SENSE IMAGING

Optimizations in Parallel MRI

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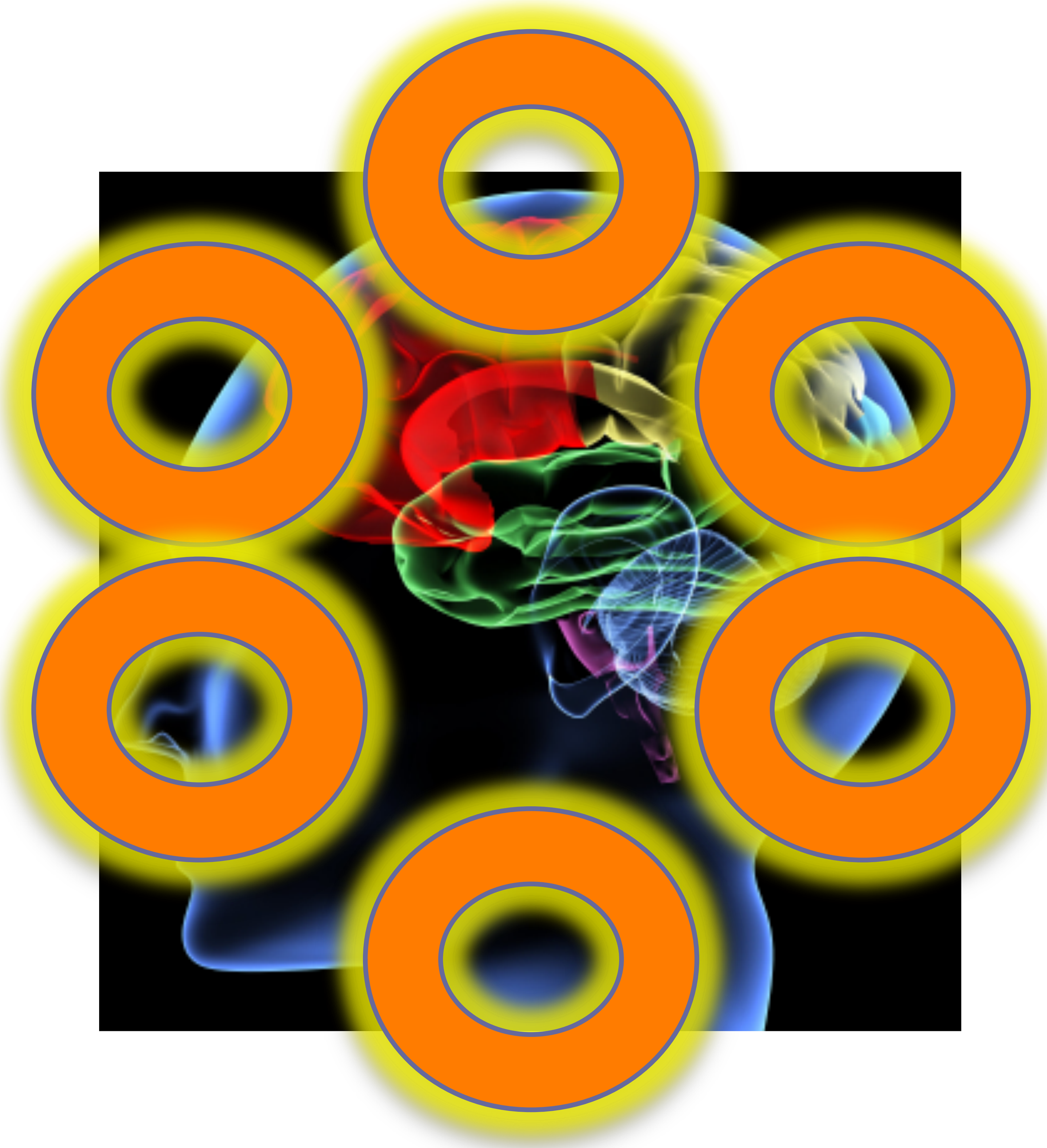
OUTLINE

- BACKGROUNDS
- SENSTIVITY PROFILING
- DATA SAMPLING OPTIMIZATION
- RESULTS & FUTURE WORK

SENSE Imaging

SENSE Imaging



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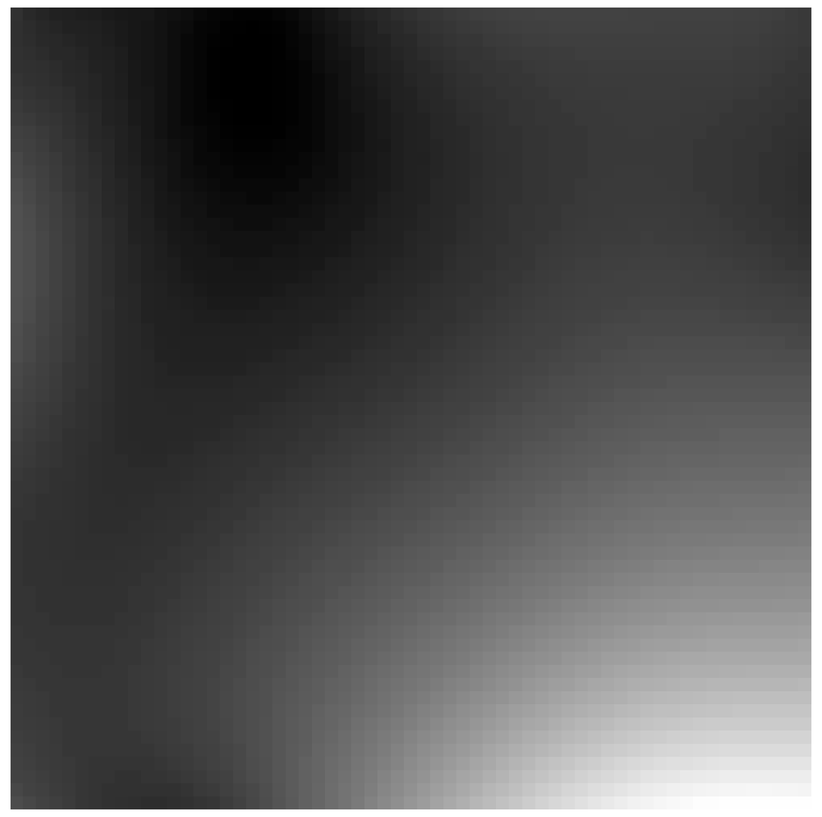

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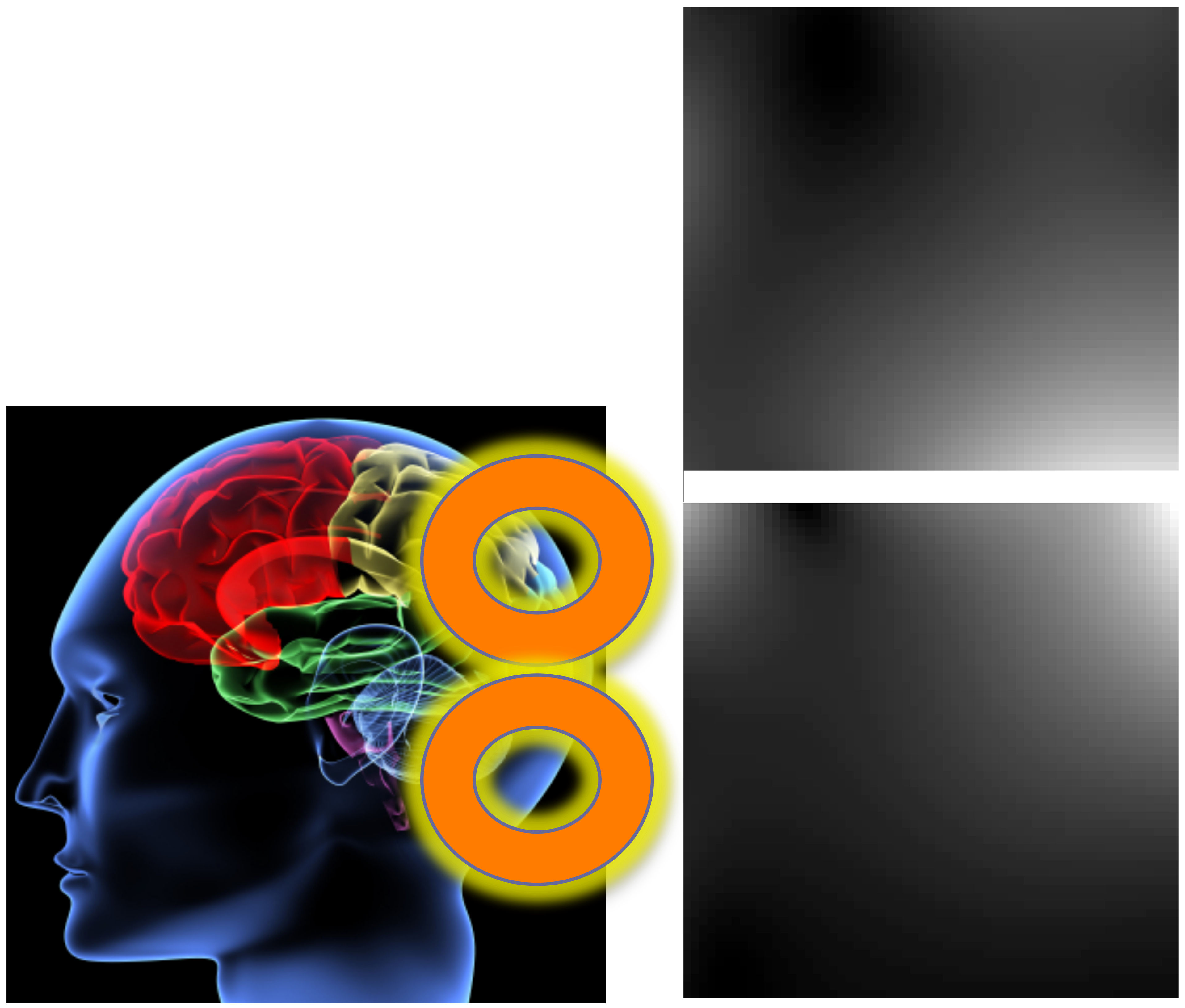

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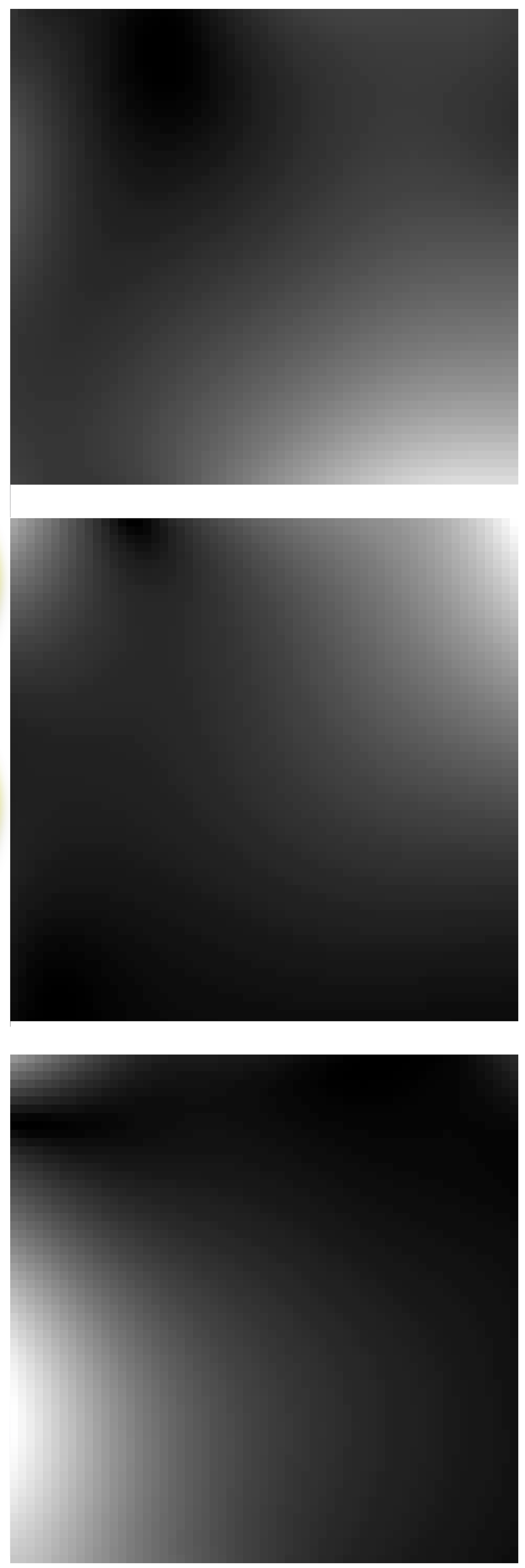

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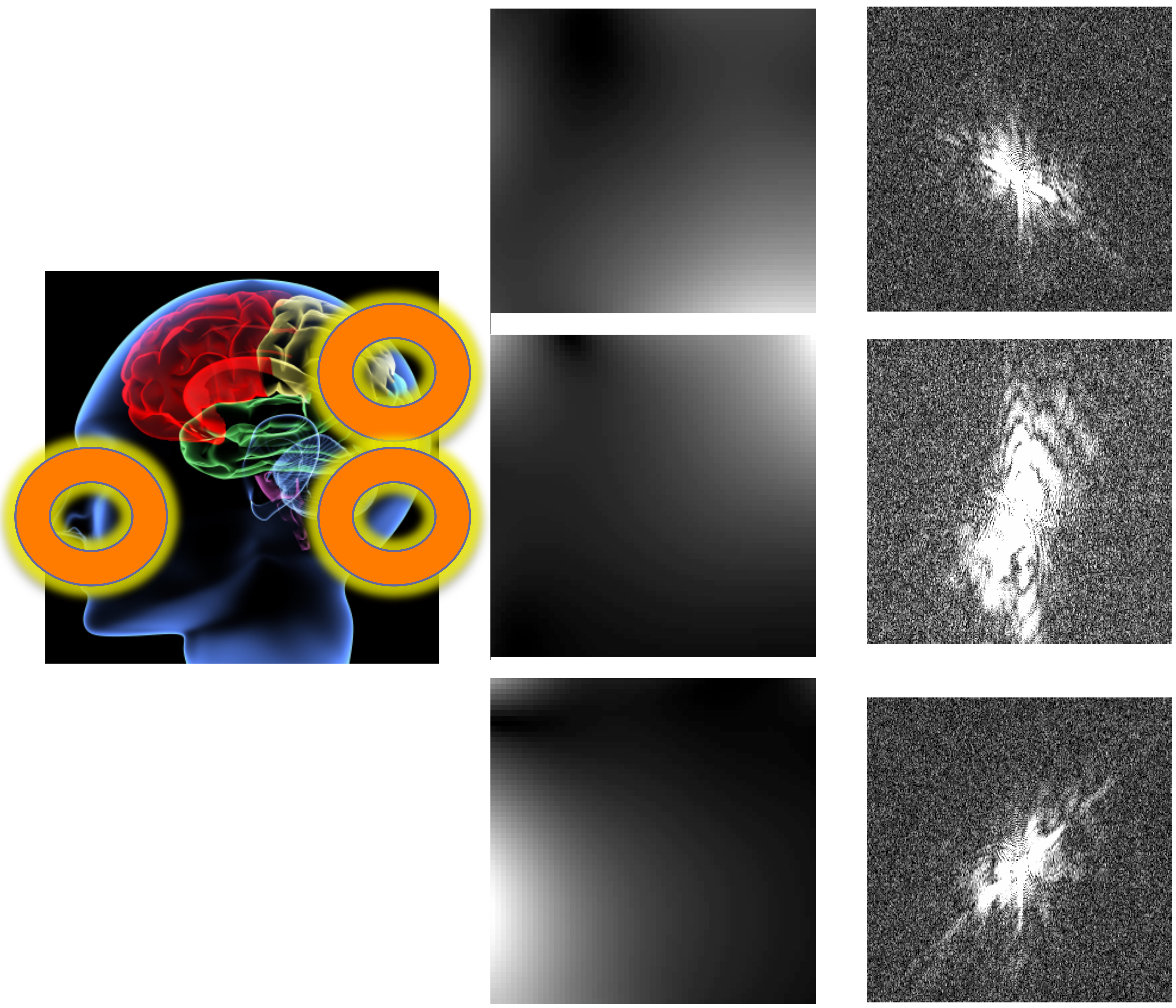

