Moving the best parts of 1985's constrained sensing reconstruction to 2011's compressed sensing reconstruction

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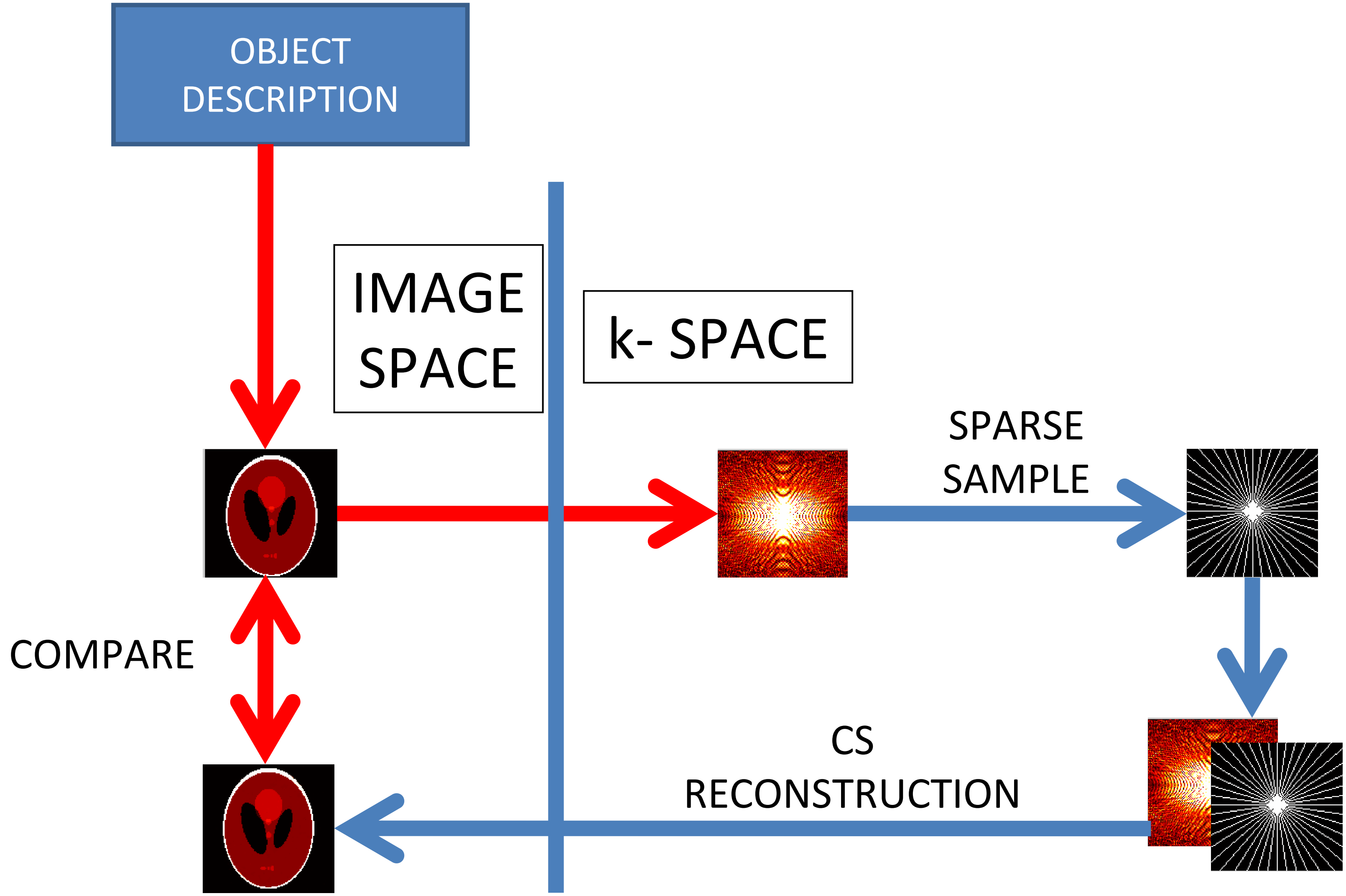
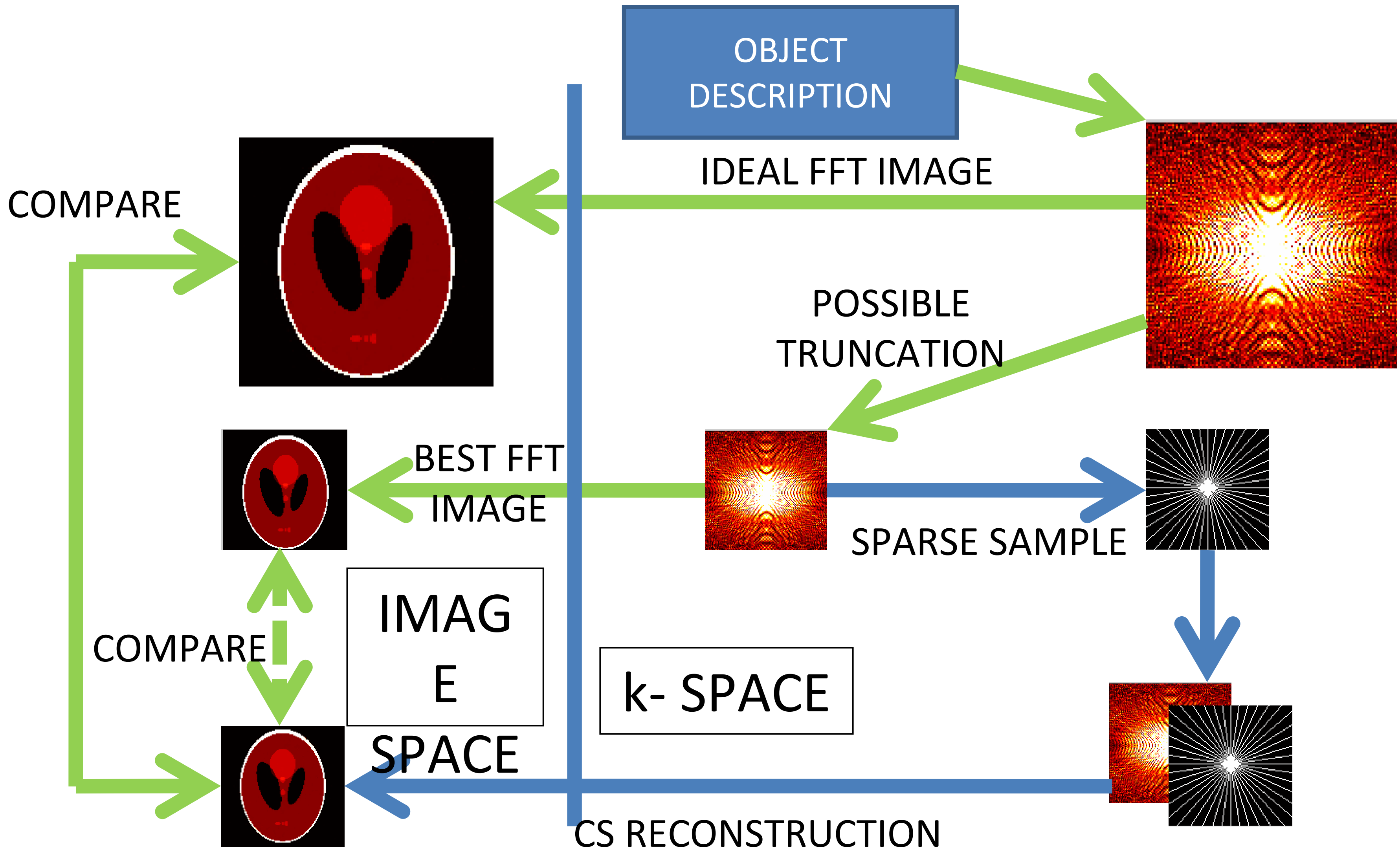
Topic to discuss

