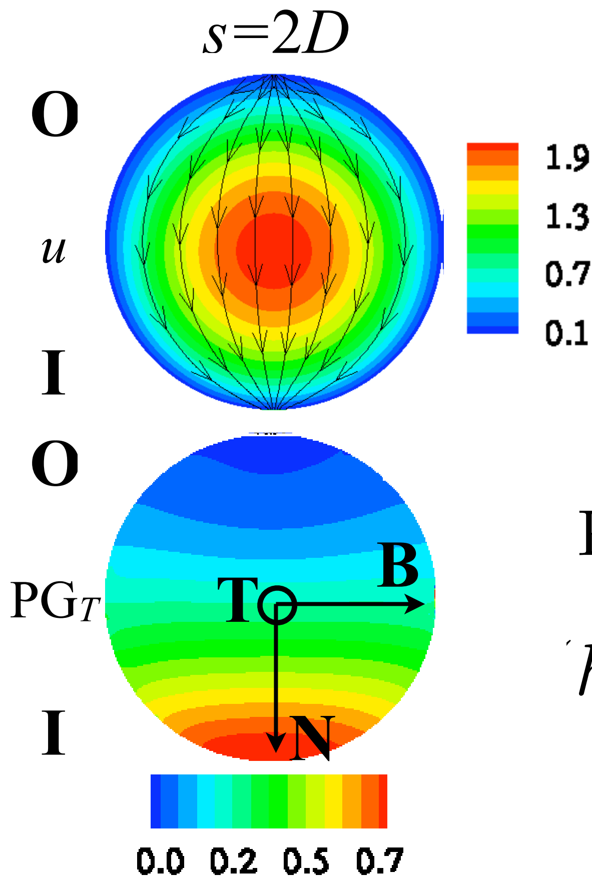
$\overline{\mathrm{TF}}_B$: driving force
$\overline{\mathrm{PG}}_B$: restoring force
$\overline{\mathrm{VF}}_B$: frictional force

$\overline{V_N}$ mainly governed by $\overline{\mathrm{CA}_N} = \overline{\mathrm{CF}_N} + \overline{\mathrm{TF}_N} + \overline{\mathrm{PG}_N} + \overline{\mathrm{VF}_N}$

$\overline{V_B}$ mainly governed by $\overline{\mathrm{CA}_B} = \overline{\mathrm{TF}_B} + \overline{\mathrm{PG}_B} + \overline{\mathrm{VF}_B}$

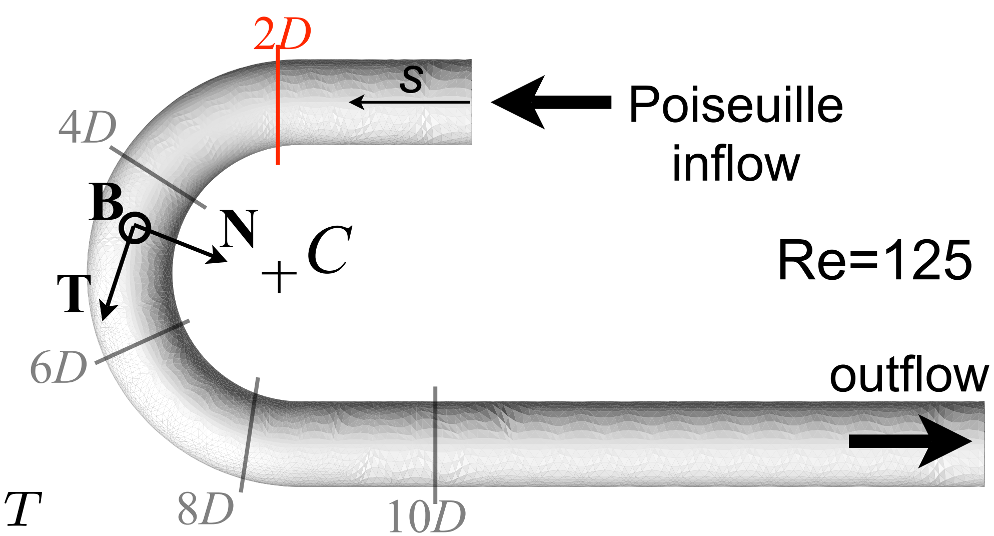
$$\overline{\xi} = \frac{1}{S}\int_S \xi \, \mathrm{d}A$$