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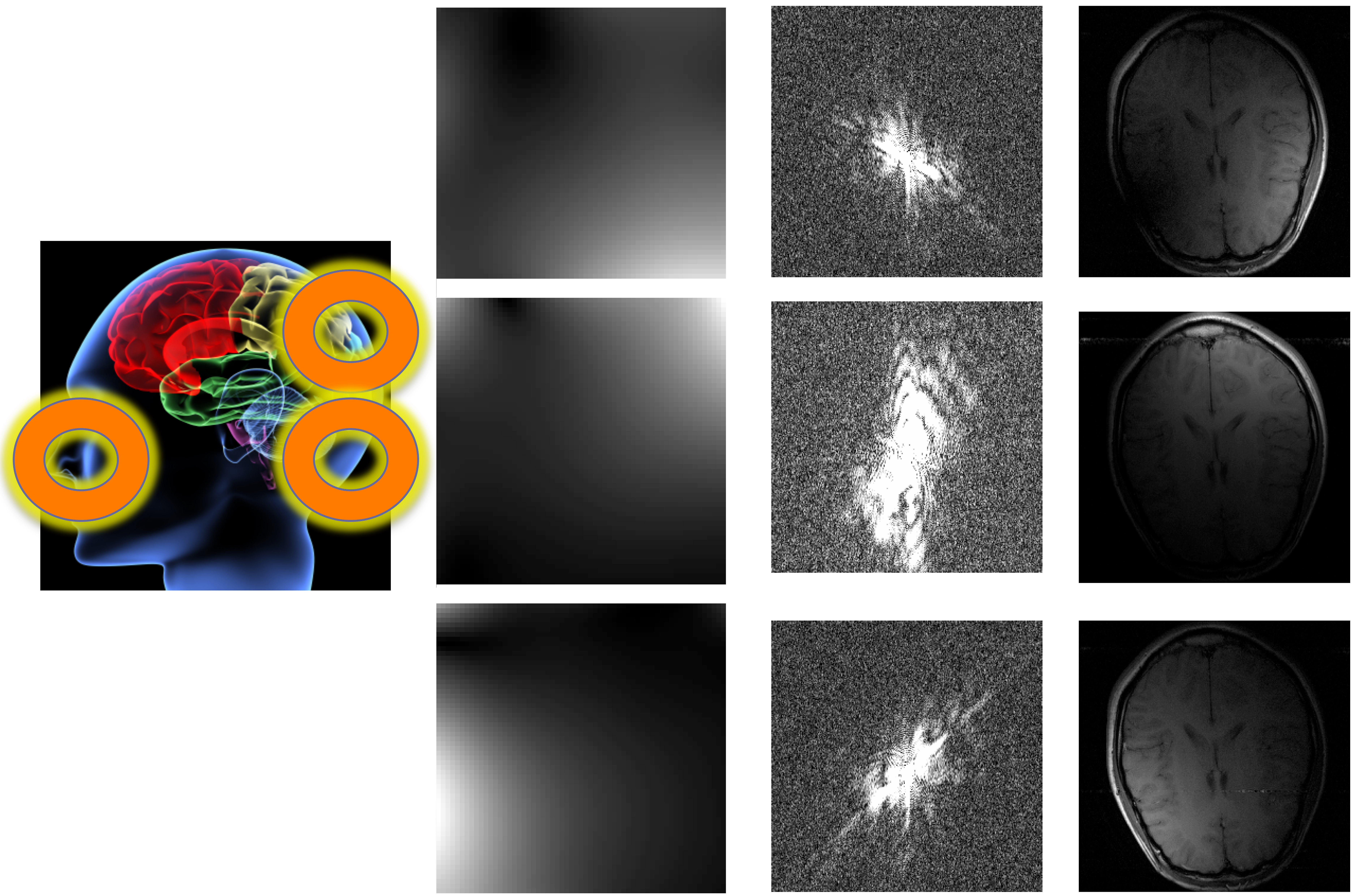

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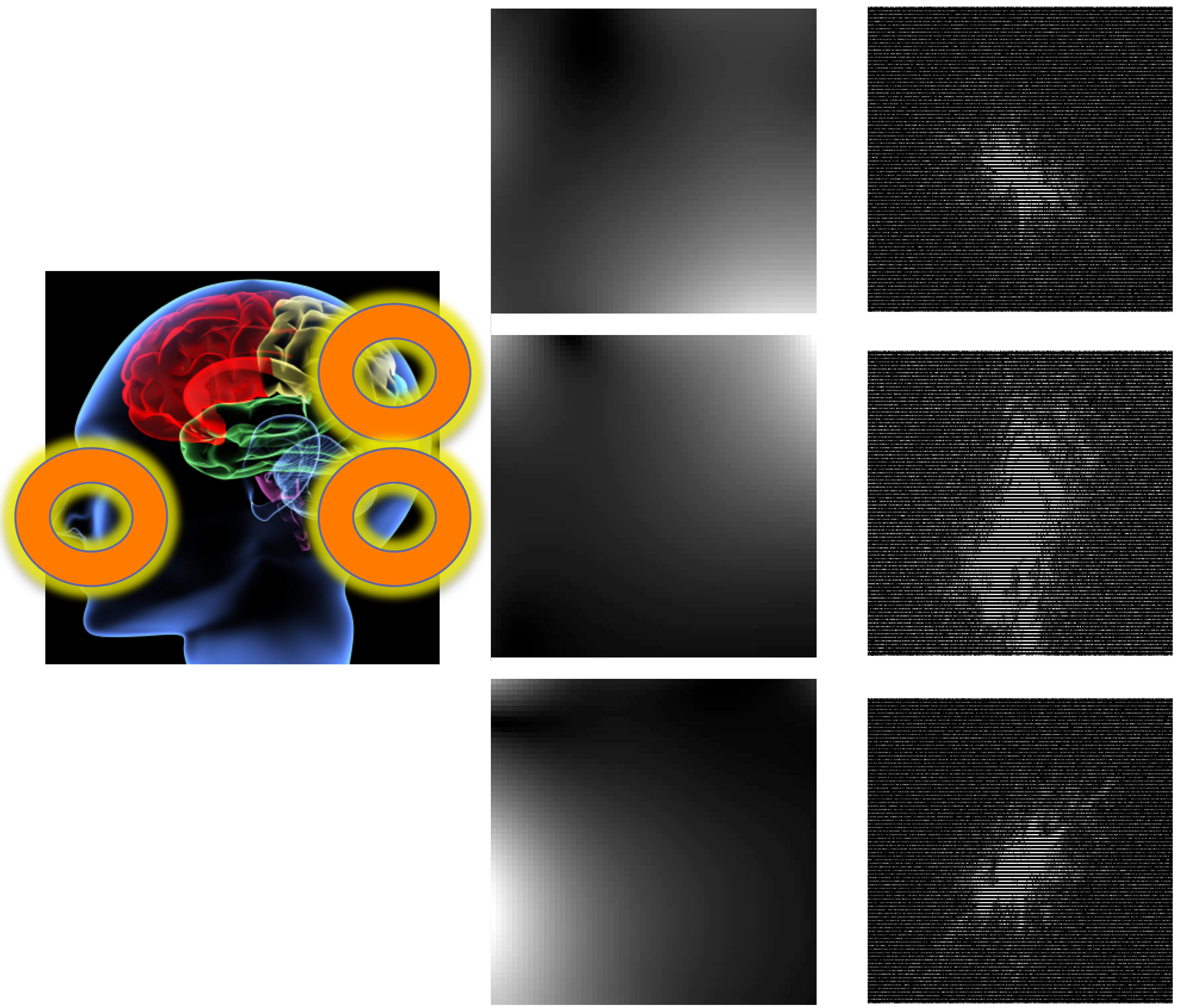

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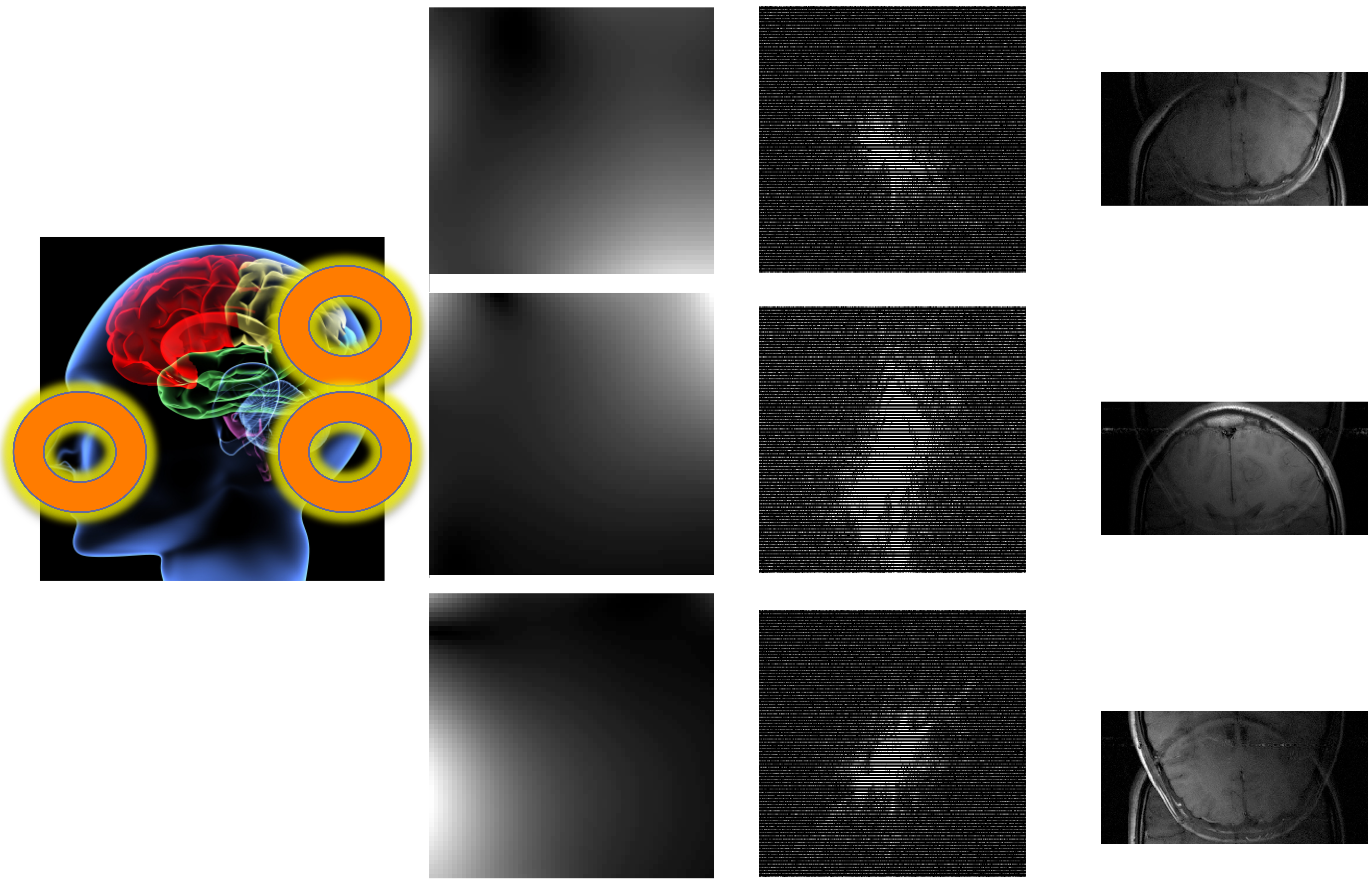

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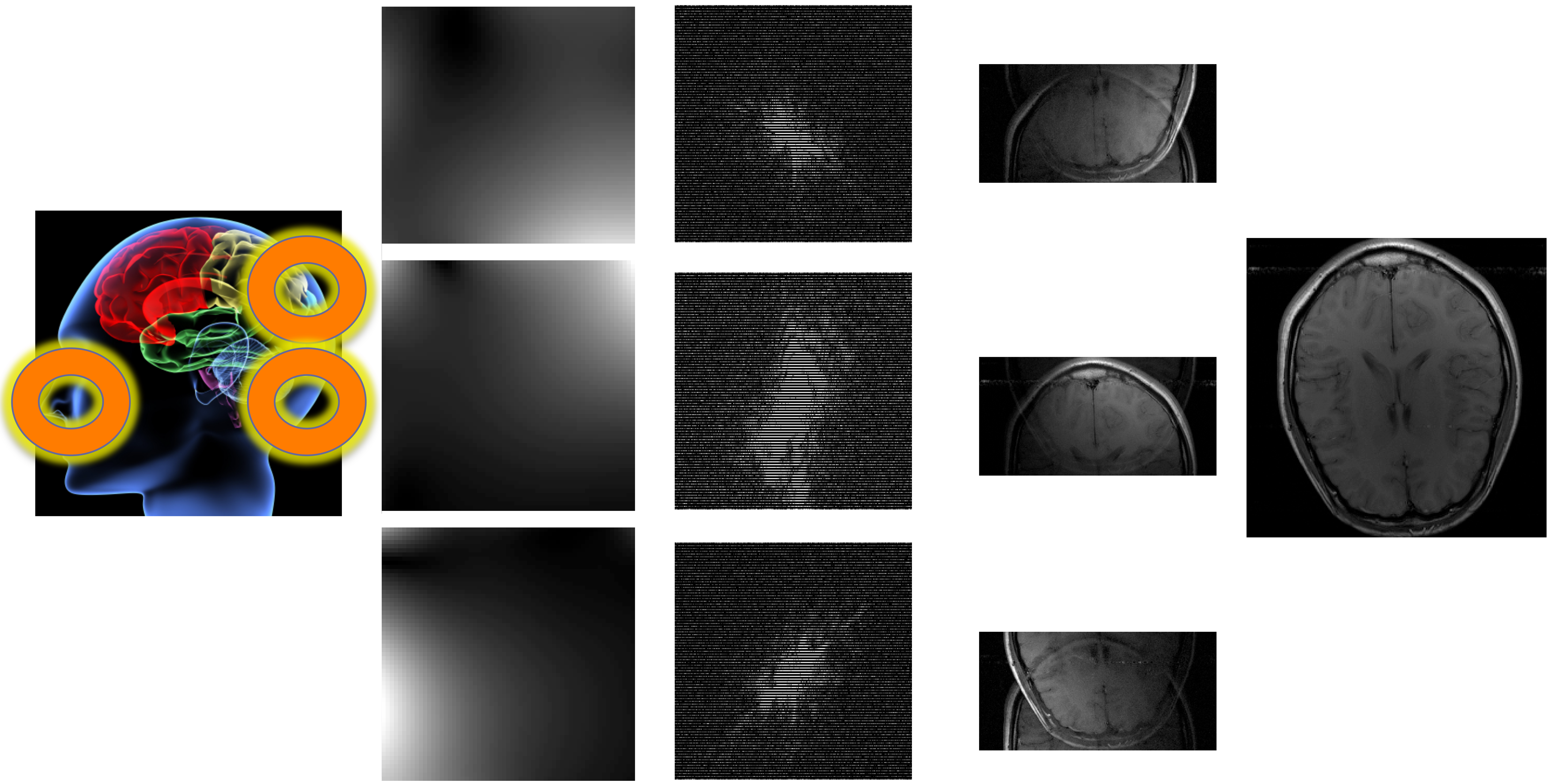