- Is MRI CS being validated the wrong way?
- Novel MRI approach for use with CS
- Impact of CS reconstruction using wrong simulated data
- Other things to try from constrained sensing reconstruction (super-resolution)

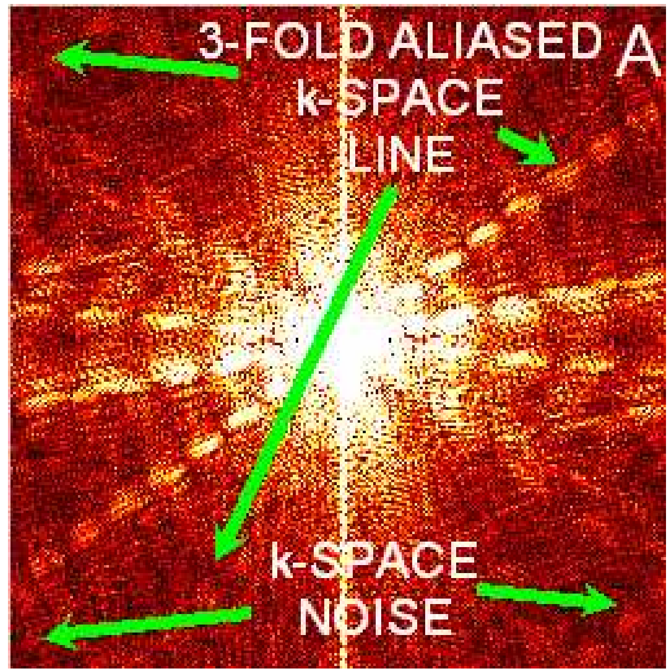
CS validation approach demonstrated in L1-Magic software package

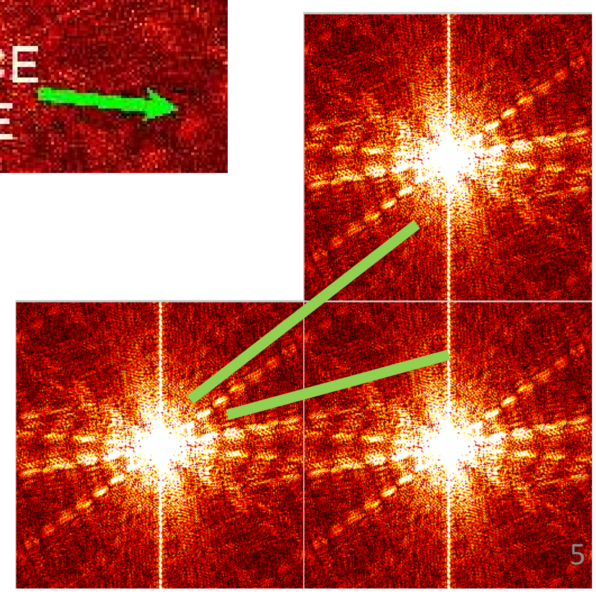


CS validation approach that best mimics true MRI data



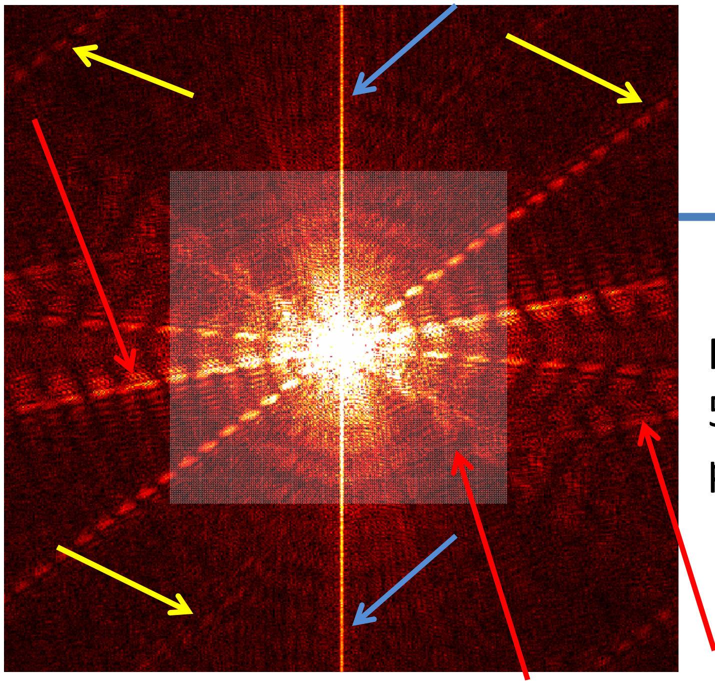
Are these validation approaches really different?





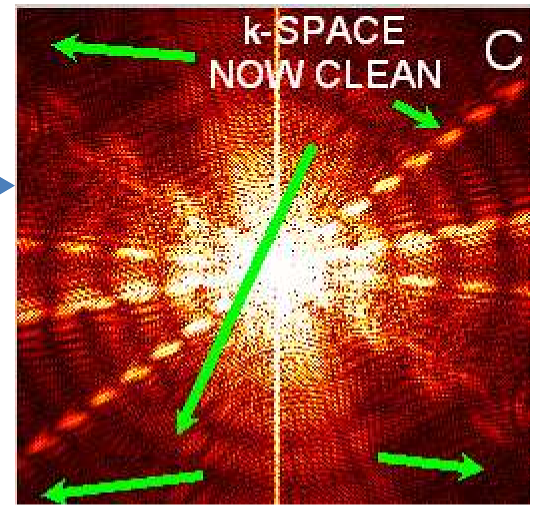
- Generate angiographic 256 x 256 phantom using techniques similar to Shepp-Logan phantom generation.
- Evidence of k-space aliasing of data
- Evidence of k-space 'mud' (noise from aliasing)

Aliasing fairly easy to ‘fix’ by generating k-space data from higher-resolution image