Axial balance of momentum



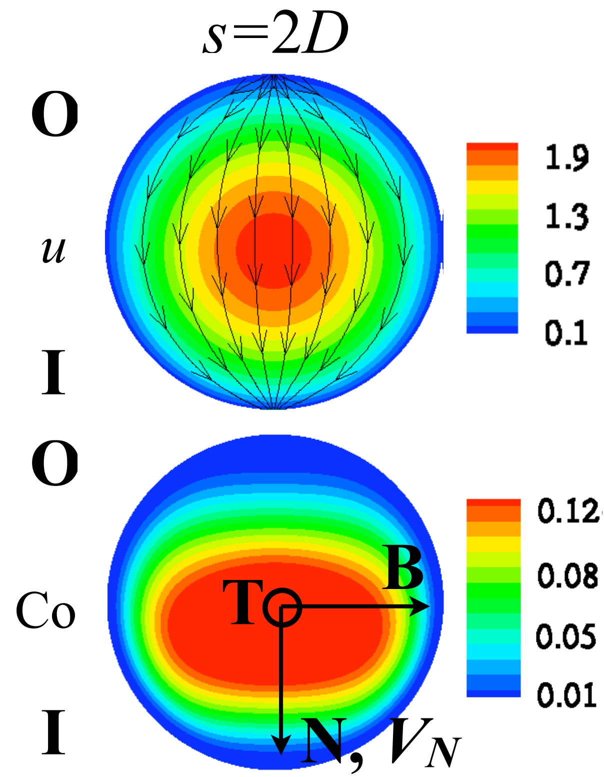
$$\mathrm{PG}_T = -\frac{1}{\rho h}\frac{\partial p}{\partial s}$$

$$h = 1 + \kappa r \sin(\theta + \phi)$$



$$\mathrm{CA}_T = \mathrm{Co} + \mathrm{PG}_T + \mathrm{VF}_T$$

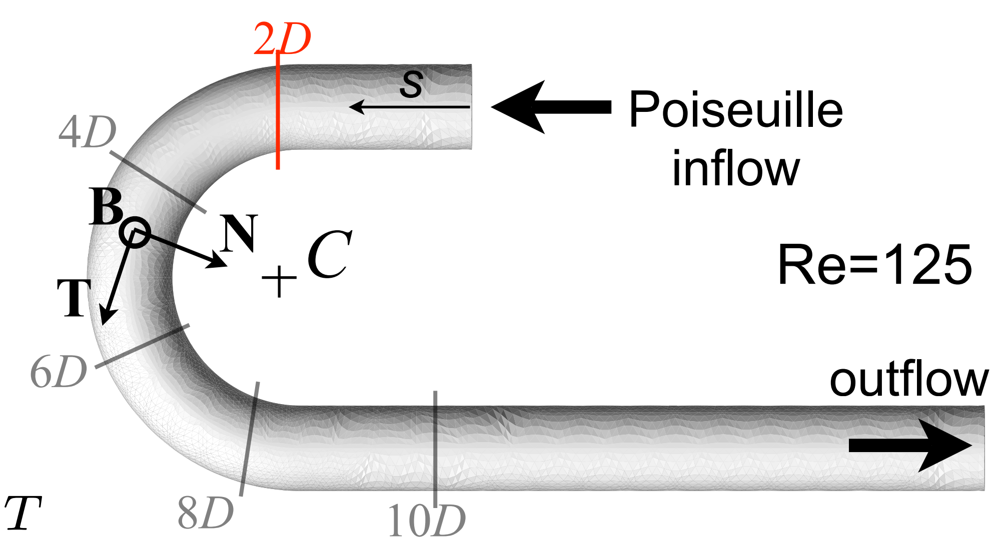
Coriolis force - Inlet



$$\mathrm{Co} = -\frac{\kappa u}{h}\left[v \sin(\theta+\phi) + w\cos(\theta+\phi)\right]$$
$$= \frac{\kappa u}{h} V_N$$