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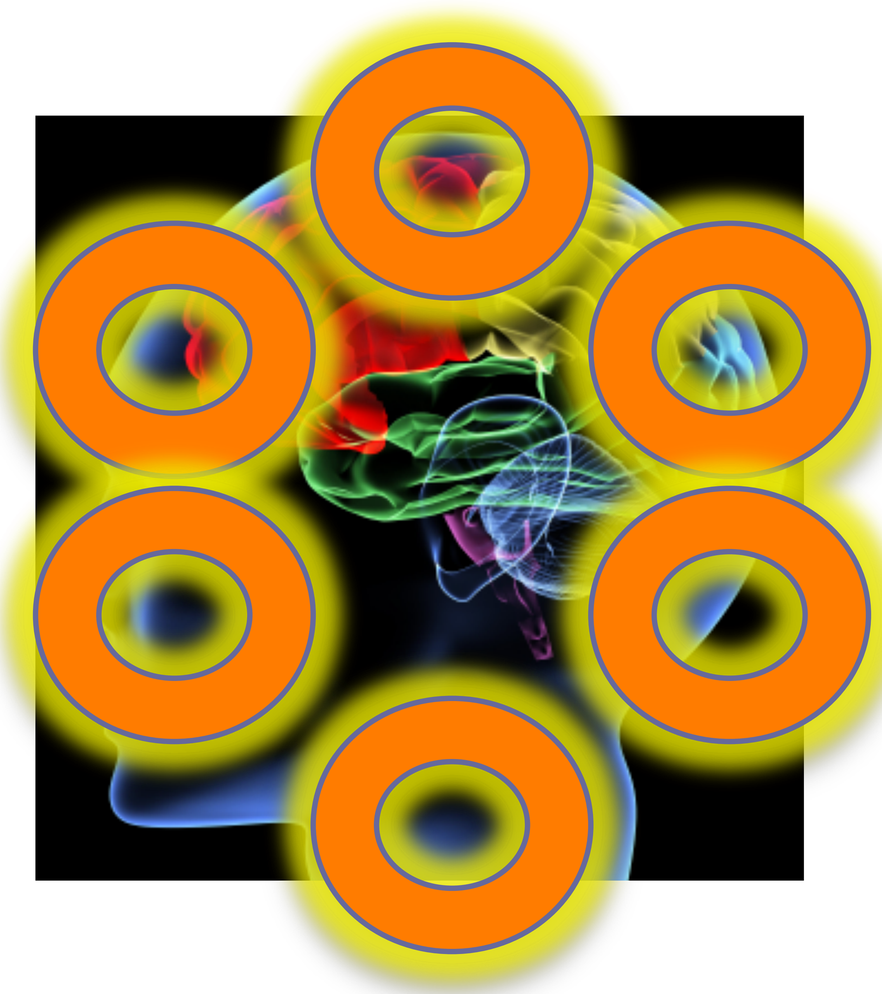
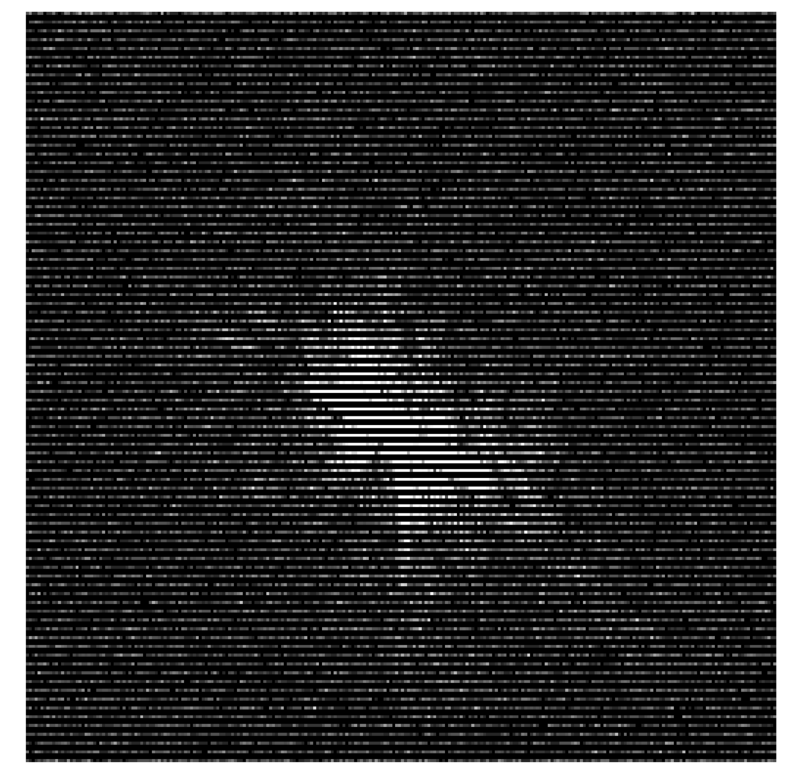
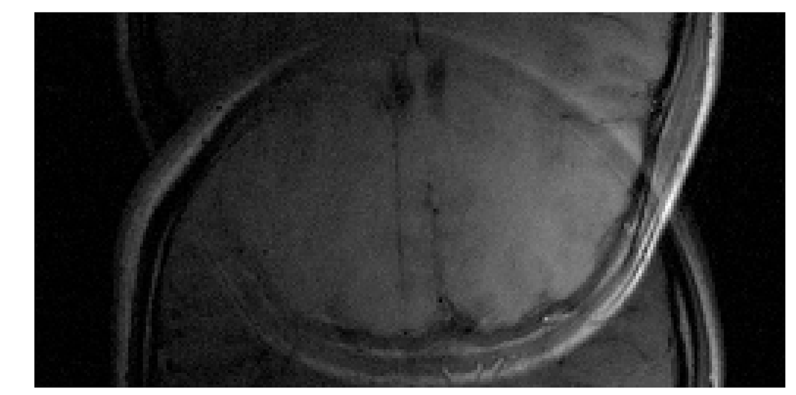
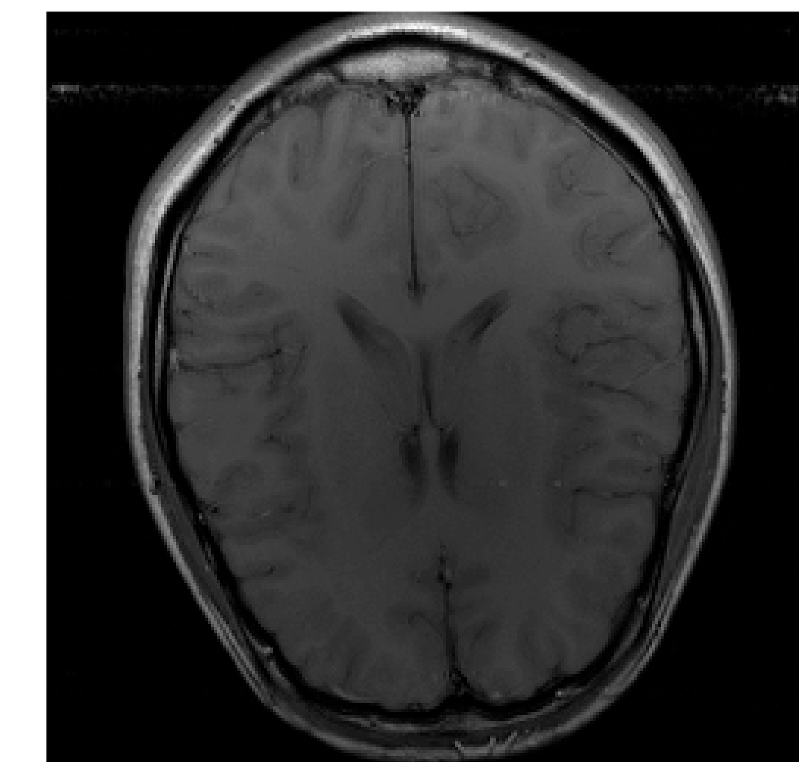
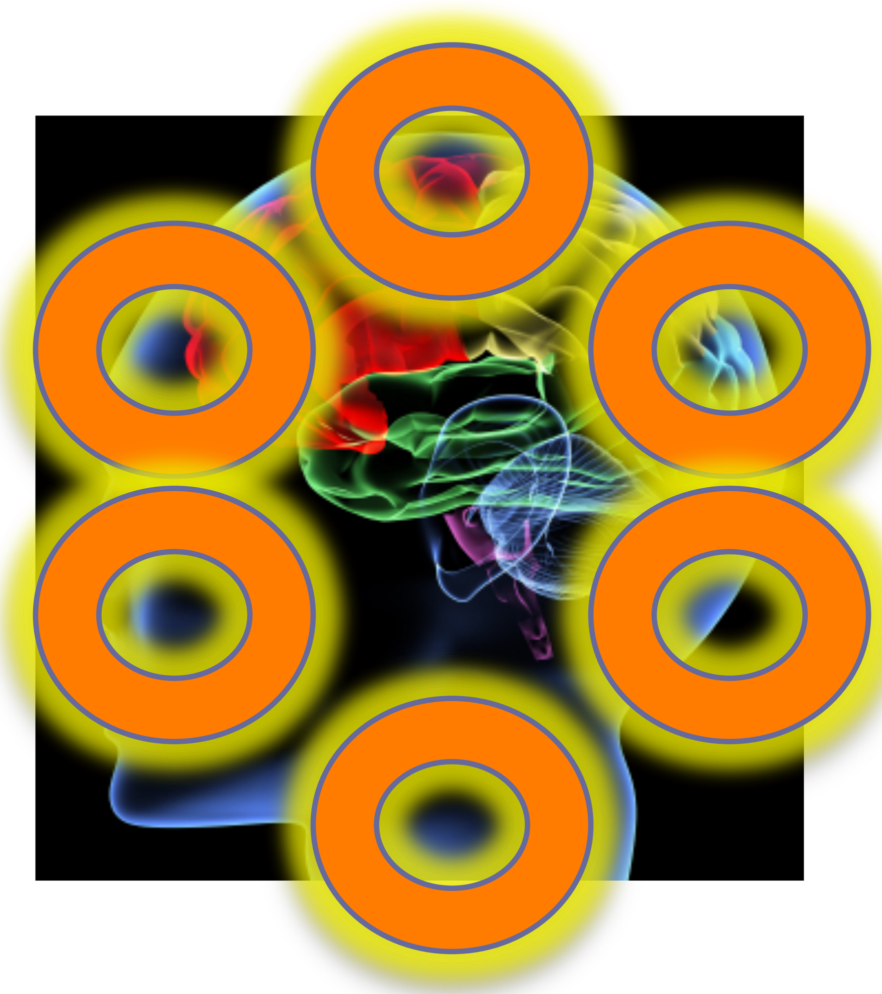
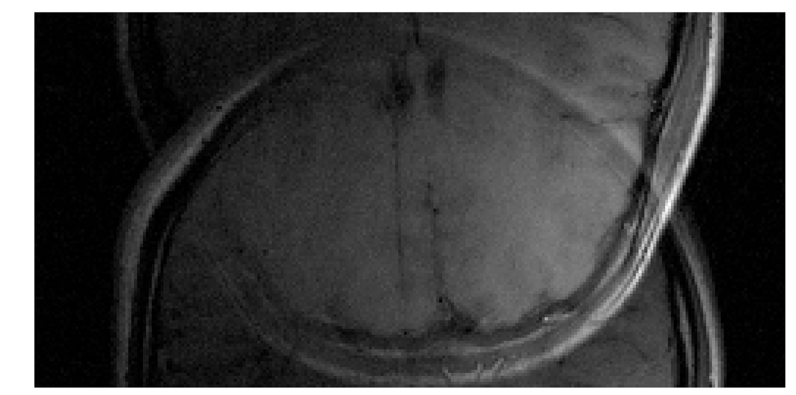
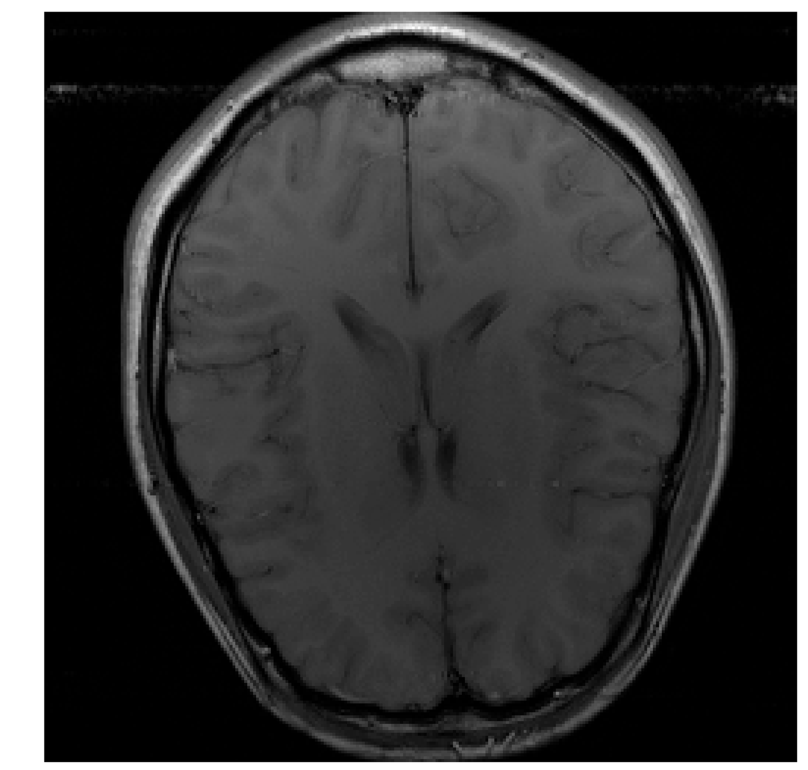


- How to calculate accurate sensitivity profile?
  - ➔ Polynomial Fitting

SENSE Imaging

- How to calculate accurate sensitivity profile?
  - ➔ Polynomial Fitting
- Is there a more efficient way under-sample k-space?
  - ➔ Semi-definite model