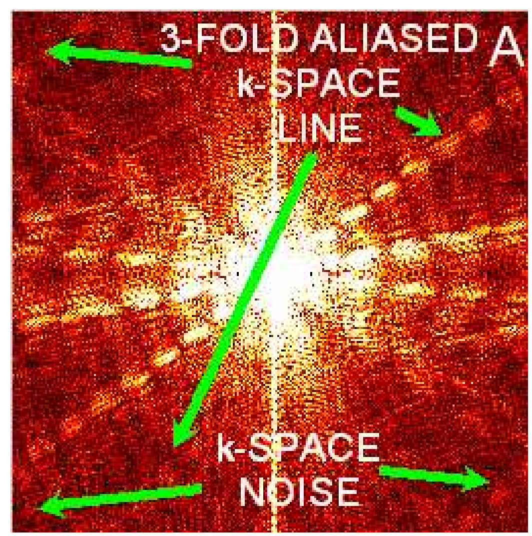


TRUNCATE

From
512 x 512
phantom



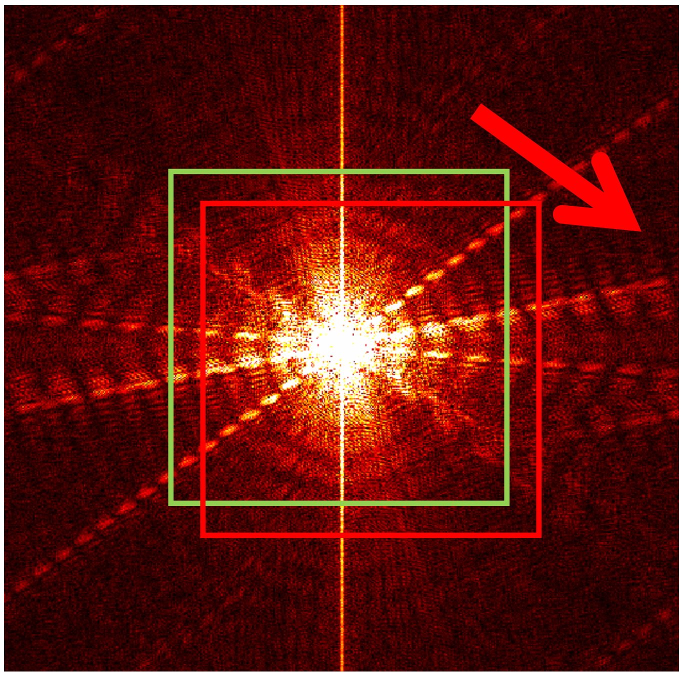
TRUNC
from
1024
x 1024
Phantom



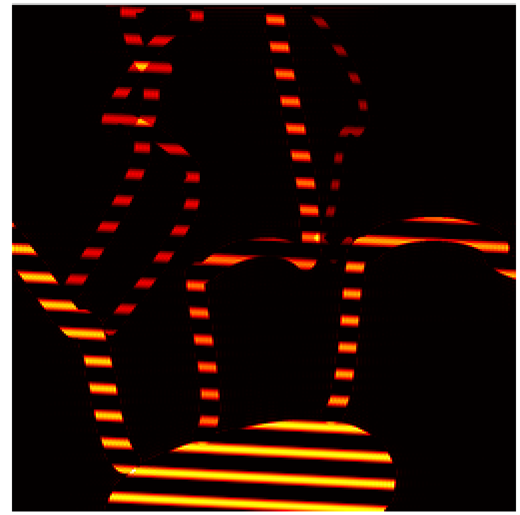
From
256
x 256
Phantom

- Calculate k-space data from high resolution phantom and then truncate.
- 512 x 512 phantom shows evidence of aliasing (mainly) outside of truncated area(ARROWS).
- Need to go to at least 1024 x 1024 resolution or calculate directly (analytically) in k-space

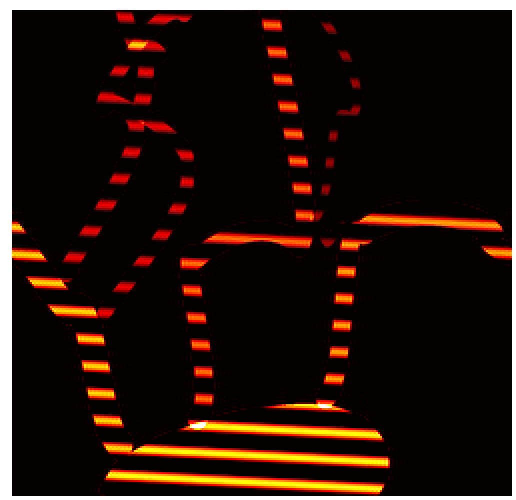
k-space offset can only be modeled via k-space data manipulation



REAL COMPONENT OF IMAGE WITH PHASE-SHIFT



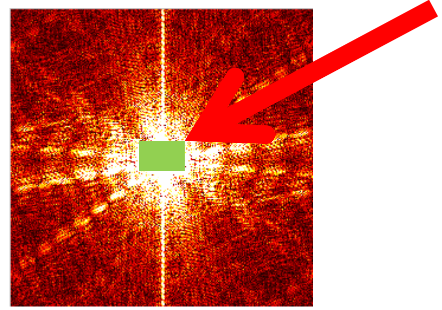
IMAGINARY COMPONENT OF IMAGE WITH PHASE-SHIFT



Modeling of k-space data by ANY algorithm works best when complexity such as this is removed from the data set

Approach to solve k-space data offset using ideas from SR reconstruction

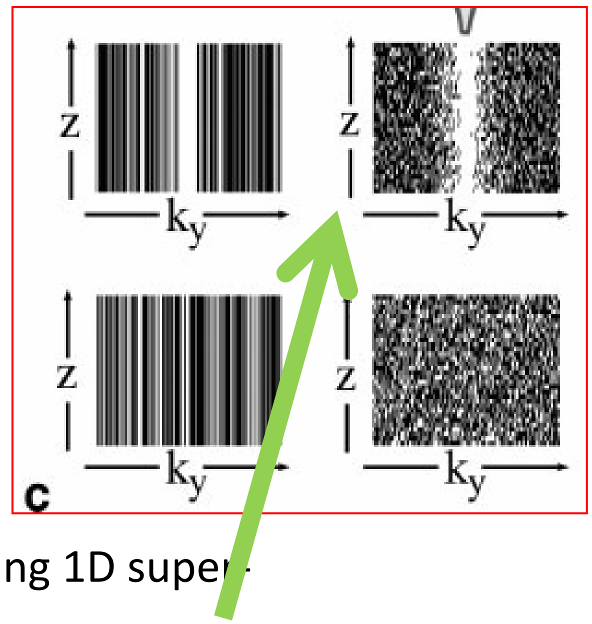
- Lustig et al. (MRM #58, 1182-95, 2007) used SMALL PARTof k-space (centre) to identify offset



- Smith et al. (IEEE TMI, #5:3, 132-9, 1986) suggest using ALL AVAILABLE DATA for phase correction

Hermitian data set $S(n)_H = (S(n) + S(-n)^*) / 2$

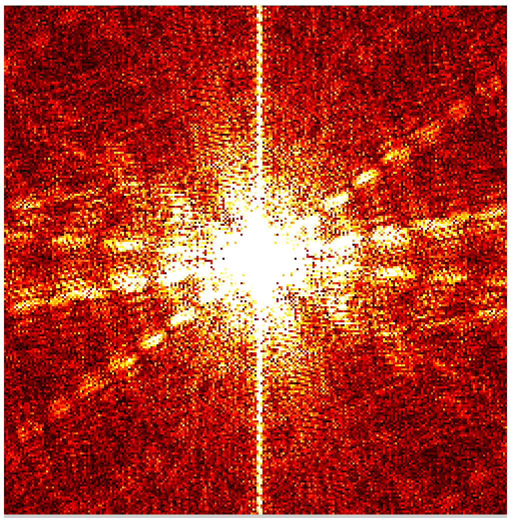
anti-Hermitian data set $S(n)_{AH} = (S(n) - S(-n)^*) / 2$



- Super reconstruction is performed on both data sets independently before images are recombined.