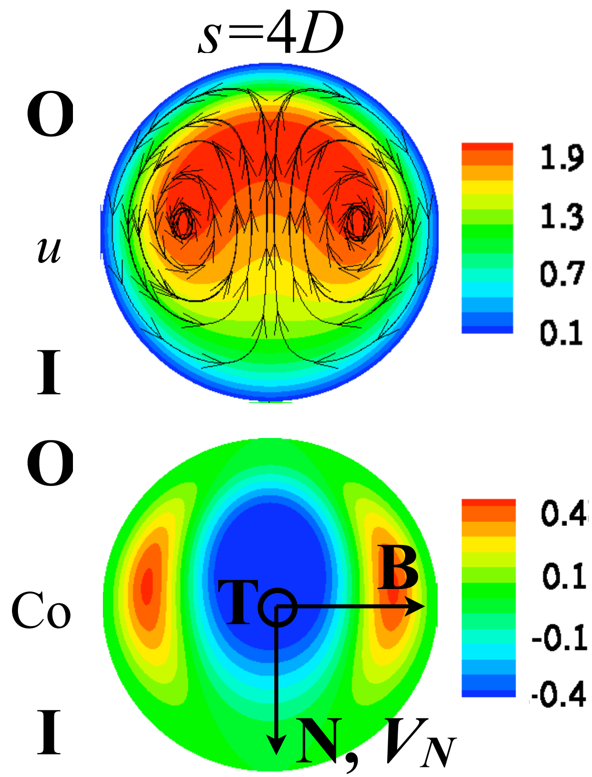
Co accelerates if V_N is centripetal

Co decelerates if V_N is centrifugal



$$\mathrm{CA}_T = \mathrm{Co} + \mathrm{PG}_T + \mathrm{VF}_T$$

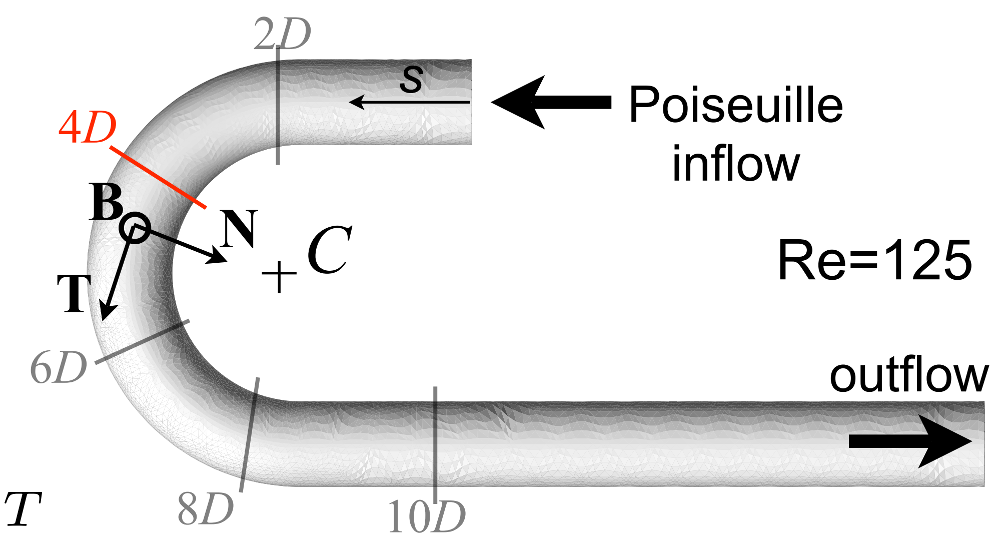
Coriolis force - Flow development



$$\mathrm{Co} = -\frac{\kappa u}{h}\left[v\sin(\theta+\phi) + w\cos(\theta+\phi)\right] = \frac{\kappa u}{h} V_N$$