Sensitivity Mapping

- Sensitivity Profiling Procedure

Fully sample body coil image & surface coil image

Fit lower resolution data to get polynomial coefficients

Interpolation of polynomials to get higher resolution sensitivity profile

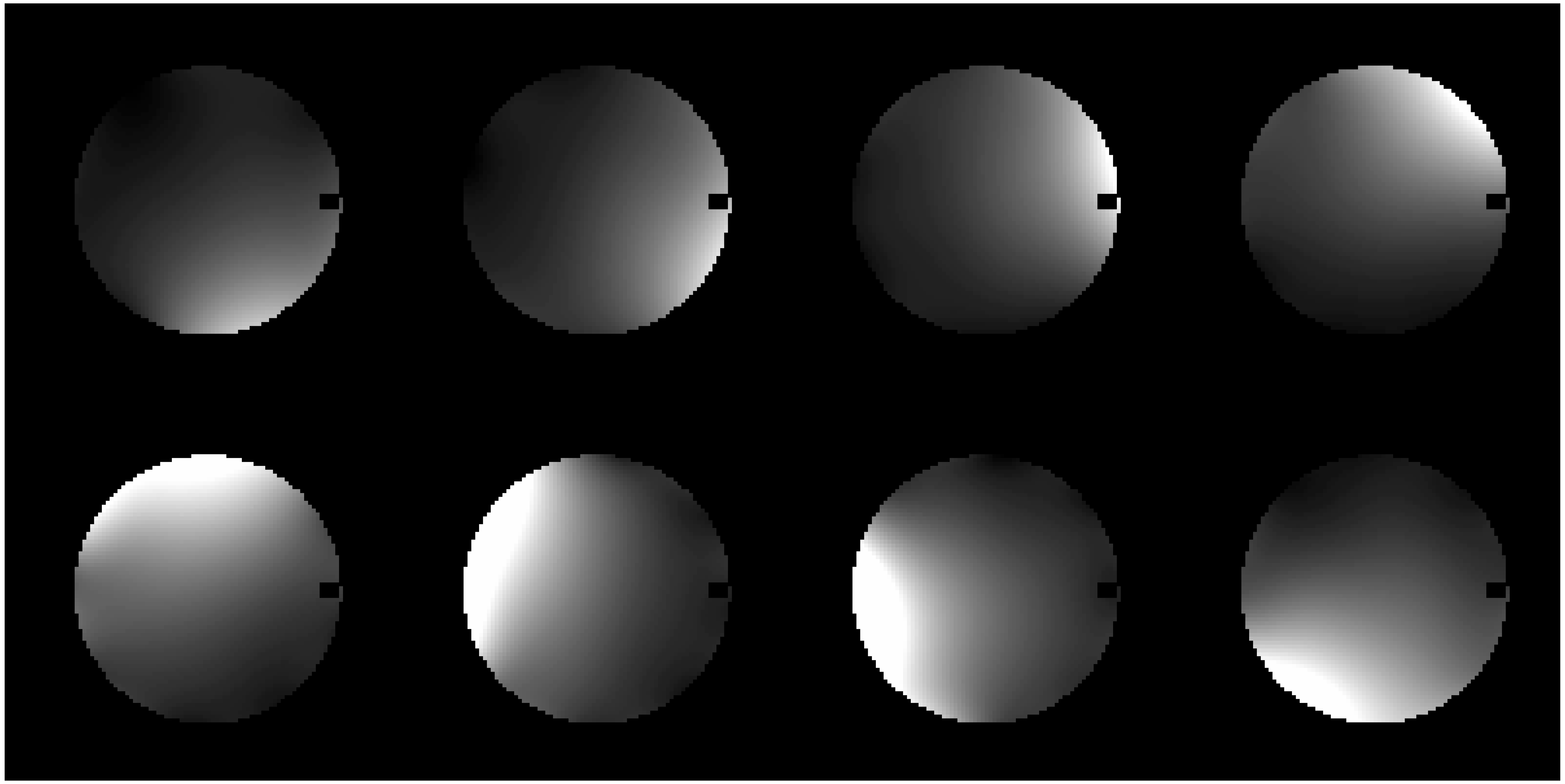
Sensitivity Mapping

$$\min_{a_{i,j}} \quad \sum (IM^{body} \cdot S_j - IM_j^{head})^2 \cdot W$$

$$s.t. \quad S_j = \sum_i a_{i,j} P_i$$

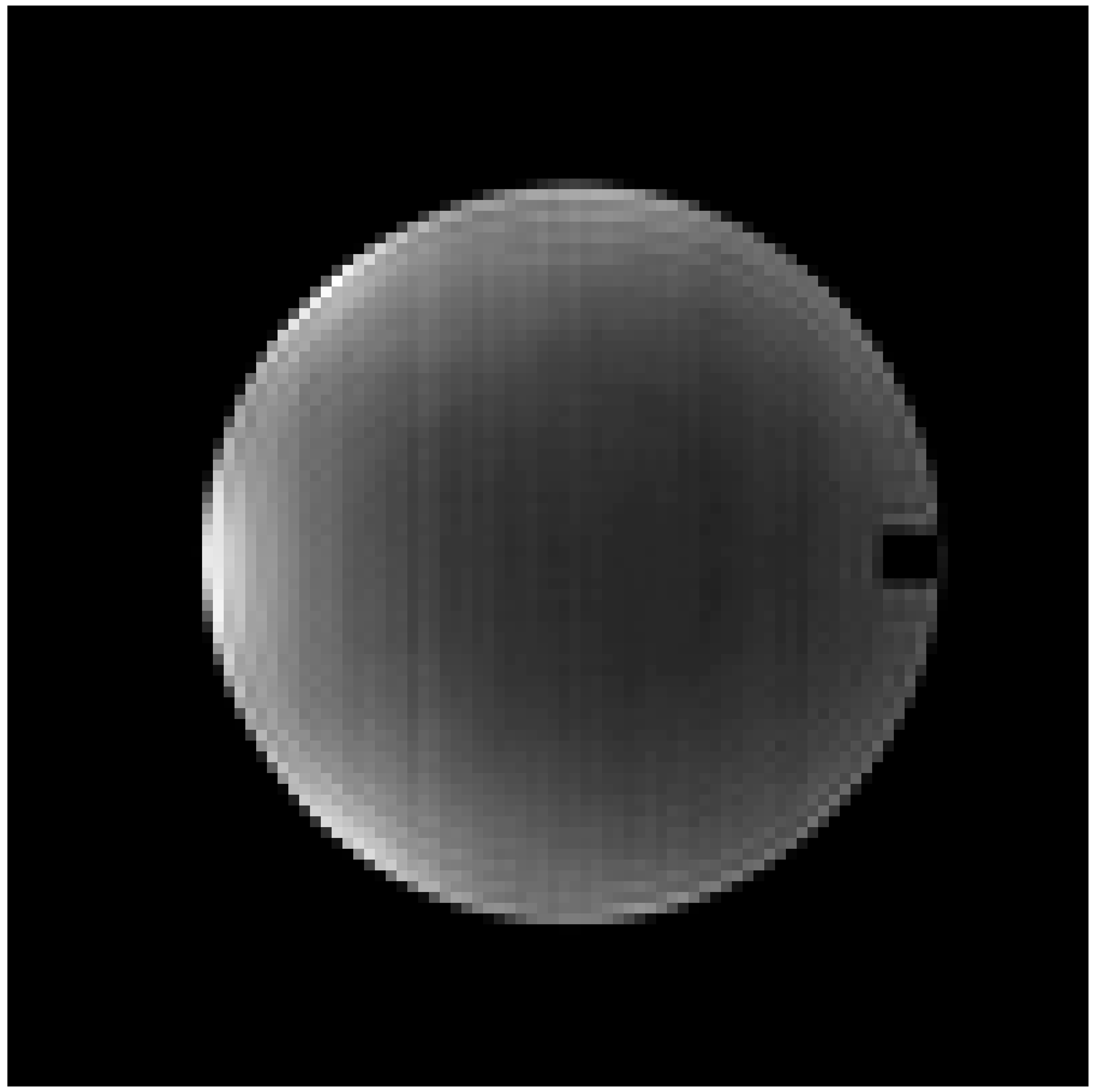
- $\{a_{i,j}\}$: coefficients of polynomials
- $\{P_i\}$: polynomial basis for fitting
- IM^{body}: low resolution body coil image
- IM^{head}: low resolution head coil images
- W: image weighting

Results & Discussions

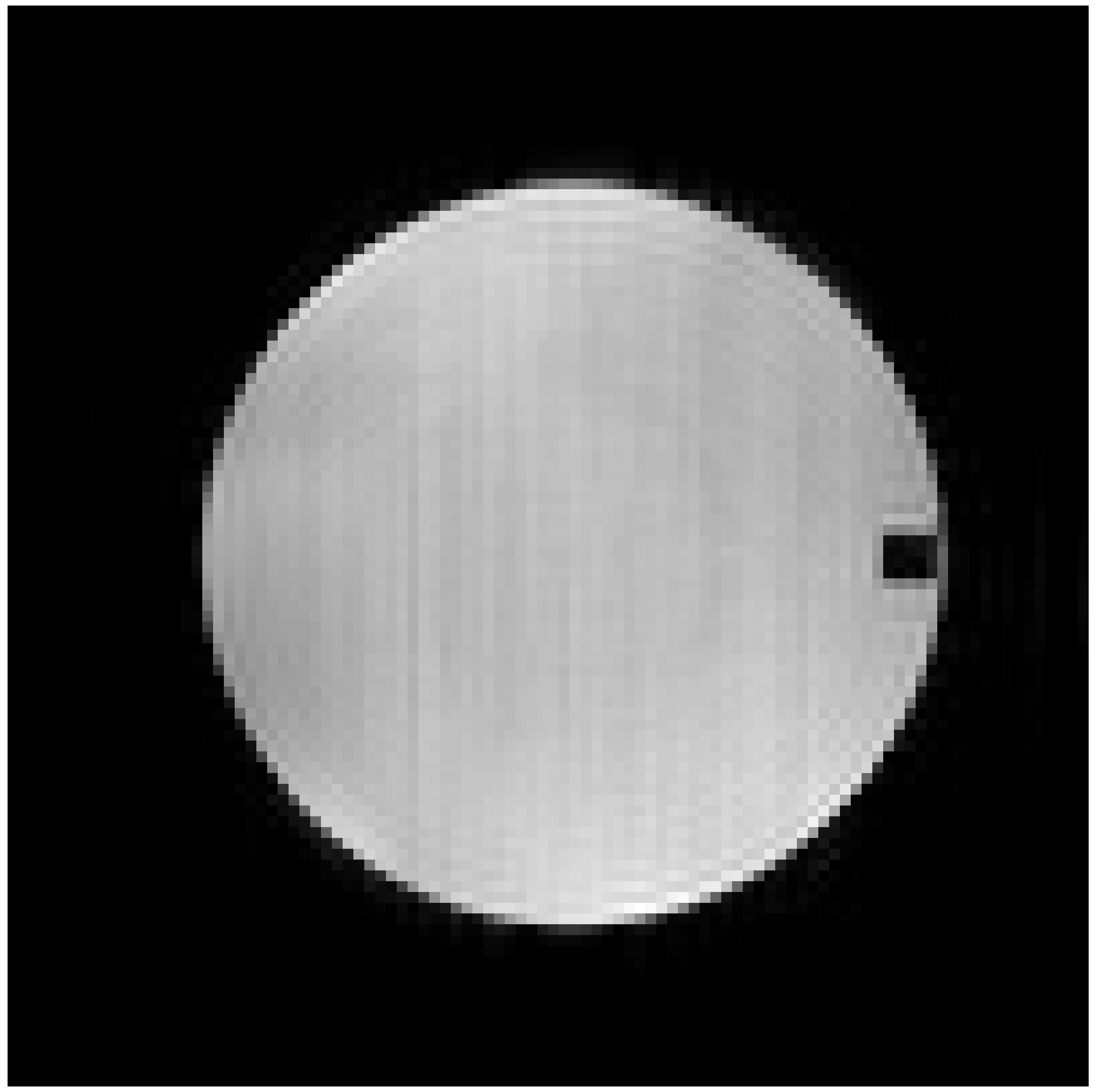


- Solved by one step Newton Method
- MATLAB R2010a
- Lower resolution dataset: 50x50x50
- Higher resolution profiling: 100x100x100

Results & Discussions



$$\sqrt{\sum IM_j^{head} \cdot IM_j^{head^*}}$$



$$\frac{\sum S_j^* \cdot IM_j^{head}}{S_j \cdot S_j^*}$$

Results & Discussions

- 3D polynomial provides sensitivity in arbitrary planes and volumes
- Sensitivity near coils is hard to fit

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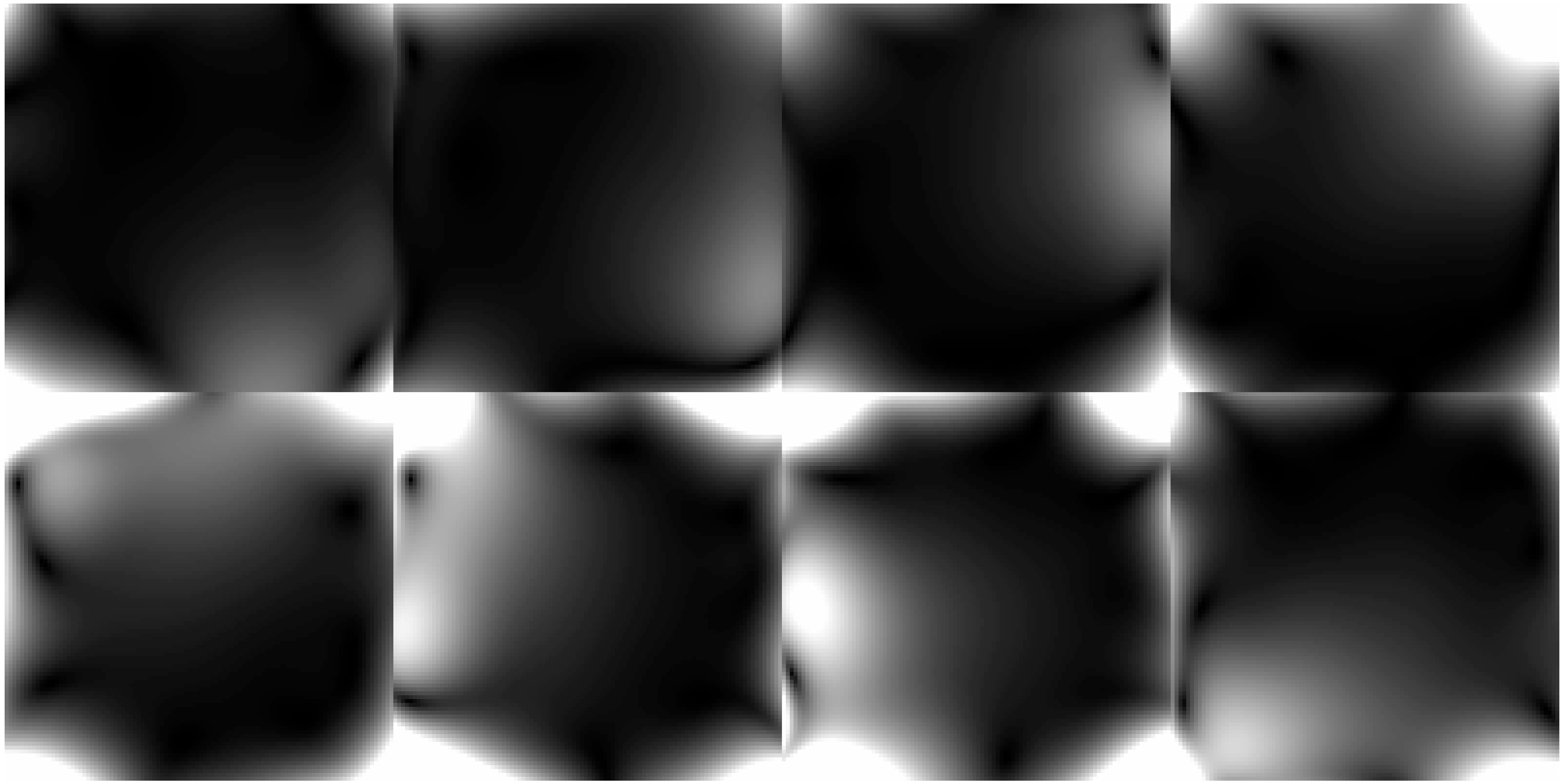
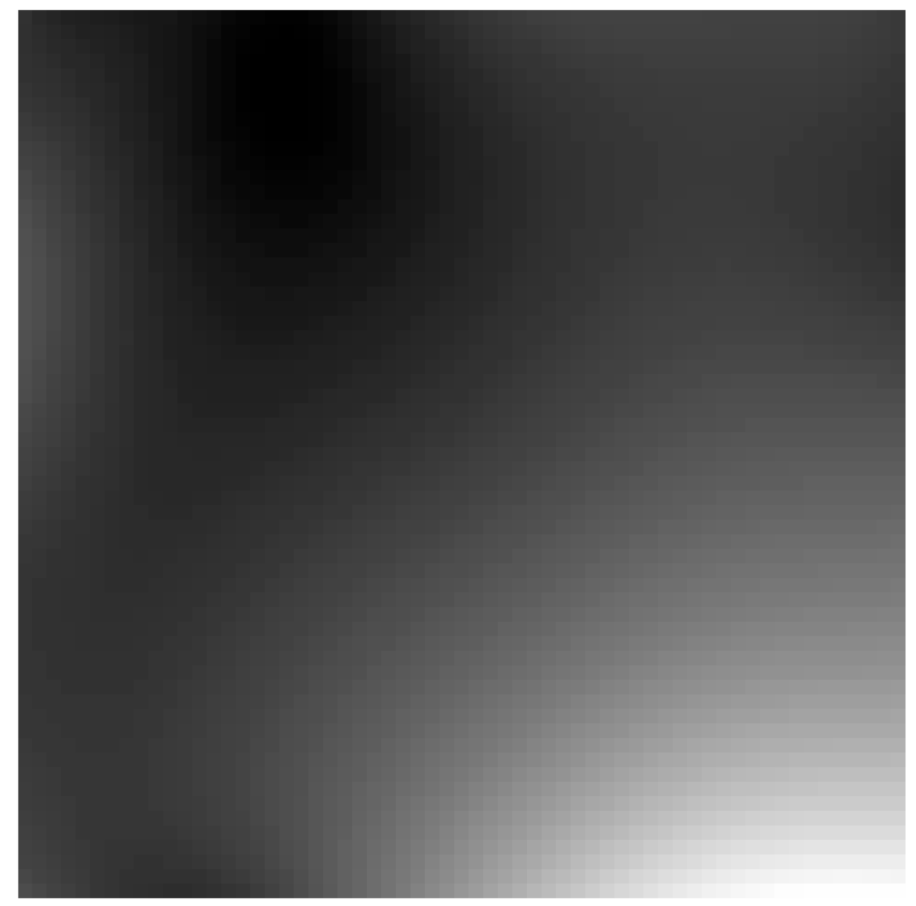