- Problem to overcome – approach only demonstrated during 1D super-resolution reconstruction. MATCHES some of Lustig's proposed symmetric CS sampling schemes,

MRI white noise
MUST be added to k-space data



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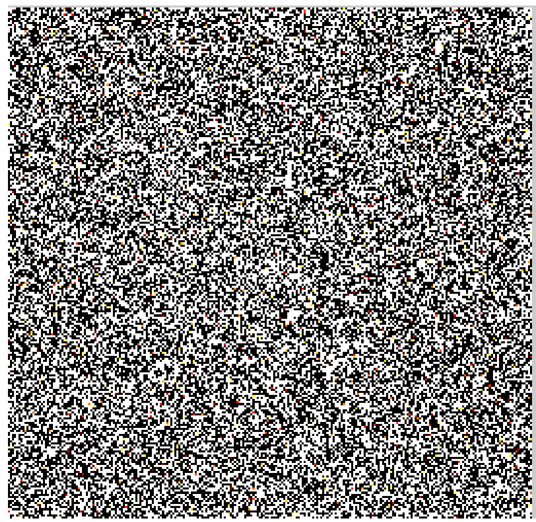
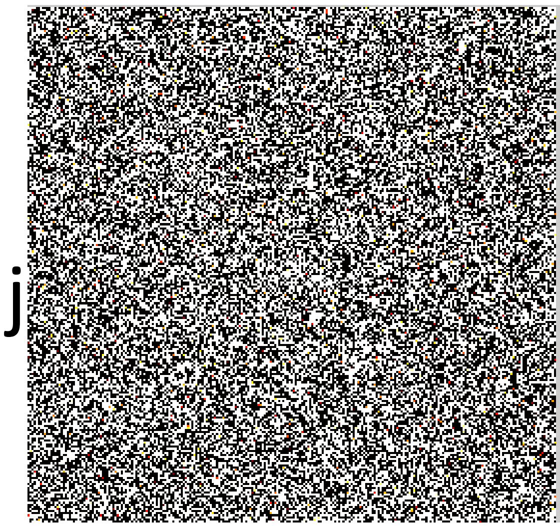
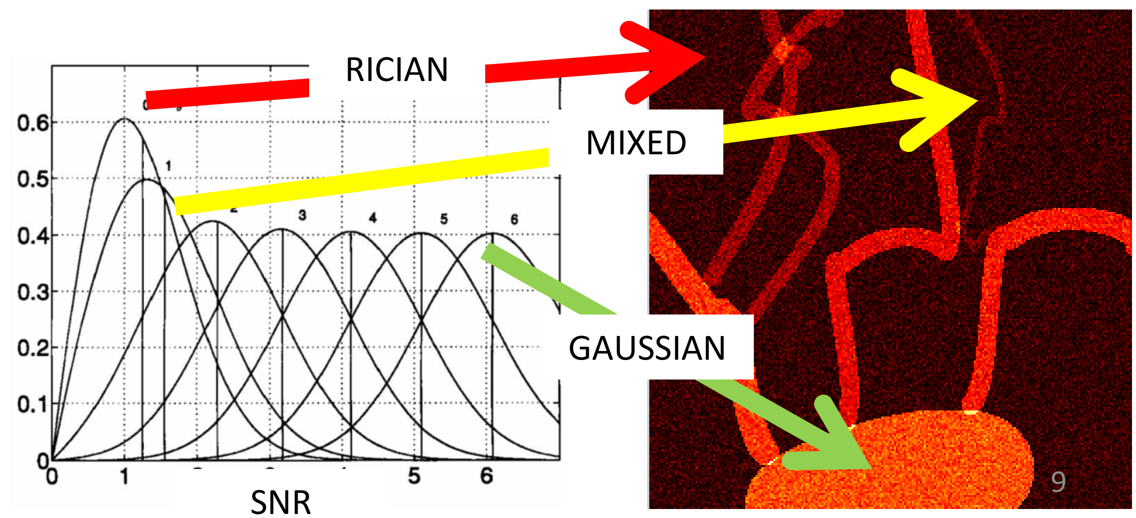


+ j



Henkleman, Med. Phys. 1985
McGibney, Med. Phys. 1993
Gudbjartsson , M.R.M. 1995
Image noise is
1.Quasi – gaussian on large intensity object
2.Mixed characteristics on lower intensity object
3.Rician on background



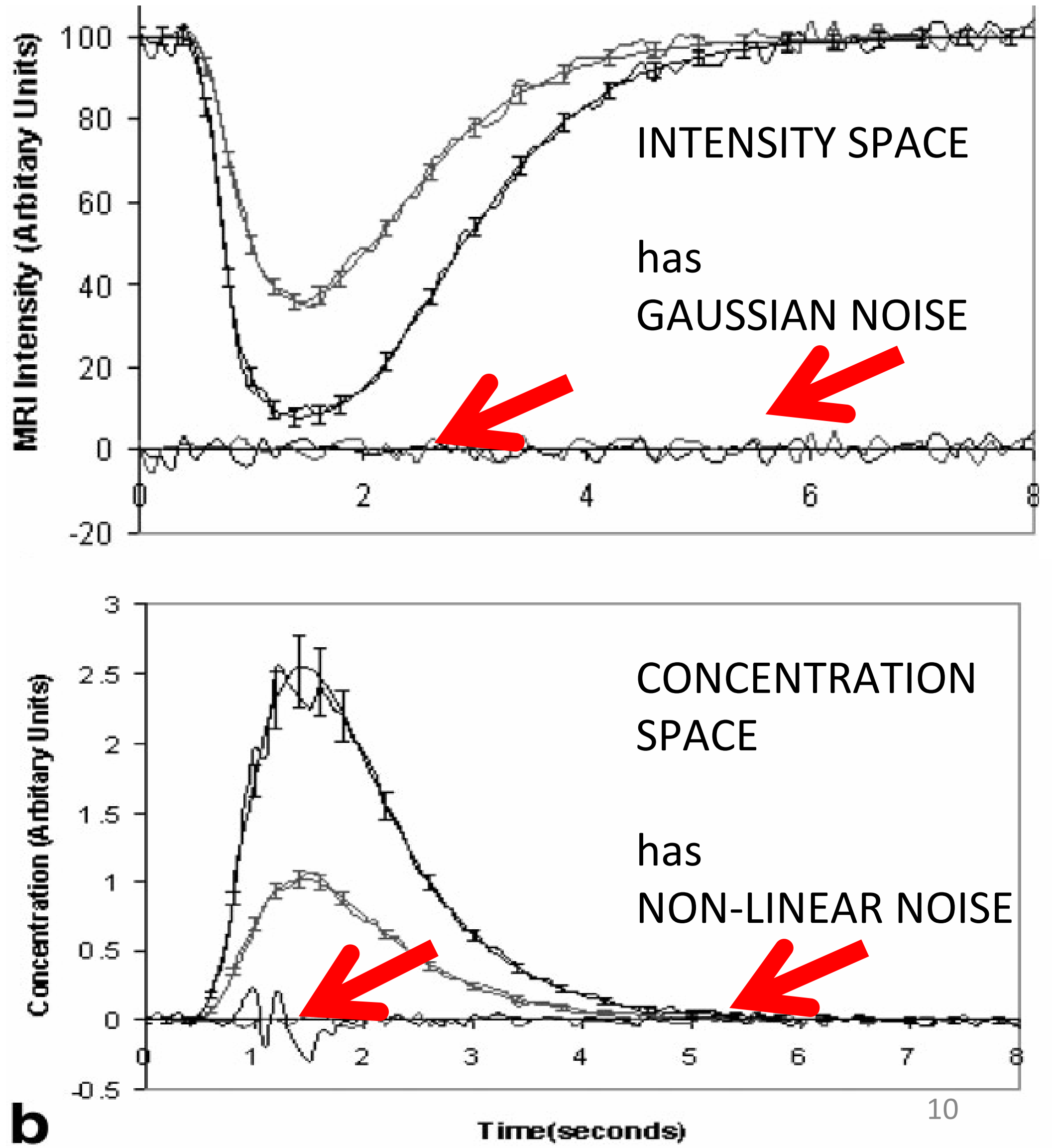
Incorrect modeling of noise is a common problem seen in literature

INCORRECT MODEL

- Generate DSC CBF concentration curves
- **Add gaussian noise (NO!)**
- Deconvolve and Analyse

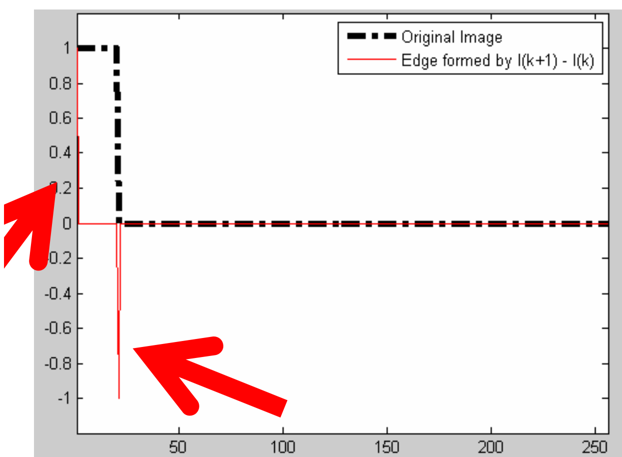
CORRECT MODEL

- Generate DSC CBF concentration curves
- **(1) Transform to Intensity space,**
- **(2) Add gaussian noise;**
- **(3) Transform back**
- Deconvolve and Analyse
- Smith et al., MRM 2001.