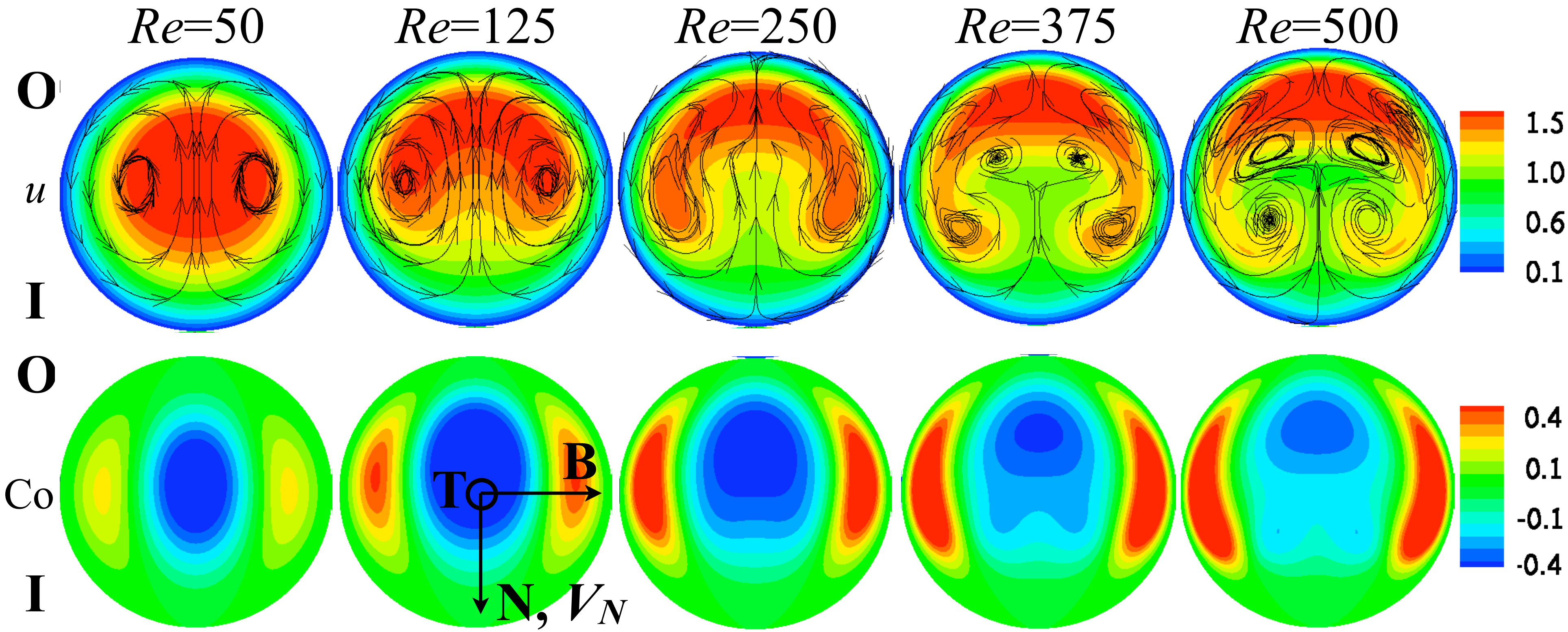
Co accelerates if V_N is centripetal

Co decelerates if V_N is centrifugal



$$\mathrm{CA}_T = \mathrm{Co} + \mathrm{PG}_T + \mathrm{VF}_T$$

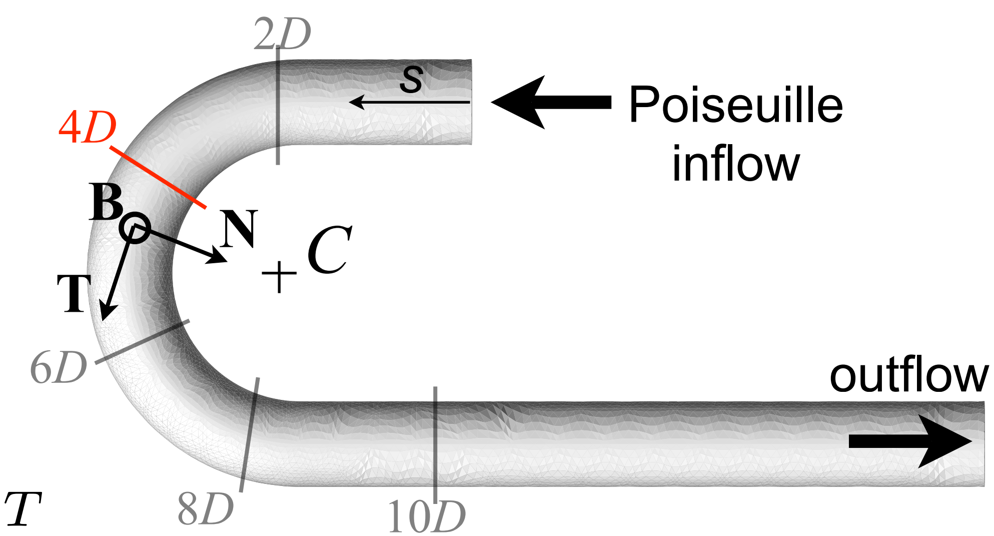
Changing velocity profiles



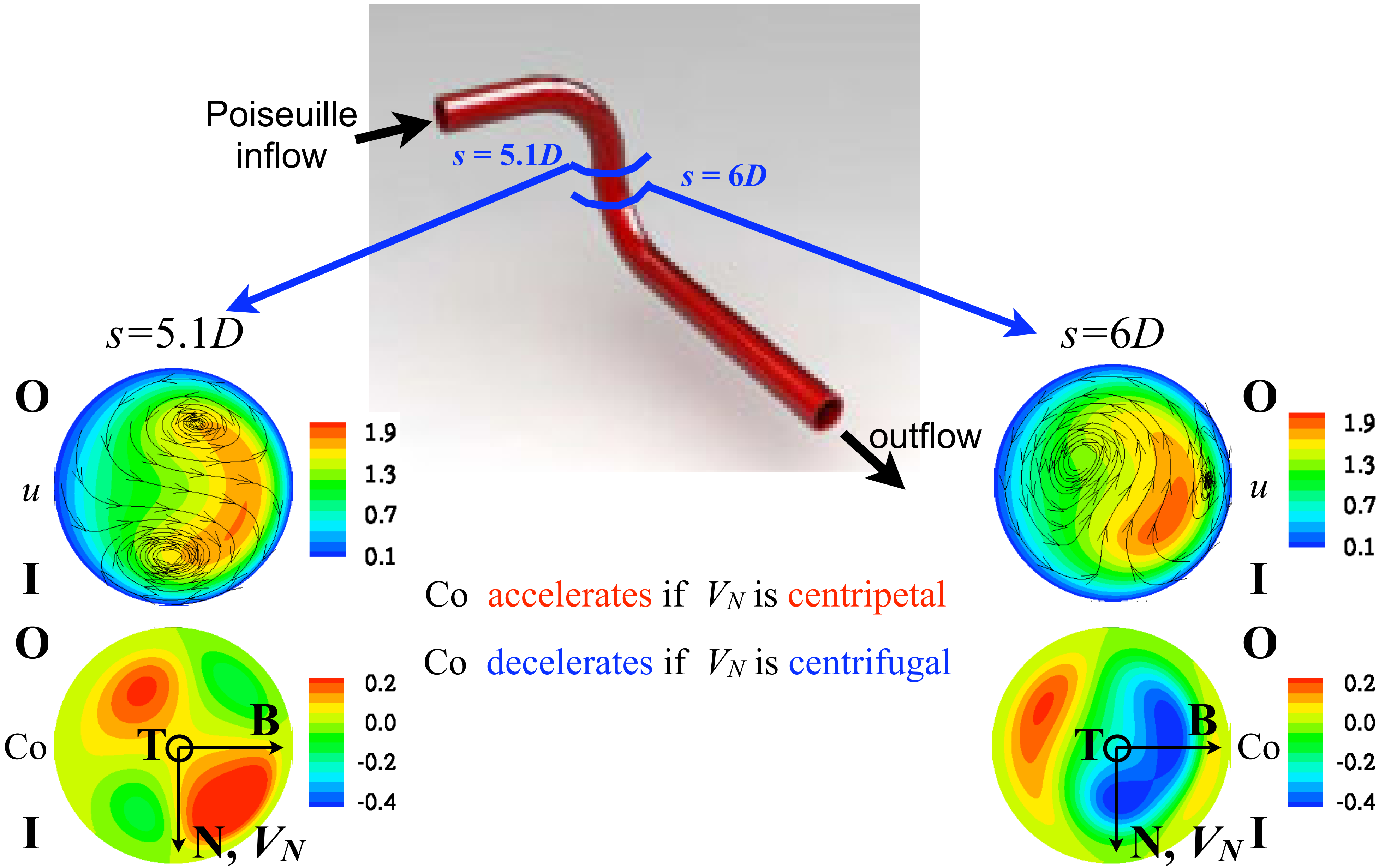
Co accelerates if V_N is centripetal