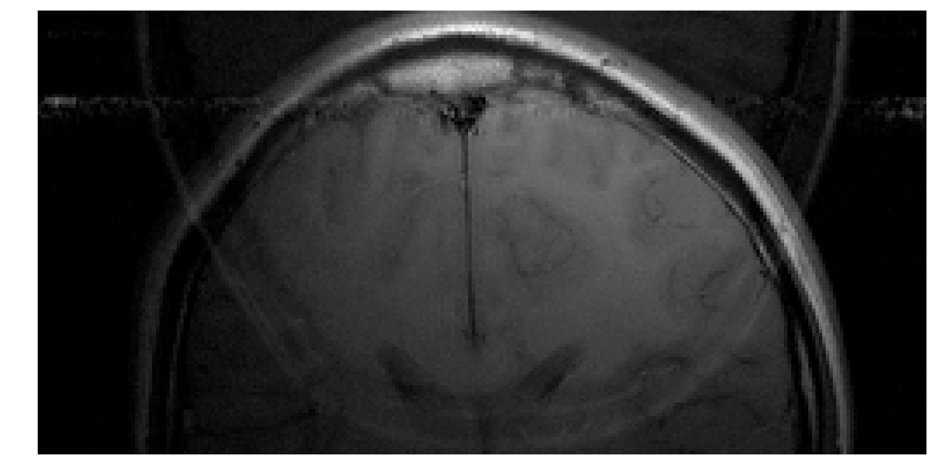
SENSE Under-sampling

ρ



S_c

$\tilde{f}_c(k)$



$m_c(x)$

$$m_c(x) = T_{ift} \bullet T_{\Pi} \bullet T_{ft}\{S_c(x) \cdot \rho(x)\}$$

$$m_c(x) = \tilde{S}_c \cdot \rho(x)$$

SENSE Under-sampling

$$M = S \cdot P$$

- Measurements

$$M = \begin{pmatrix} m_1 & m_2 & \ldots & m_c \end{pmatrix}^T$$

- Proton density

$$P = \begin{pmatrix} \rho_1 & \rho_2 & \ldots & \rho_n \end{pmatrix}^T$$

- Sensitivity encoding matrix

$$S = \begin{pmatrix} \tilde{S}_1 & & & \\ & \tilde{S}_2 & & \\ & & \ddots & \\ & & & \tilde{S}_c \end{pmatrix}$$

SENSE Under-sampling

- Reconstruction

$$P = (S^T \cdot S)^{-1} \cdot S^T \cdot M$$

- Measurement with errors

$$\tilde{P} = (S^T \cdot S)^{-1} \cdot S^T \cdot (M + \varepsilon)$$

- $(S^T S)^{-1} S^T$ has the effect of magnifying noise
- Worst case of noise magnification is determined by largest eigenvalue of $(S^T S)^{-1}$ $\rightarrow$ smallest eigenvalue of $S^T S$

SENSE Under-sampling

- worst case of noise magnification is determined by largest eigenvalue of $(S^T S)^{-1}$ ➔ maximize the smallest eigenvalue of $S^T S$

$$\max_{r_1, r_2} \quad \lambda$$

$$s.t. \quad S^T \cdot S - \lambda I \succ 0$$

- Size of S depends on r1,r2 ➔ lowest SNR point has worst noise magnification

SENSE Under-sampling

- Under-sampling factor = 2
- Size of S: 4x4

$$M = \begin{pmatrix} s_{1,1} & & s_{1,3} & \\ & s_{1,2} & & s_{1,4} \\ s_{2,1} & & s_{2,3} & \\ & s_{2,2} & & s_{2,4} \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{pmatrix}$$

- Under-sampling factor = 4
- Size of S: 2x4

$$M = \begin{pmatrix} s_{1,1} & s_{1,2} & s_{1,3} & s_{1,4} \\ s_{2,1} & s_{2,2} & s_{2,3} & s_{2,4} \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{pmatrix}$$

SENSE Under-sampling

- Under-sampling factor = 2
- Size of S: 4x4

$$M = \begin{pmatrix} s_{1,1} & & s_{1,3} & \\ & s_{1,2} & & s_{1,4} \\ s_{2,1} & & s_{2,3} & \\ & s_{2,2} & & s_{2,4} \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{pmatrix}$$

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SENSE Under-sampling

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$$M = \begin{pmatrix} s_{1,1} & s_{1,2} & s_{1,3} & s_{1,4} \\ s_{2,1} & s_{2,2} & s_{2,3} & s_{2,4} \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \end{pmatrix}$$

- Worst SNR case

$$M = \begin{pmatrix} s_{1,1} & s_{1,3} \\ s_{2,1} & s_{2,3} \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_3 \end{pmatrix}$$

SENSE Under-sampling

- Nonlinear Semi-definite problem

$$\max_{r_1, r_2} \quad \lambda$$

$$s.t. \quad S^T \cdot S - \lambda I \succeq 0$$

- Linear Trust Region step

$$\max_{\Delta r_1, \Delta r_2} \quad \lambda$$

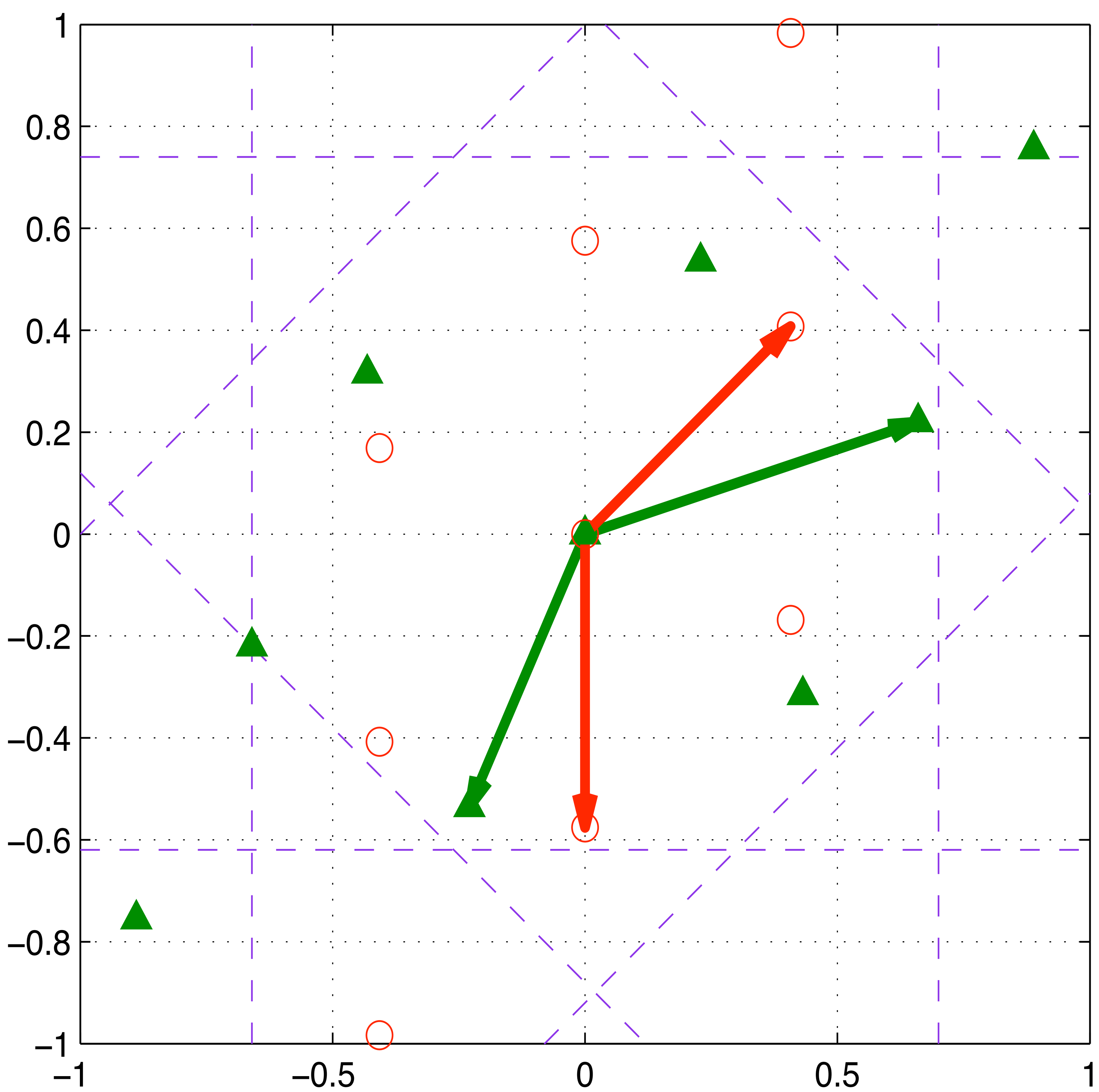
$$s.t. \quad \tilde{S}^T \cdot \tilde{S} + \frac{\partial (S^T \cdot S)}{\partial \tilde{r}_1} \cdot \Delta r_1 + \frac{\partial (S^T \cdot S)}{\partial \tilde{r}_2} \cdot \Delta r_2 - \lambda I \succeq 0$$