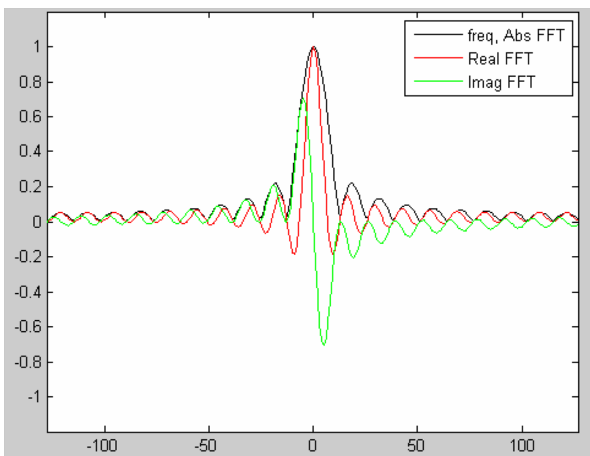


Applying CS to wrong data set?
My novel approach

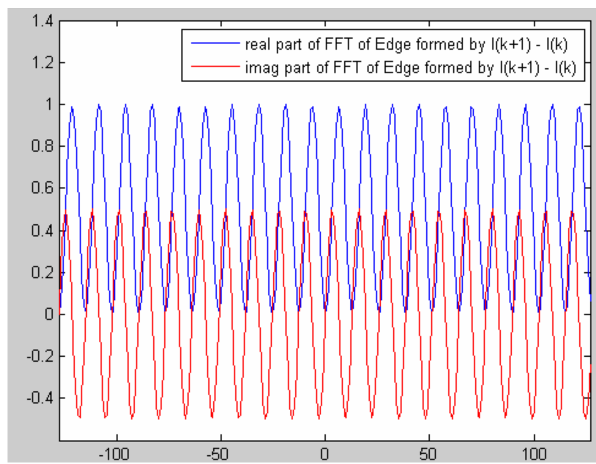
– You need the data sparse in one domain to get CS to work
– Use (1D) simulation of box-car is not sparse
– Using the edges of the box-car is sparse



ORIGINAL (BLACK)
AND EDGE (RED) IMAGES



K-SPACE DATA
FOR NORMAL IMAGE

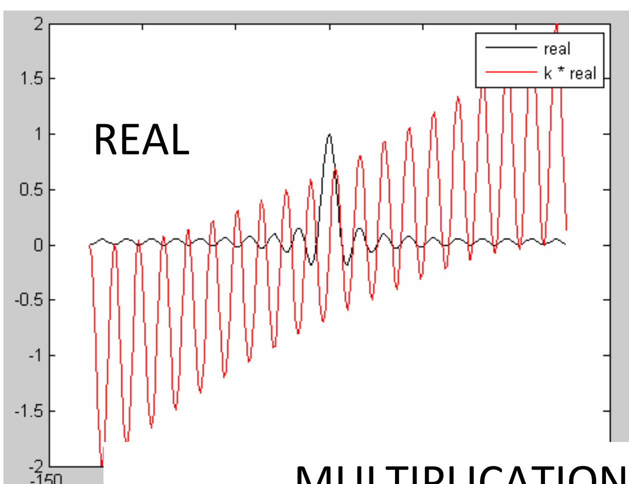


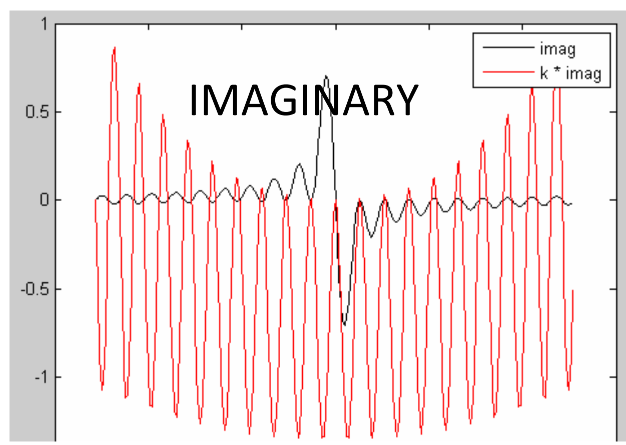
K-SPACE DATA
FOR EDGE IMAGE

Problems to solve with validating algorithms this sort of data

- How can you generate ‘EDGE’ k-space data experimentally?

- ANSWER: Multiple original k-space data of NON-EDGE image by k.
  – Important to avoid aliasing in original k-space data





MULTIPLICATION IS NOT ‘PERFECT’ BECAUSE OF
RESIDUAL ALIASING PRESENT WHEN USING 256 pt LINE IMAGE

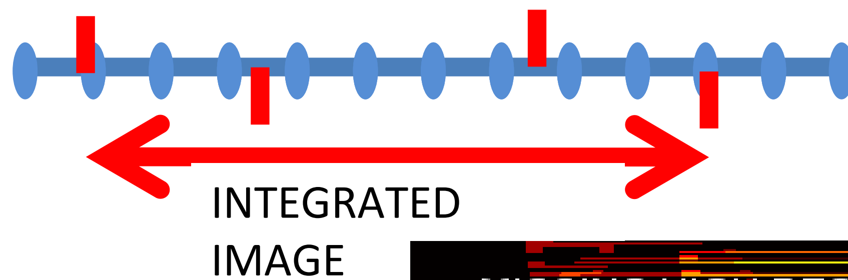
Problems to overcome when using ‘edge’ k-space data

- Multiplication by k enhances high frequency noise components
  - Especially true for ‘wide’ image components which are ‘narrow’ in frequency domain
  - SOLUTION: Fit wide and narrow separately?

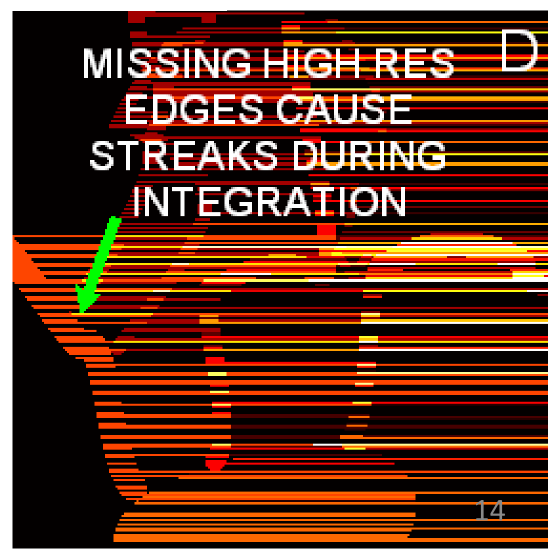
- In super-resolution image reconstruction using TERA (Smith, 1990) only needed a few data points in edge k-space to recognize edges since ‘sinusoids’ are easy to model.