Co decelerates if V_N is centrifugal

$$\mathrm{CA}_T = \mathrm{Co} + \mathrm{PG}_T + \mathrm{VF}_T$$



Coriolis force in double bends



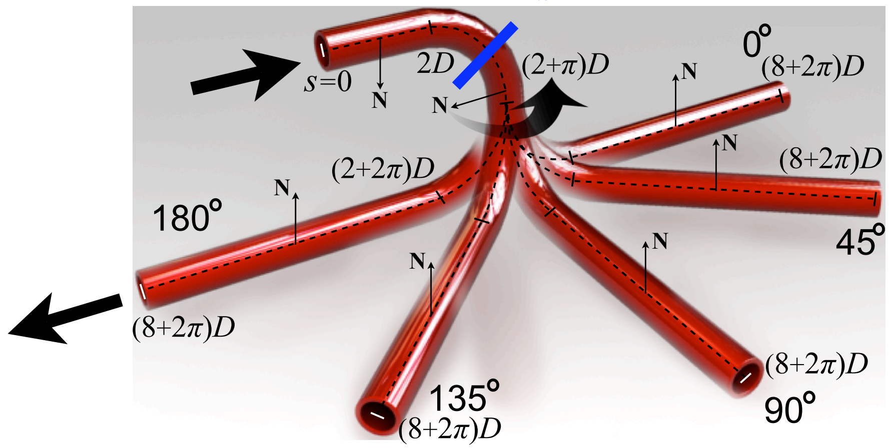
Co accelerates if V_N is centripetal

Co decelerates if V_N is centrifugal

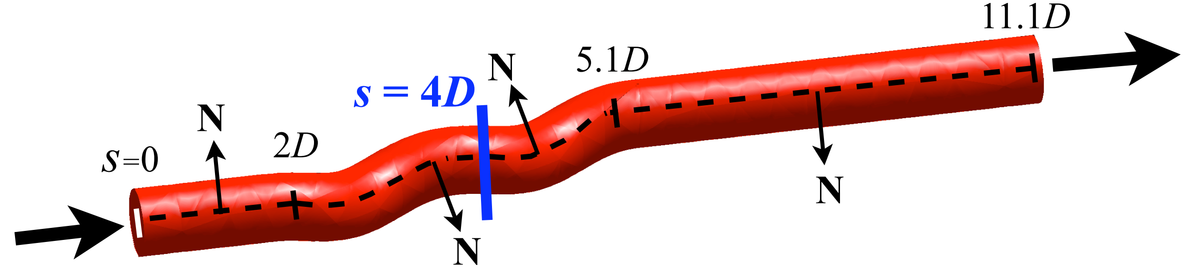
$$\mathrm{CA}_T = \mathrm{Co} + \mathrm{PG}_T + \mathrm{VF}_T$$