$$|\Delta r_1| \le \varepsilon$$

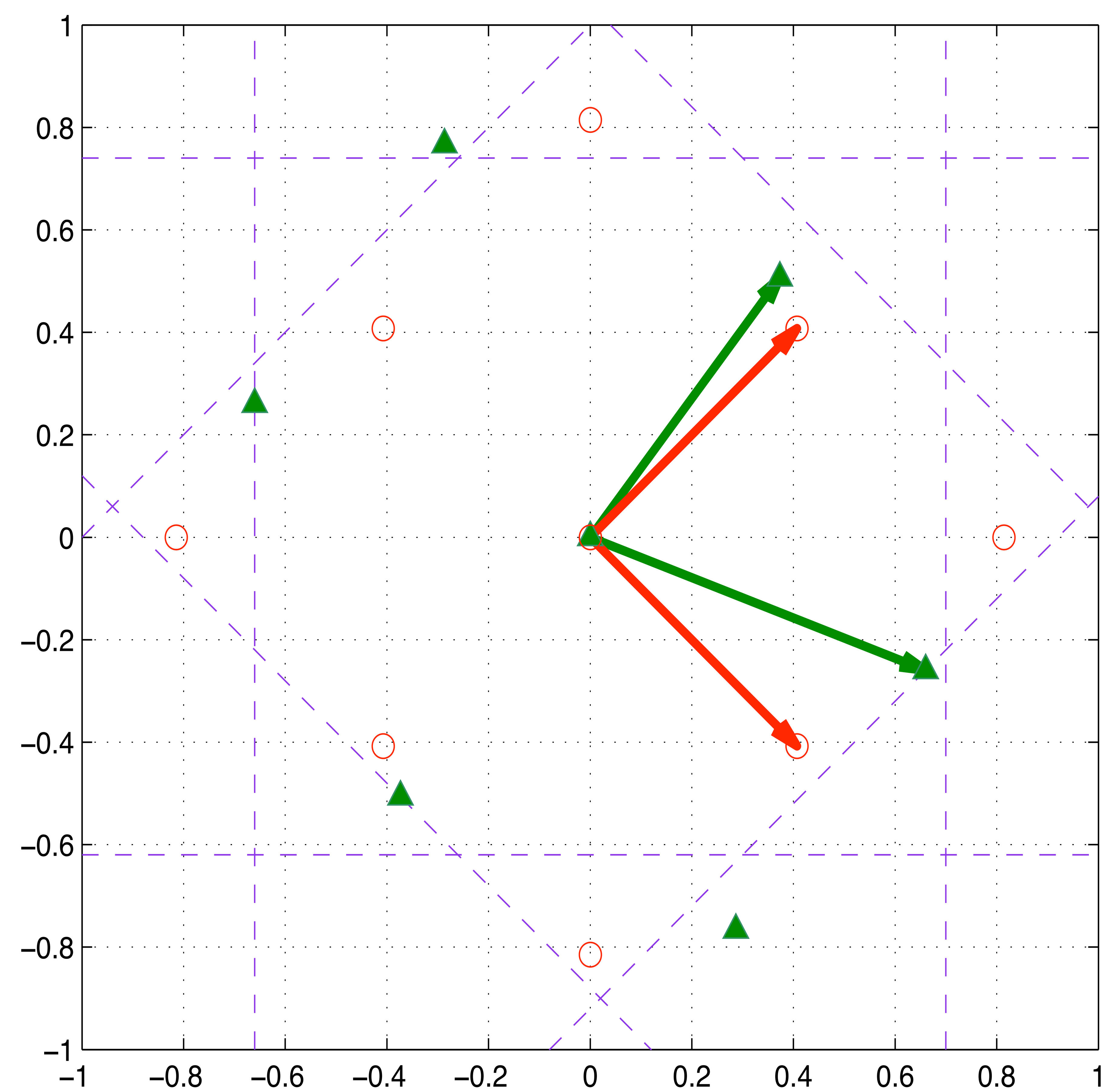
$$|\Delta r_2| \le \varepsilon$$

Results & Discussions

- Solved by SeDuMi_1.2.1
- MATLAB R2010a
- Iterative trust region method



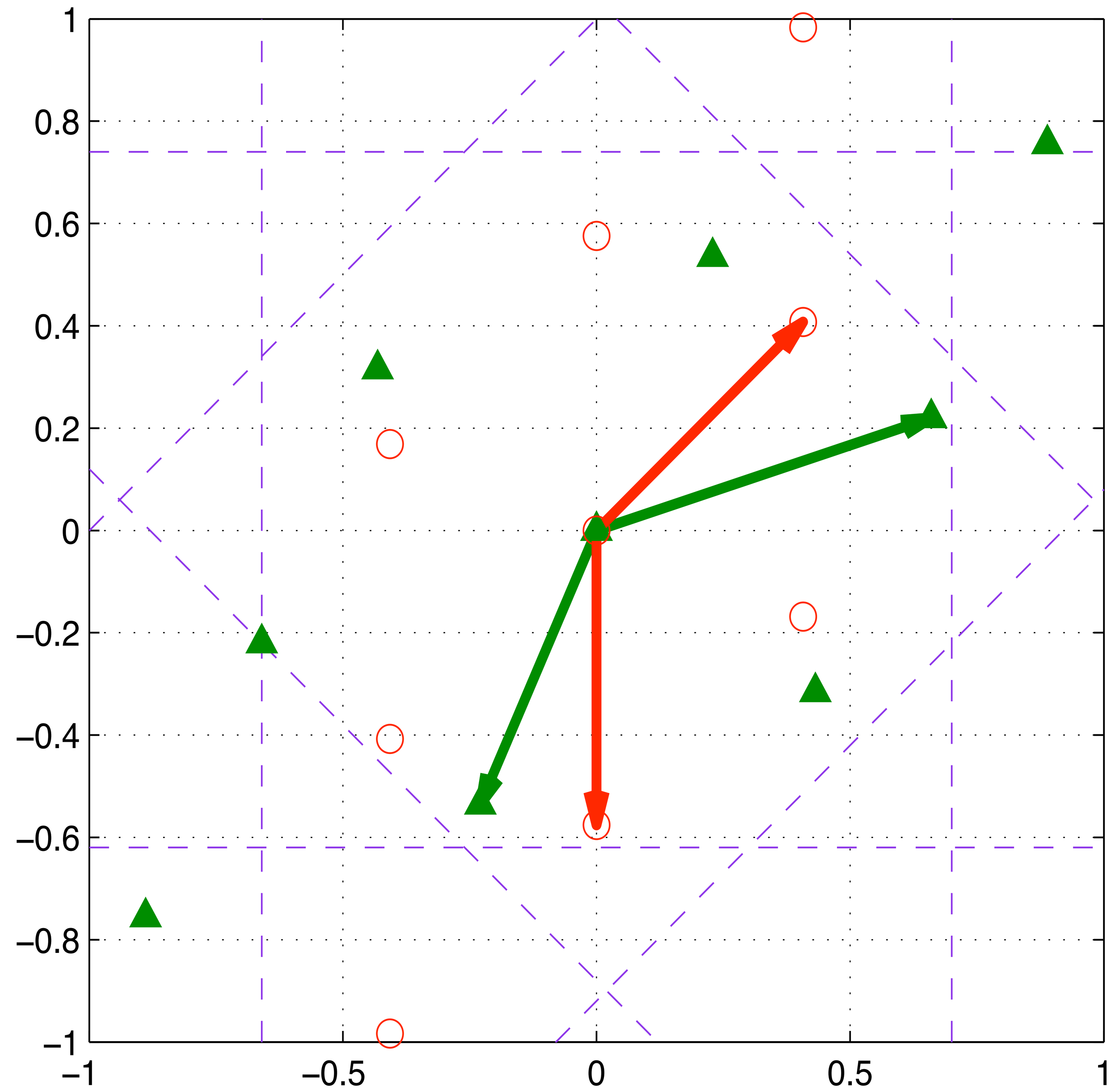
case 1

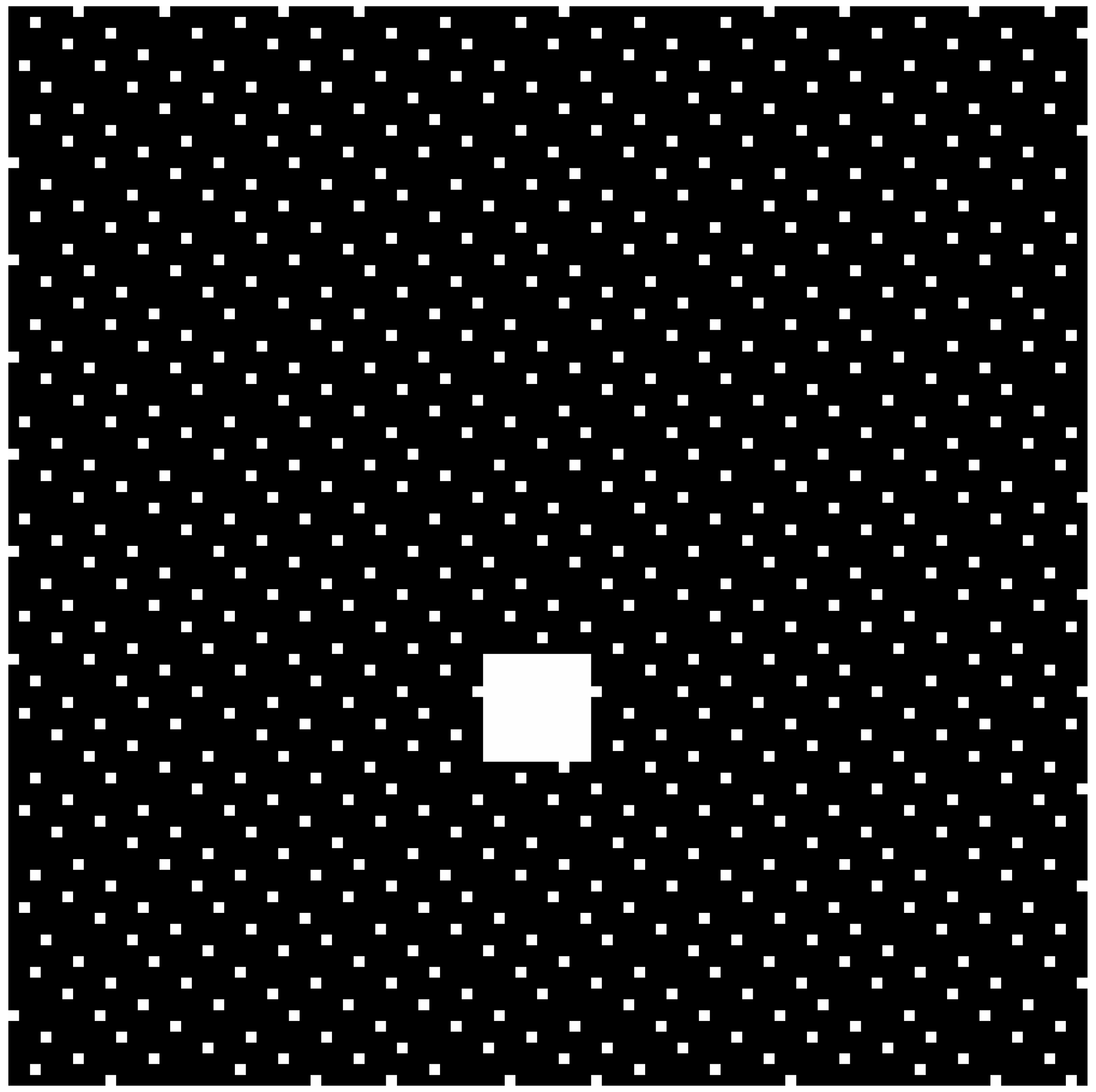
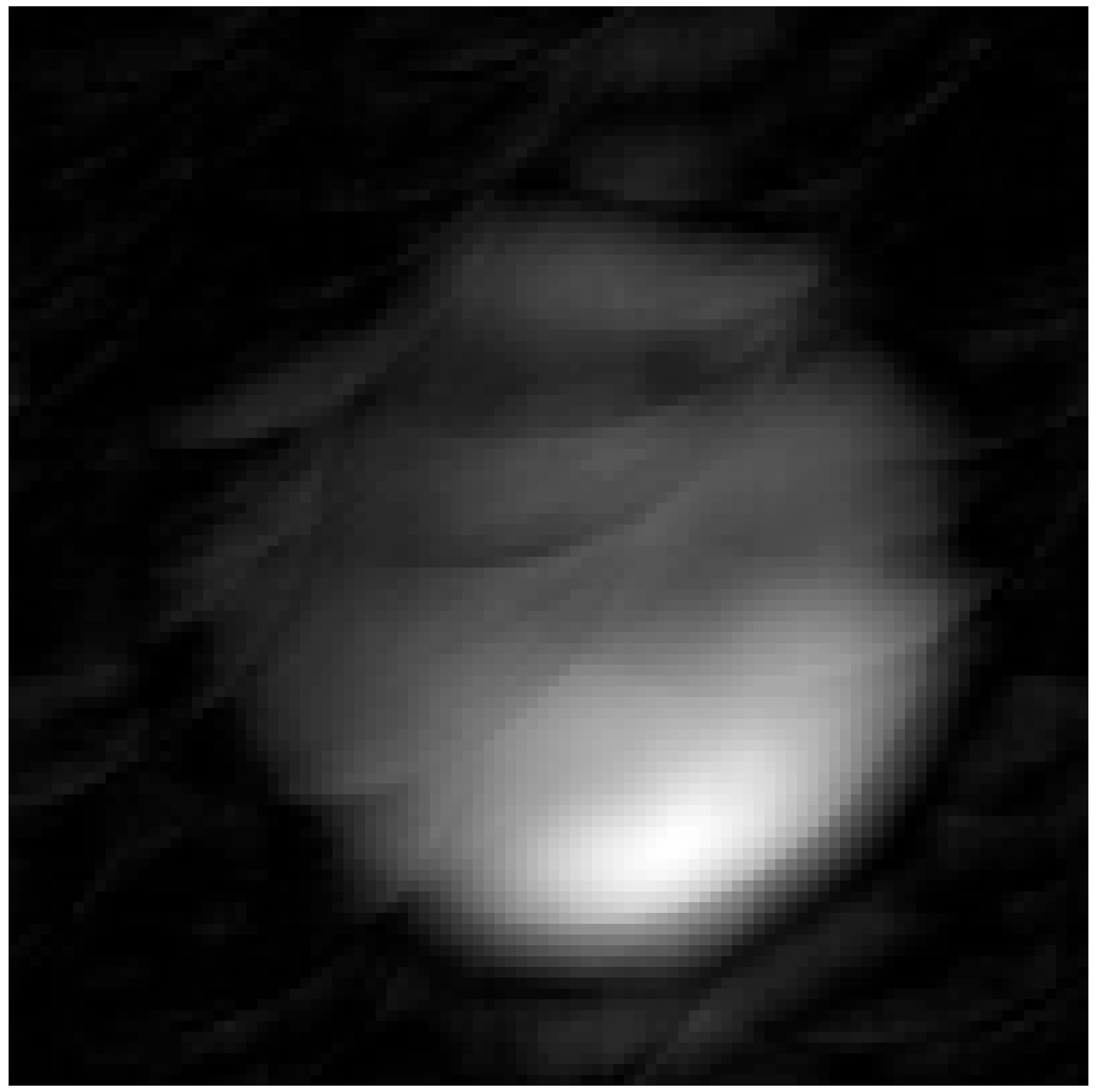


case 2

Results & Discussions

Case 1



- Condition number improvement: 121.88 v.s. 24.58
- Sampled data: 8.5%

Results & Discussions

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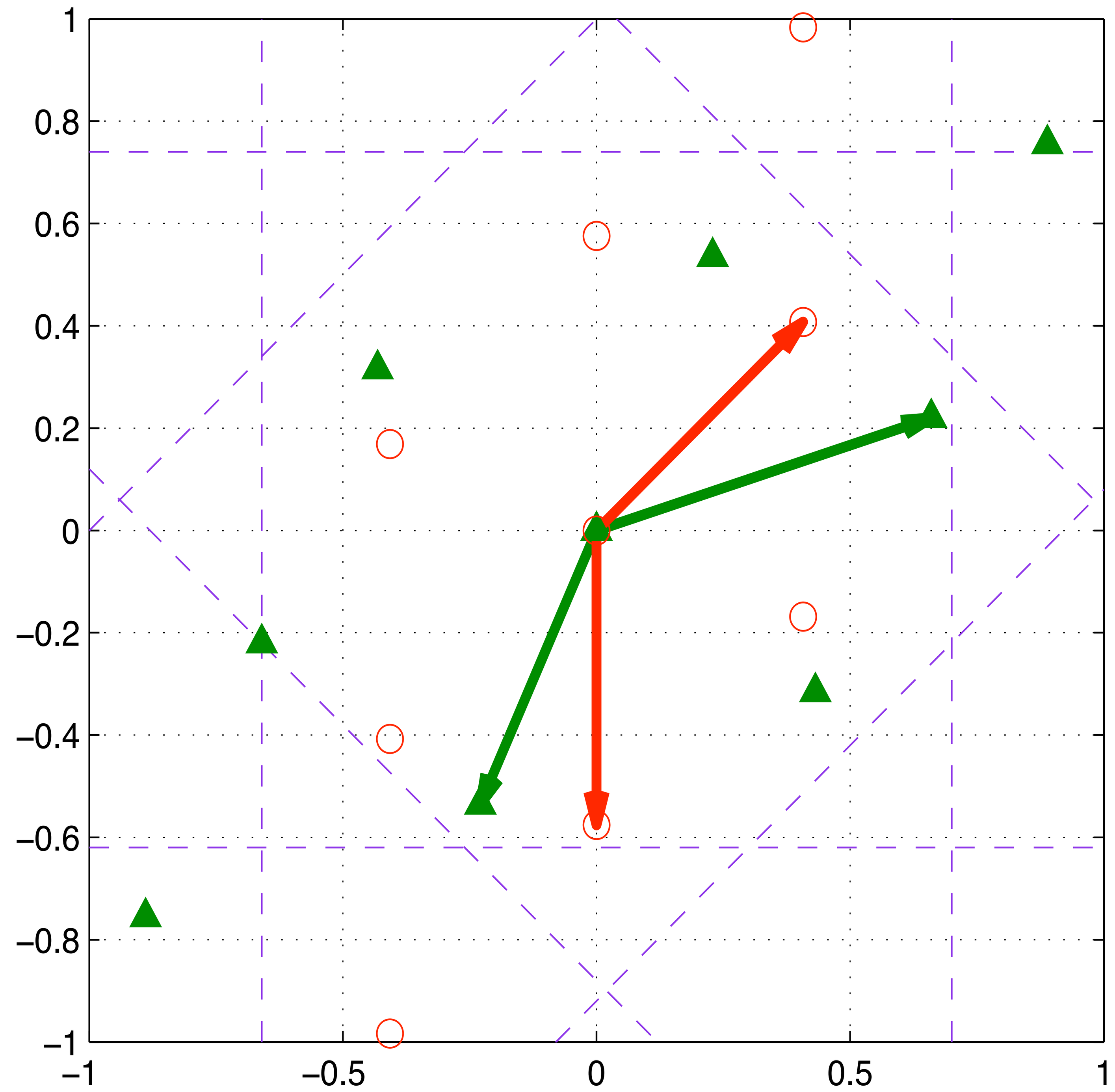