Modeling the data edges can work 'too well' with phantoms

- Frequency components have form
  $A \cos (2 \pi F (k-1) / N) + j B \sin (2 \pi F (k-1) / N)$
  - *Problem if F is NOT an integer* as image edges fall between sample points so numerical integration fails



- Solution with *super-resolution reconstruction* was its inherent ability to zoom data *x 16 or more* times PLUS ability to apply *DFT matching* and *pole pulling* to control image instabilities in reliable fashion



M. R. Smith and S. T. Nichols, Proceedings of 10th Annual Meeting Society of MRM, San Francisco, #2, 749, 1991

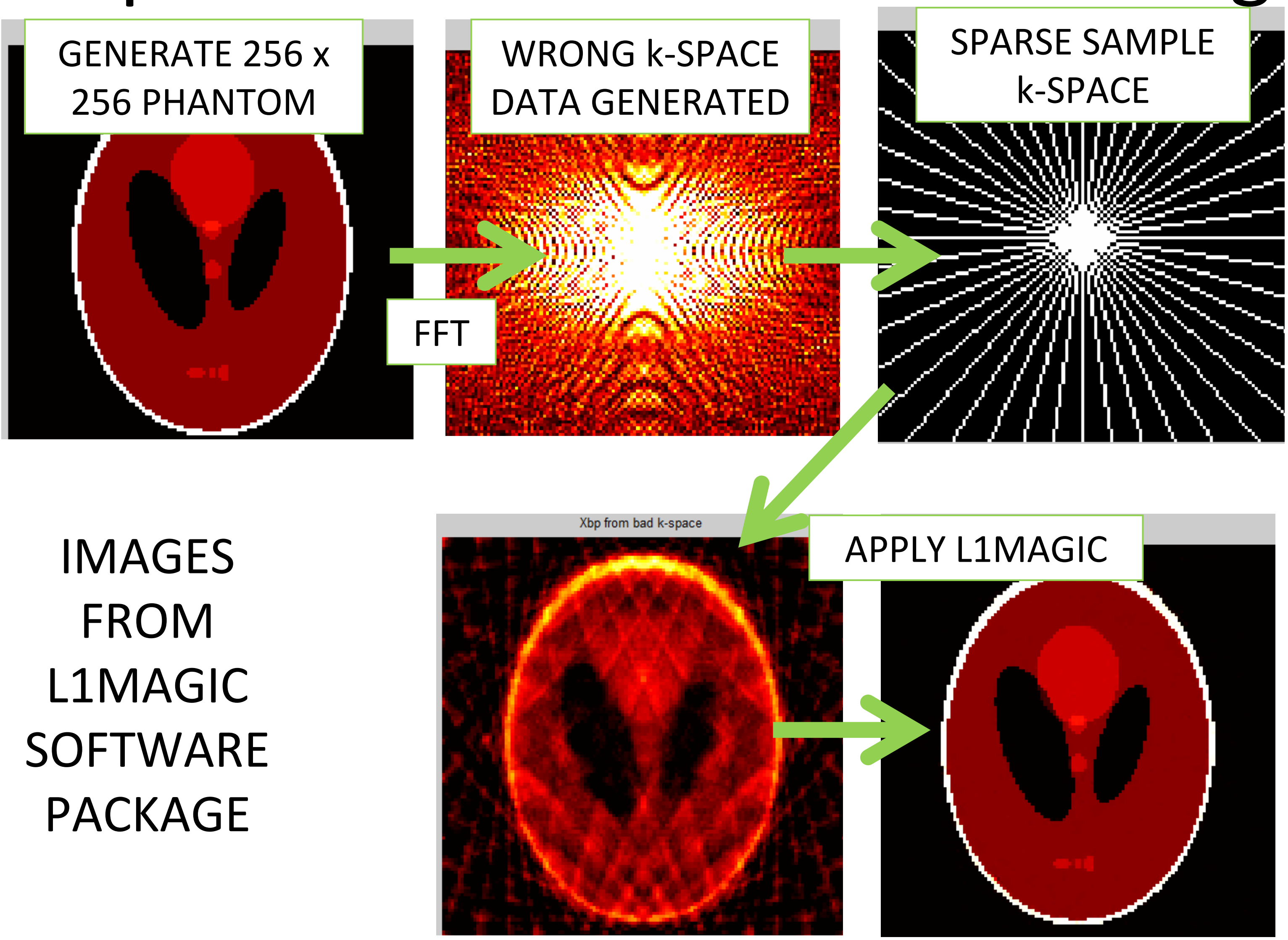
Where are we at in trying these SR ideas with CS reconstruction?

- ‘Correct’ validation versus ‘Incorrect’ validation examined for L1MAGIC software completed
  - Working on Lustig re-validation – different from L1magic

- Components
  - Experimentally gather limited k-space (on a 256 x 256)
  - Apply L1-magic
  - Generate CS-image (256 x 256)

- Validation
  - Compare CS-image to what you would have got if you had gathered all the 256 x 256 k –space data