Effect of torsion on WSS

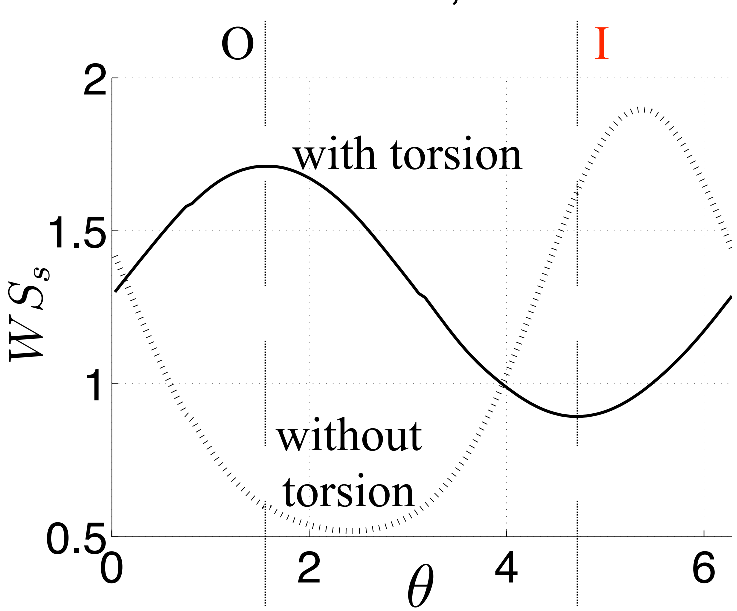
Without torsion



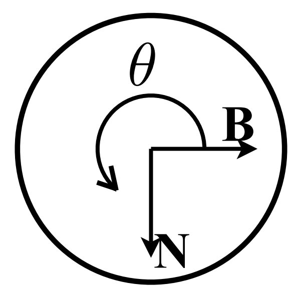
With torsion



Axial WSS, $s = 4D$

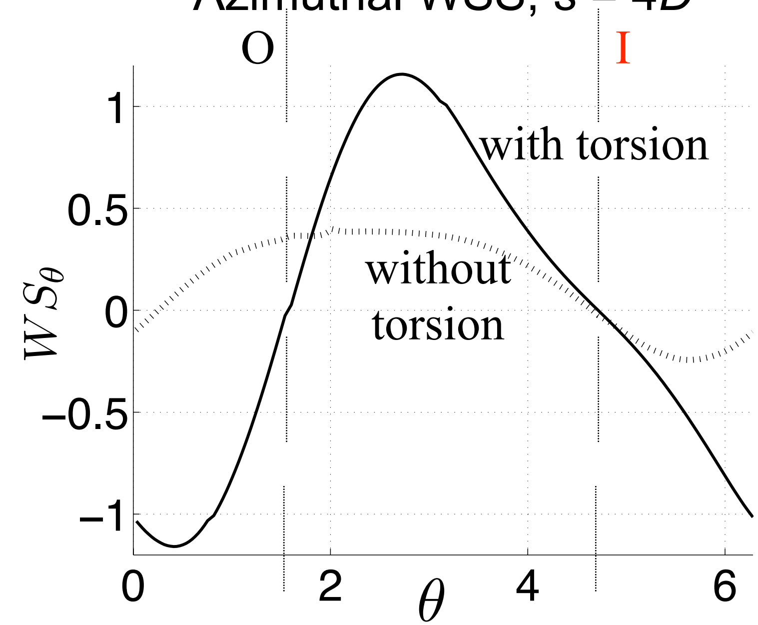


O: outer wall

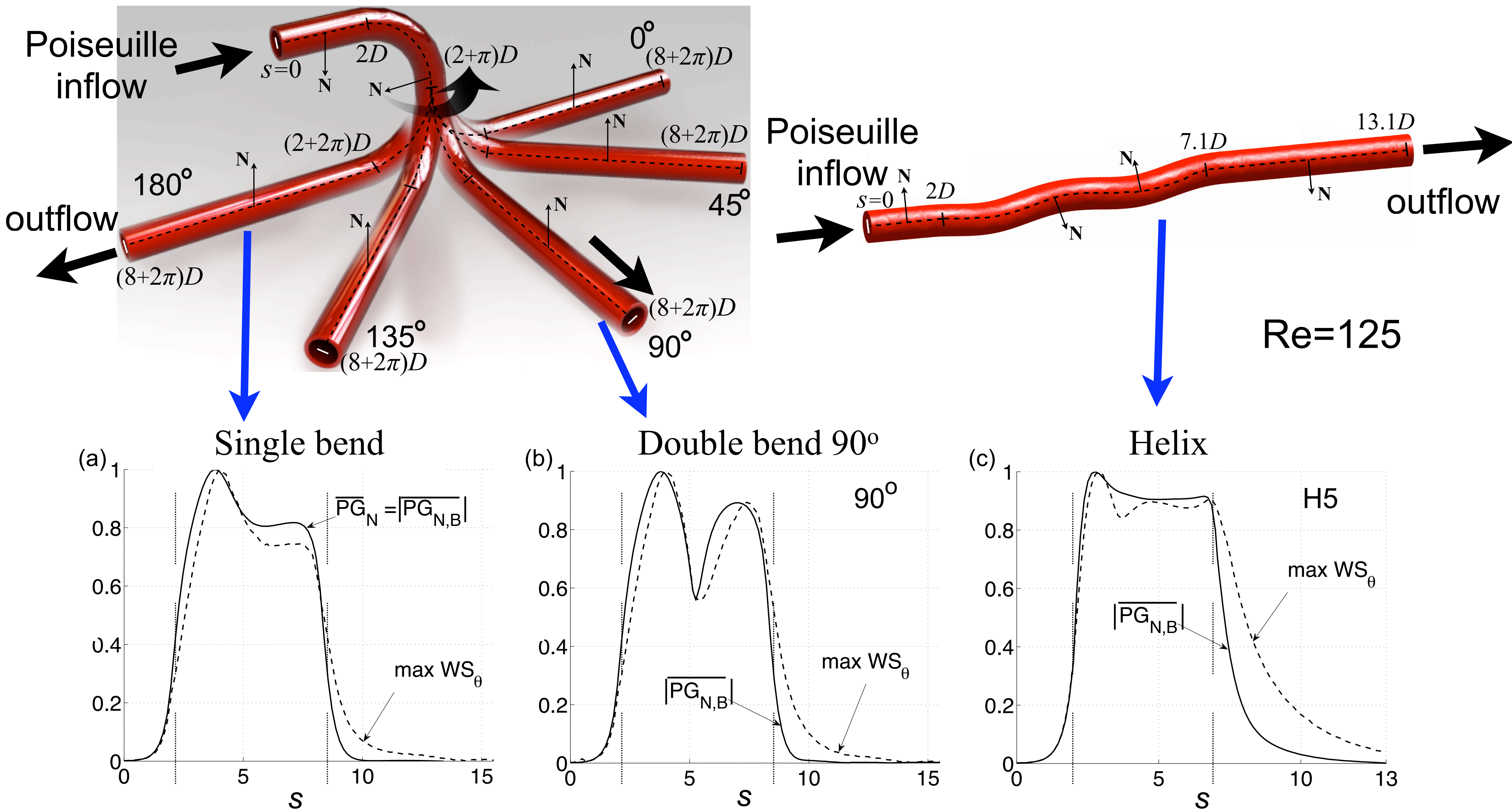


I: inner wall

Azimuthal WSS, $s = 4D$

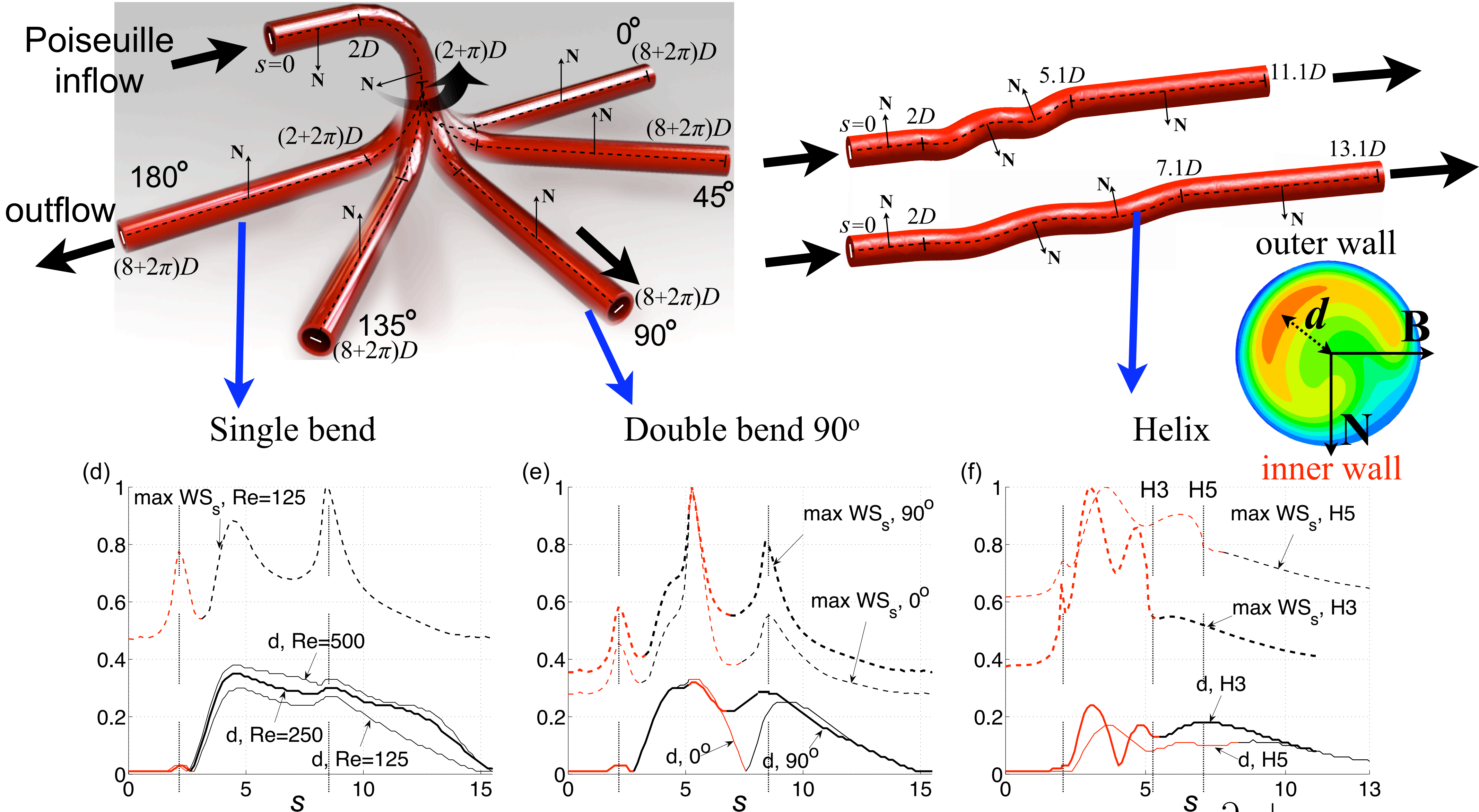


Azimuthal wall stresses and PG forces



$$|\overline{\mathrm{PG}}_{N,B}| = \sqrt{(\overline{\mathrm{PG}}_N)^2 + (\overline{\mathrm{PG}}_B)^2}$$ correlates well with $$WS_\theta = -\nu\rho \frac{\partial w}{\partial r}\Big|_{r=D/2}$$

Axial wall stresses and peak velocities



Radial offset, d, of peak axial velocity correlates well with $WS_s = -\nu\rho \frac{\partial u}{\partial r}\Big|_{r=D/2}$

Conclusions

- Effect of vessel curvature and torsion on blood flow from a local linear momentum perspective
- Roles assigned to in-plane forces and accelerations based on the physics of underdamped oscillations
- The centrifugal force generates normal motions
- The torsional force couples normal and binormal motions, enhancing in-plane mixing and reducing azimuthal WSS
- The Coriolis force links normal motions to axial accelerations that shape the velocity profile
- Quantification of the level of flow development and flow coupling across different bends