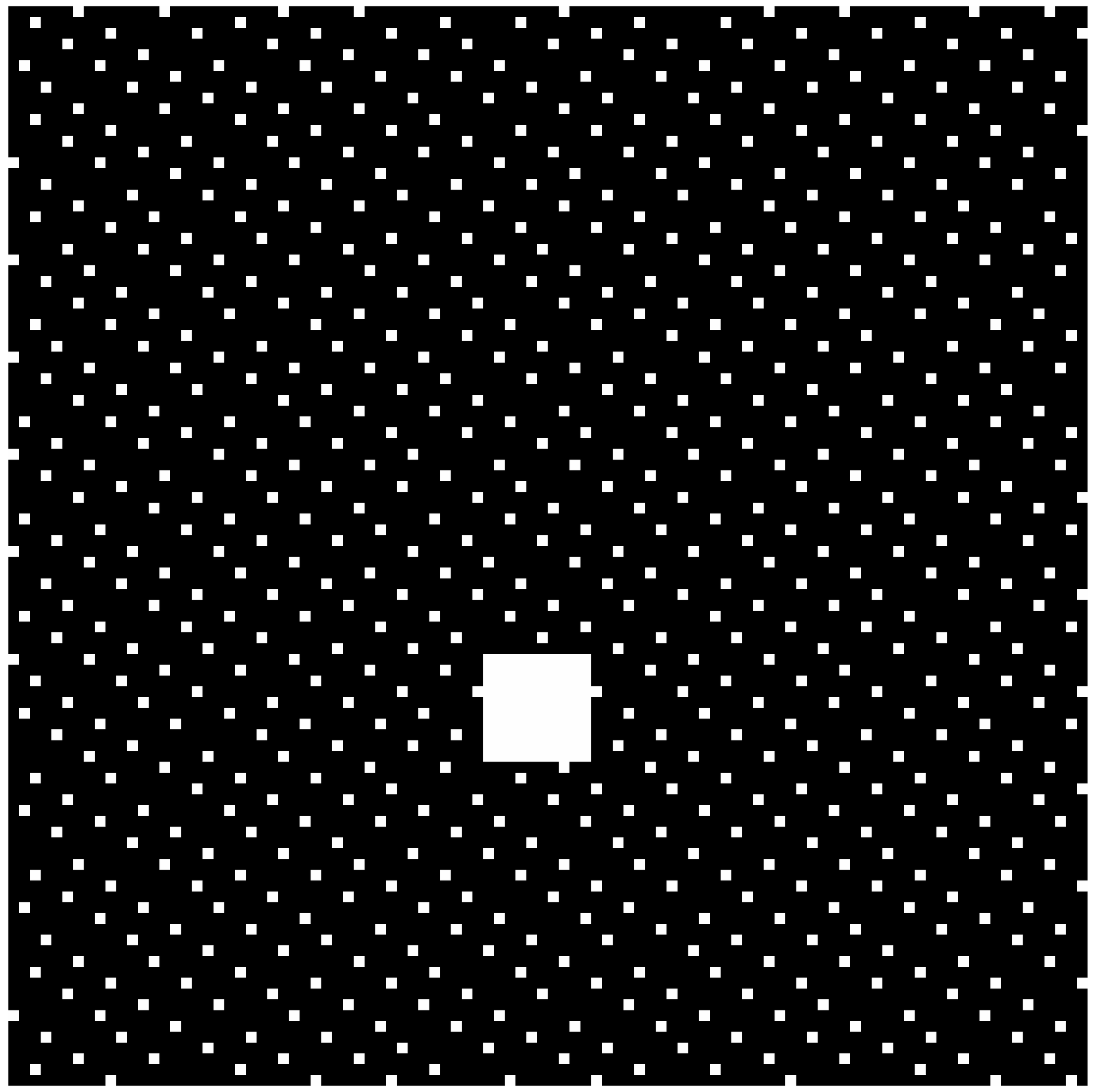

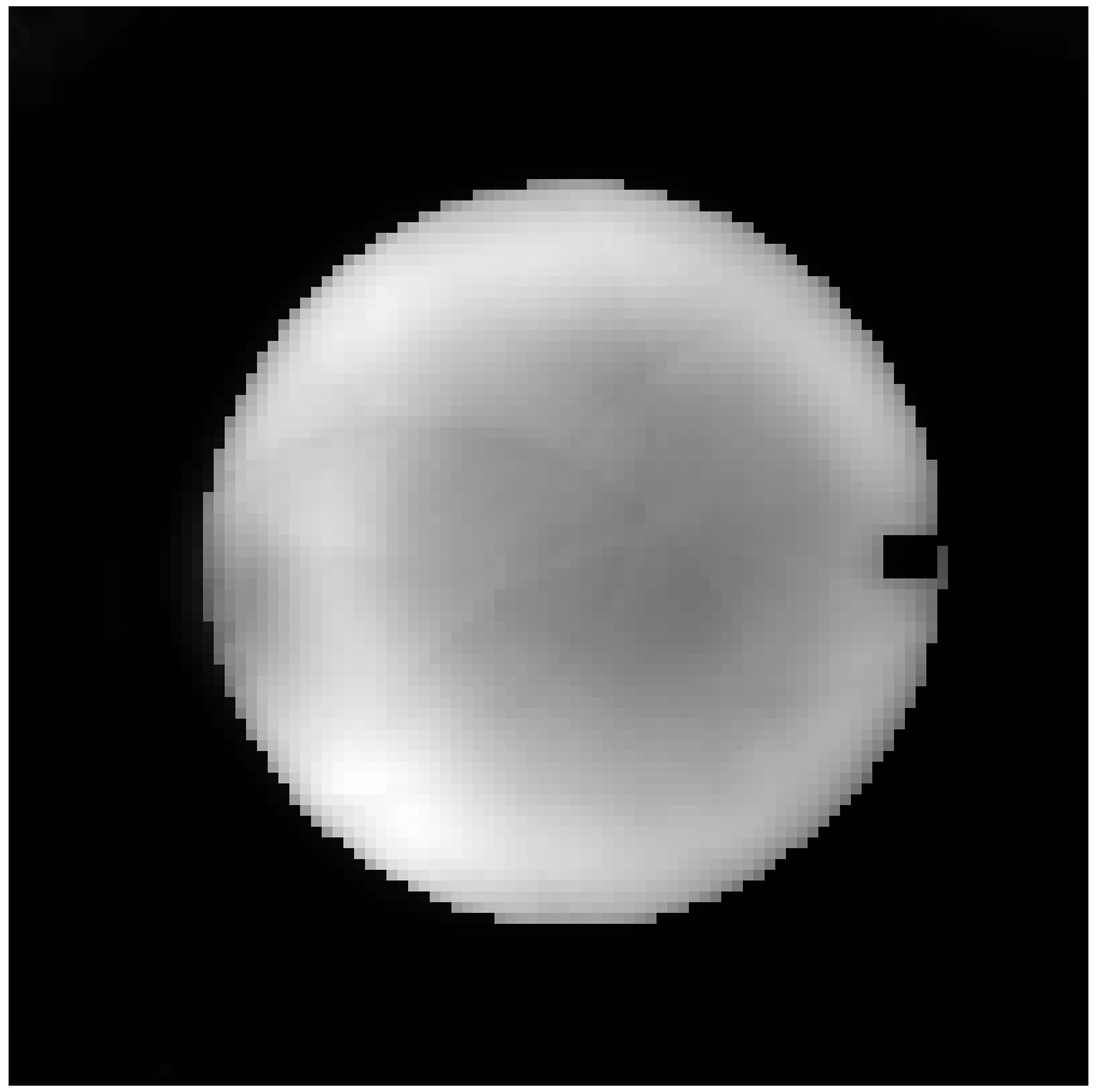

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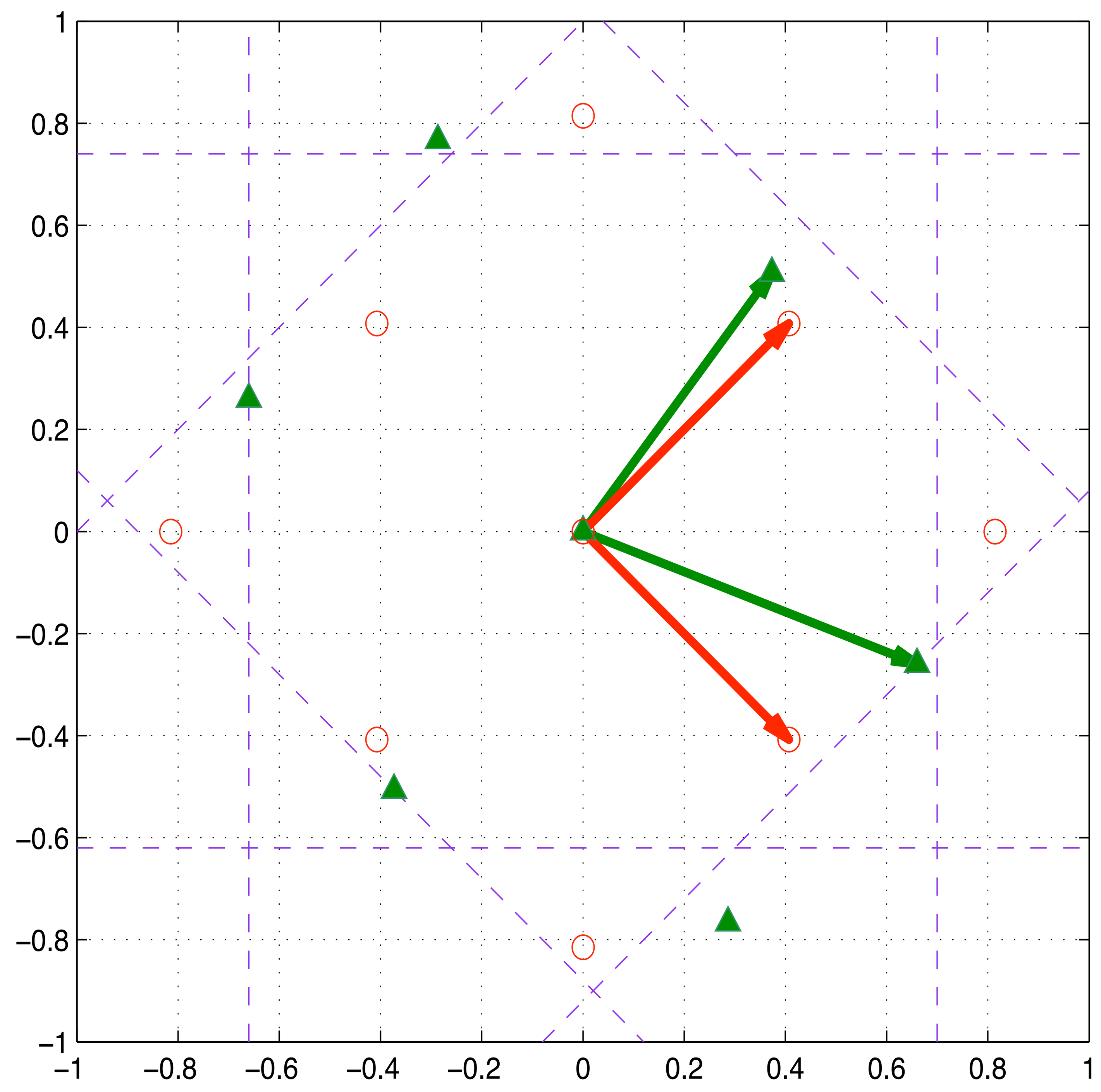
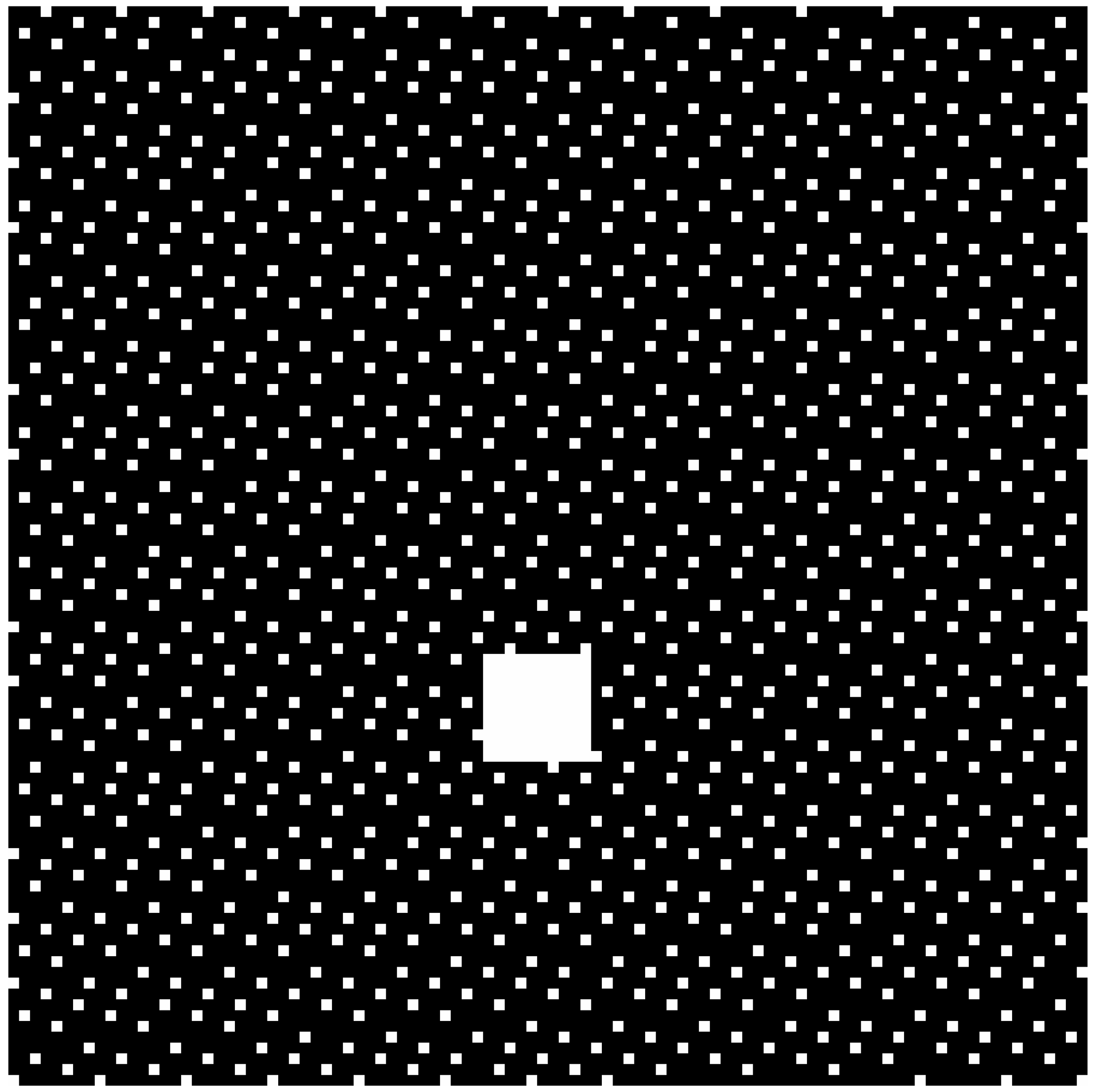
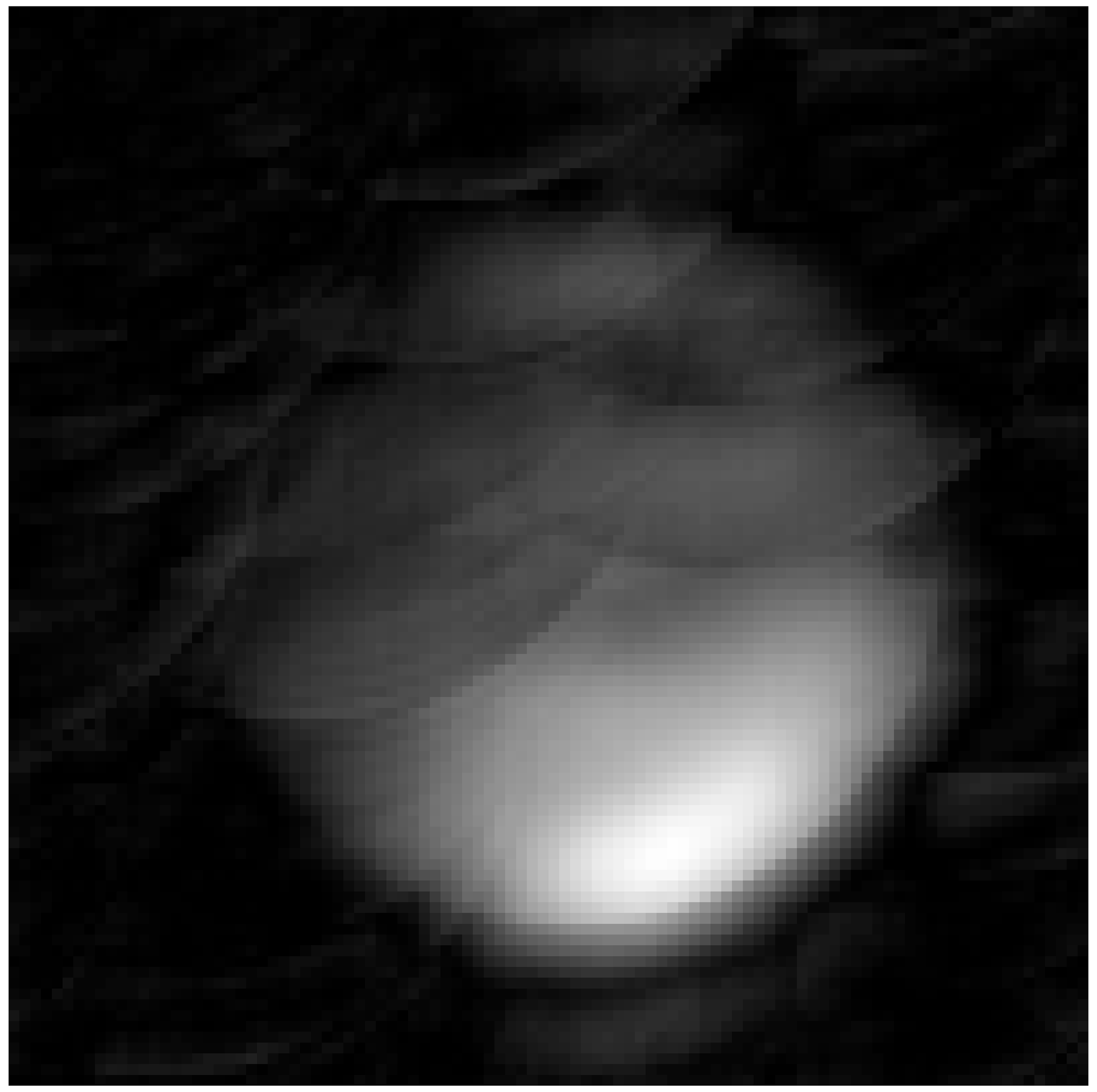
Results & Discussions