This is the INCORRECT validation procedure for constrained imaging

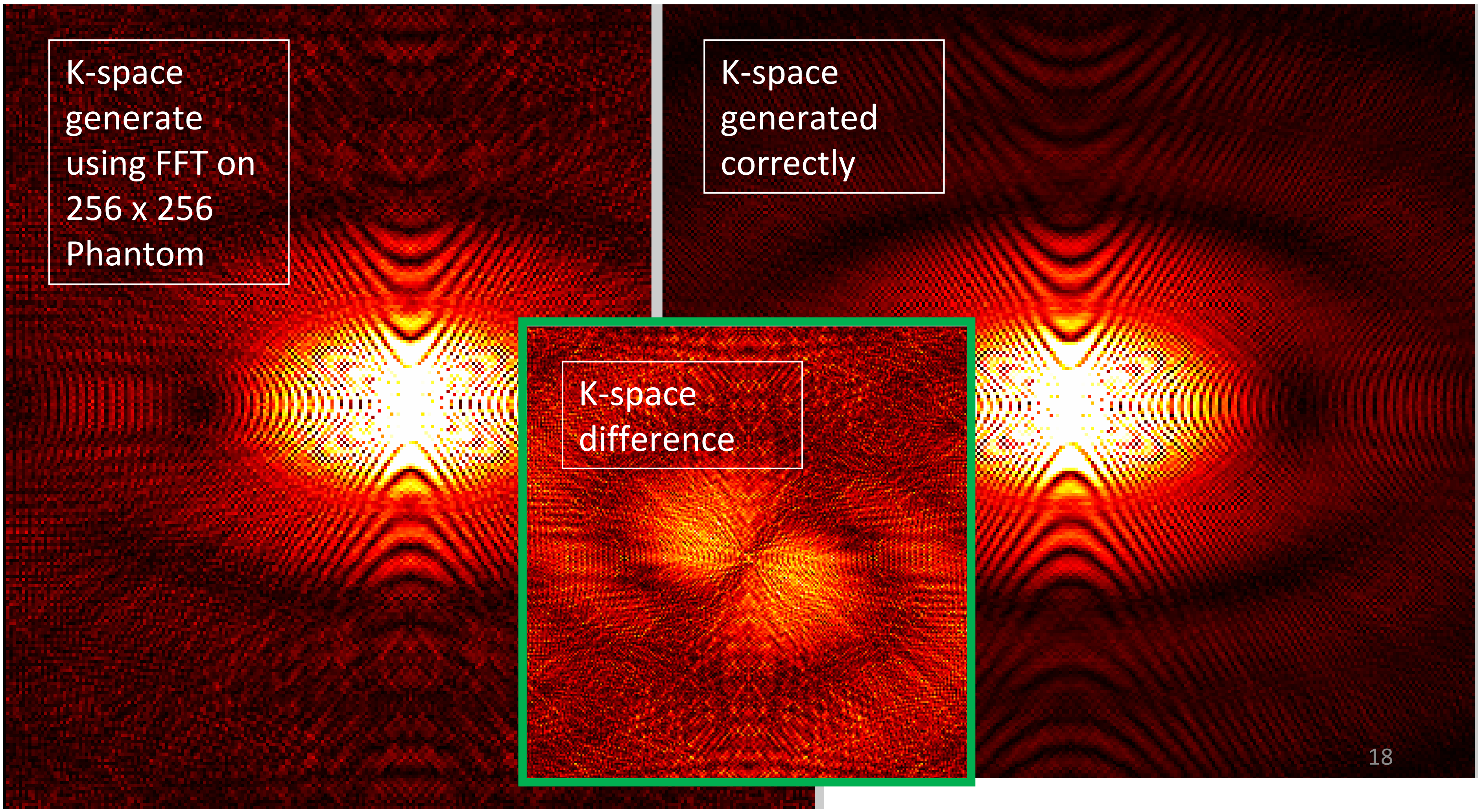


Validation is wrong as we are not simulating the correct k-space data

- Components
  - Experimentally gather limited k-space (on a 256 x 256)
  - Apply L1-magic (or other reconstruction algorithm)
  - Generate CS-image (256 x 256)

- If we tune our CS algorithms for the wrong sort of simulated data – then the algorithm may not be correctly tuned for the true experimental data.
  - If we tune our algorithms for the correct experimental data, what will be reviewer's comments?

Differences in the k-space data that will be sparsely sampled during CS validation



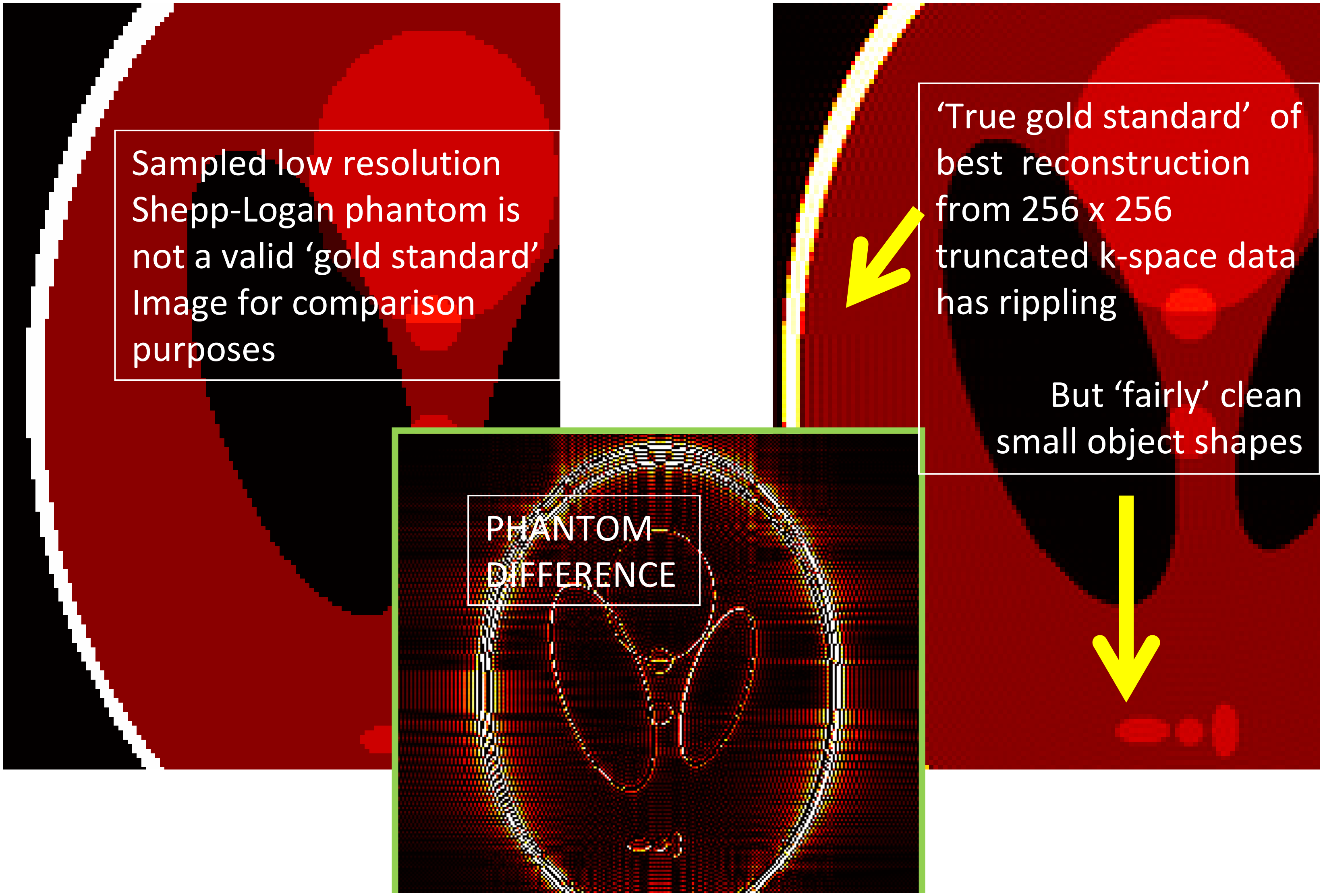
So what happens if we use the correct data in CS reconstruction

- It will either work to bring back the 'gold standard' image or it will not work
  - Is the truncated image the correct 'gold standard' image?
  - Reviewers often insist on 'exact and unrealistic' phantoms which means 'good' algorithms that work well experimentally are discarded

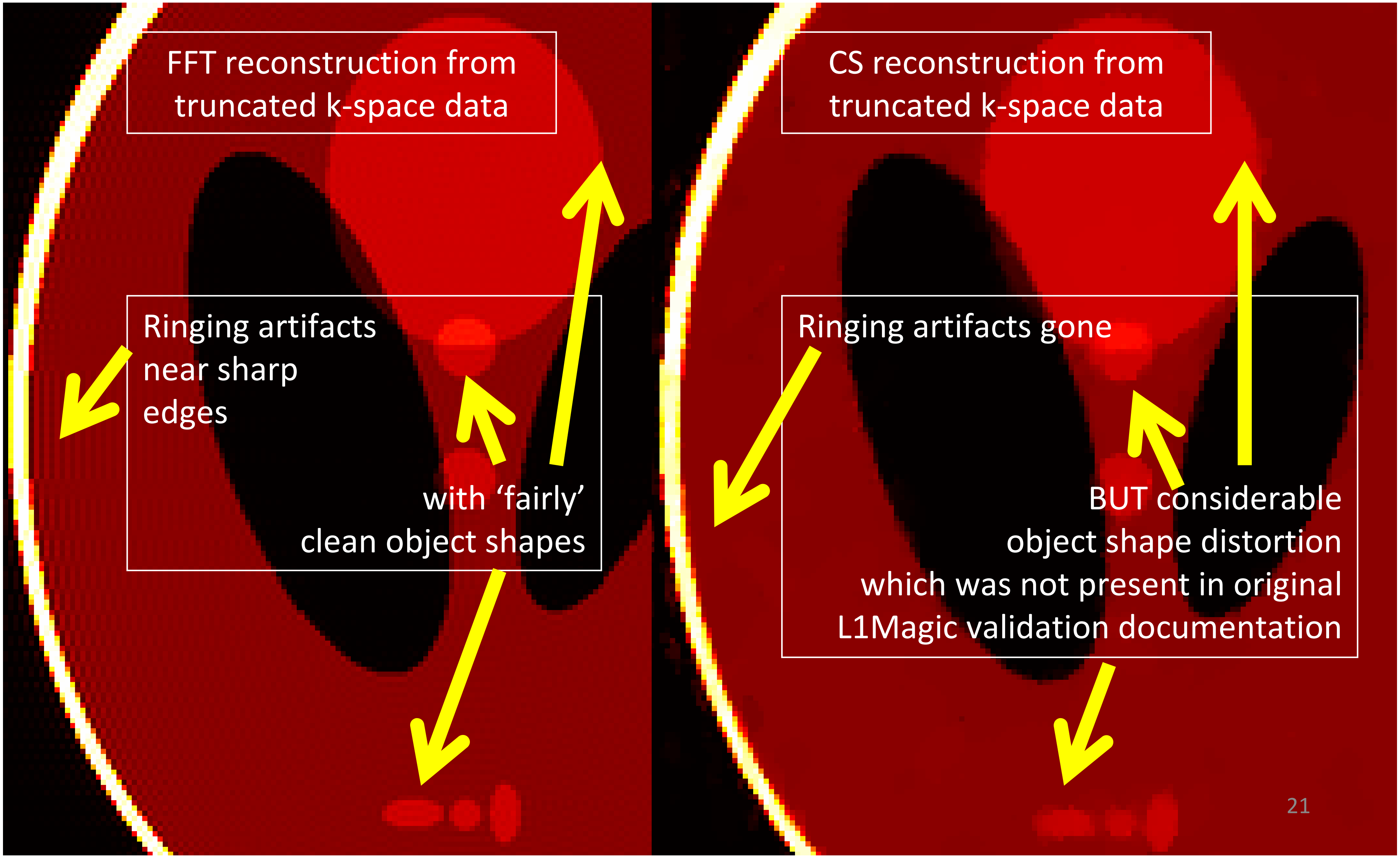
- If the L1MAGIC algorithm does not work on 'real' k-space data (truncated with offsets causing image phase shifts) then we must investigate whether
  - Minor tuning of algorithm required
  - Major tuning of algorithm required

Differences in phantom images

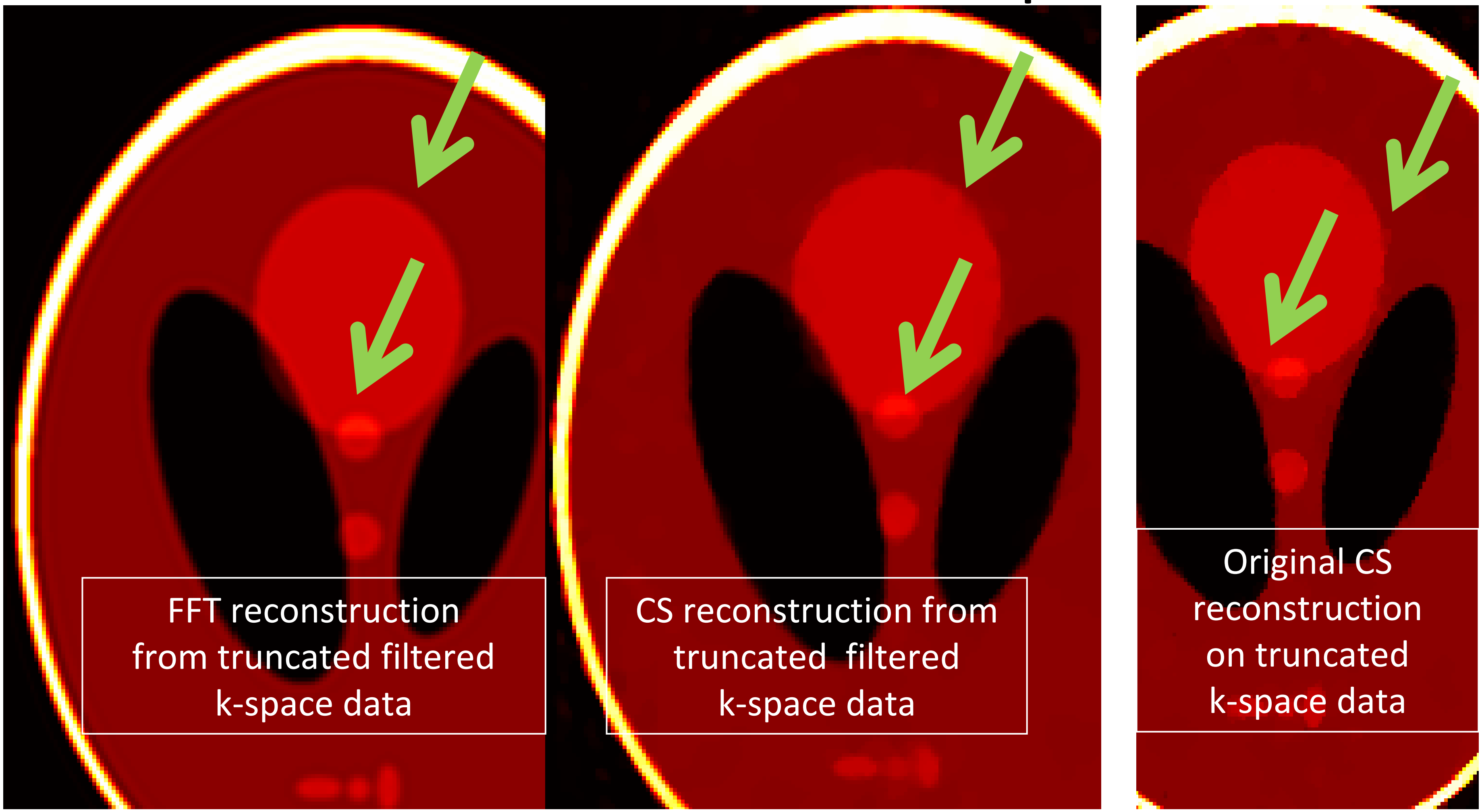
Which ‘gold standard’ acceptable to reviewers?



CS applied to truncated data



If the k-space data is fermi-filtered, does the CS reconstruction improve?



Lots of possible future directions

- DEFINE '*DOES THE RECONSTRUCTION IMPROVE*?'
  - K-space difference metrics and Visual Difference Predictors
- STRANGE IMAGES; *MISSING DATA REINTRODUCED, NOT MISSING NOISE*.
  - Determine image then move back (FFT) into frequency domain. Now add back gaussian random noise on the new points of k-space data before IFFT.
- REVALIDATE LUSTIG'S CS RECONSTRUCTION APPROACH.
- B0 AND GRADIENT ISSUES MEANS THAT THE PEAK OF K-SPACE DATA IS NOT CENTRED AT (0, 0) UNLIKE THE IDEAL PHANTOM.
  - Solution from super-resolution -- Reconstruct Hermitian and Anti-Hermitian components of image and recombine.
- INVESTIGATE ISSUES OF 'EDGE GENERATION' RECONSTRUCTION
  - Issues involving multiplying experimental k-space data by k
- OVERLAP BETWEEN SPARSE SAMPLING AND SUPER-RESOLUTION
  - Can they be usefully combined?

Conclusion

- Much may be transferred from early attempts to reduce k-space data requirement started with partial Fourier transforms (late ’70s) and super-resolution (SR) algorithms (late ’80’s).

- Current constrained sensing (CS) algorithms are being incorrectly validated (i.e. assumed to be working correctly) using Shepp-Logan like phantoms.

- Have suggested a number of solutions and new approaches that could be moved from SR to CS.

- Initial investigations under way already show that current CS algorithms need to be modified to correctly handle the ‘true’ characteristics of experimental k-space