Case 2



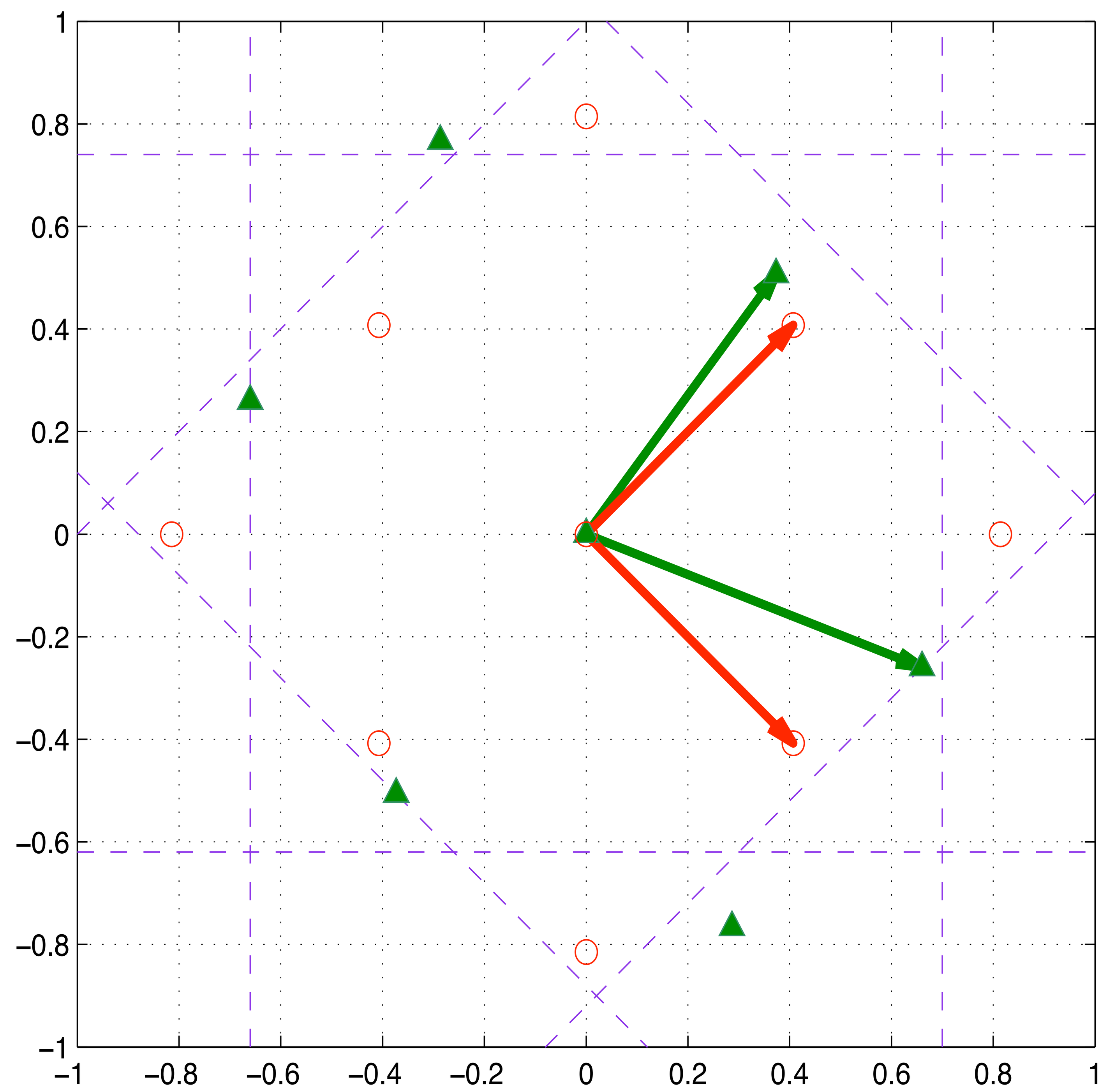


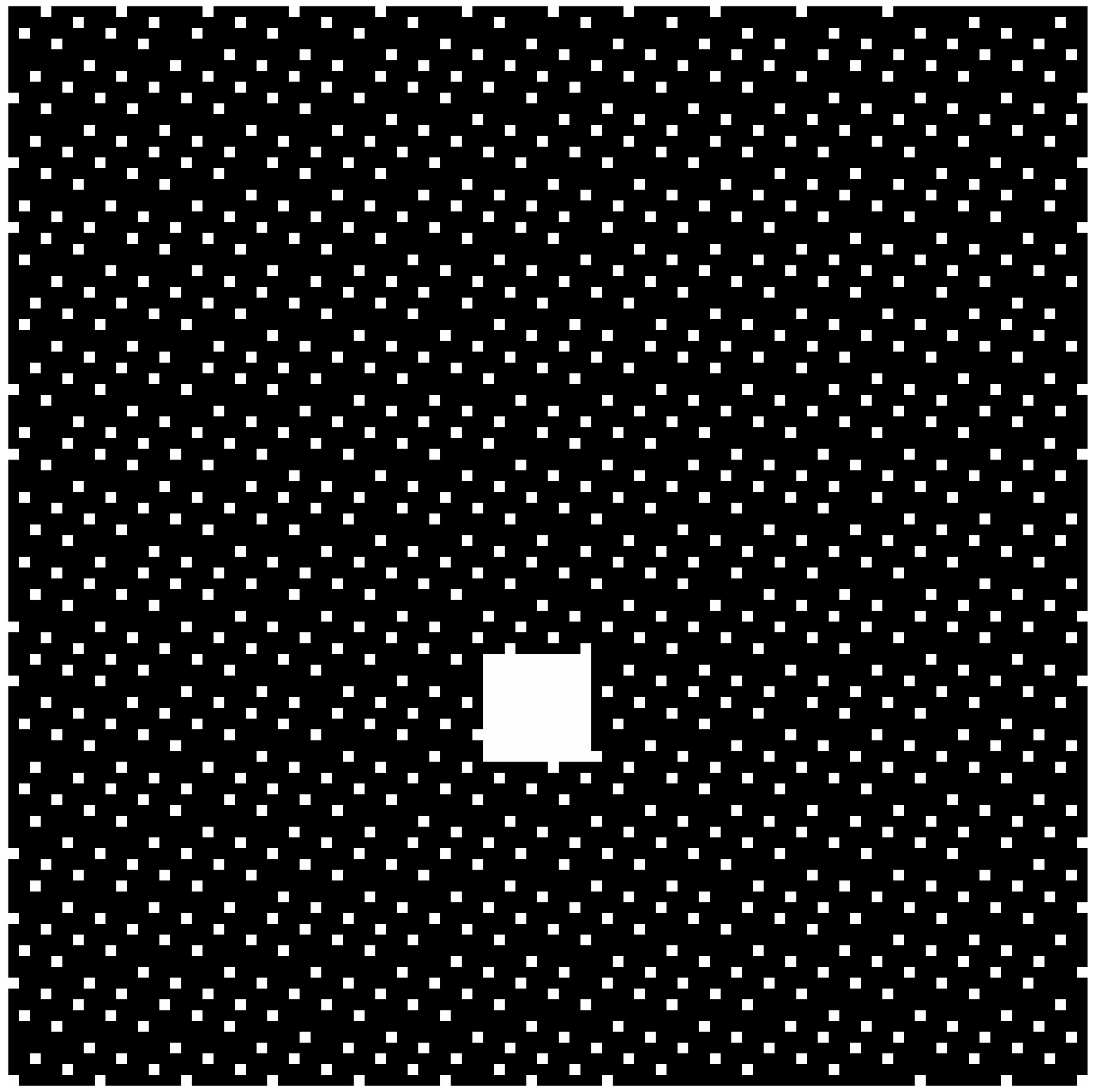


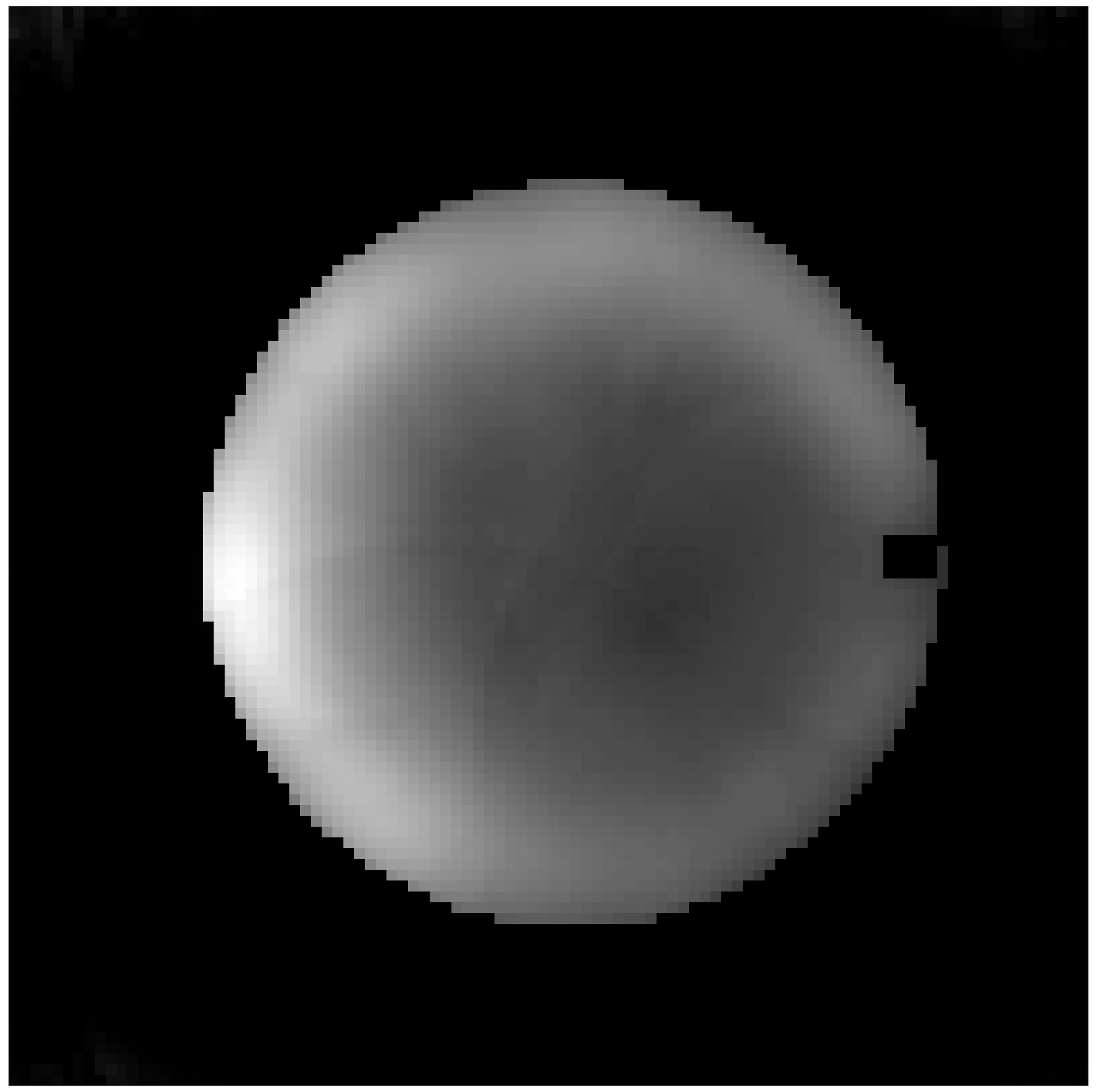
- Condition number improvement:
  12.95 v.s. 8.5563
- Sample data: 11.7%

Results & Discussions

Case 2



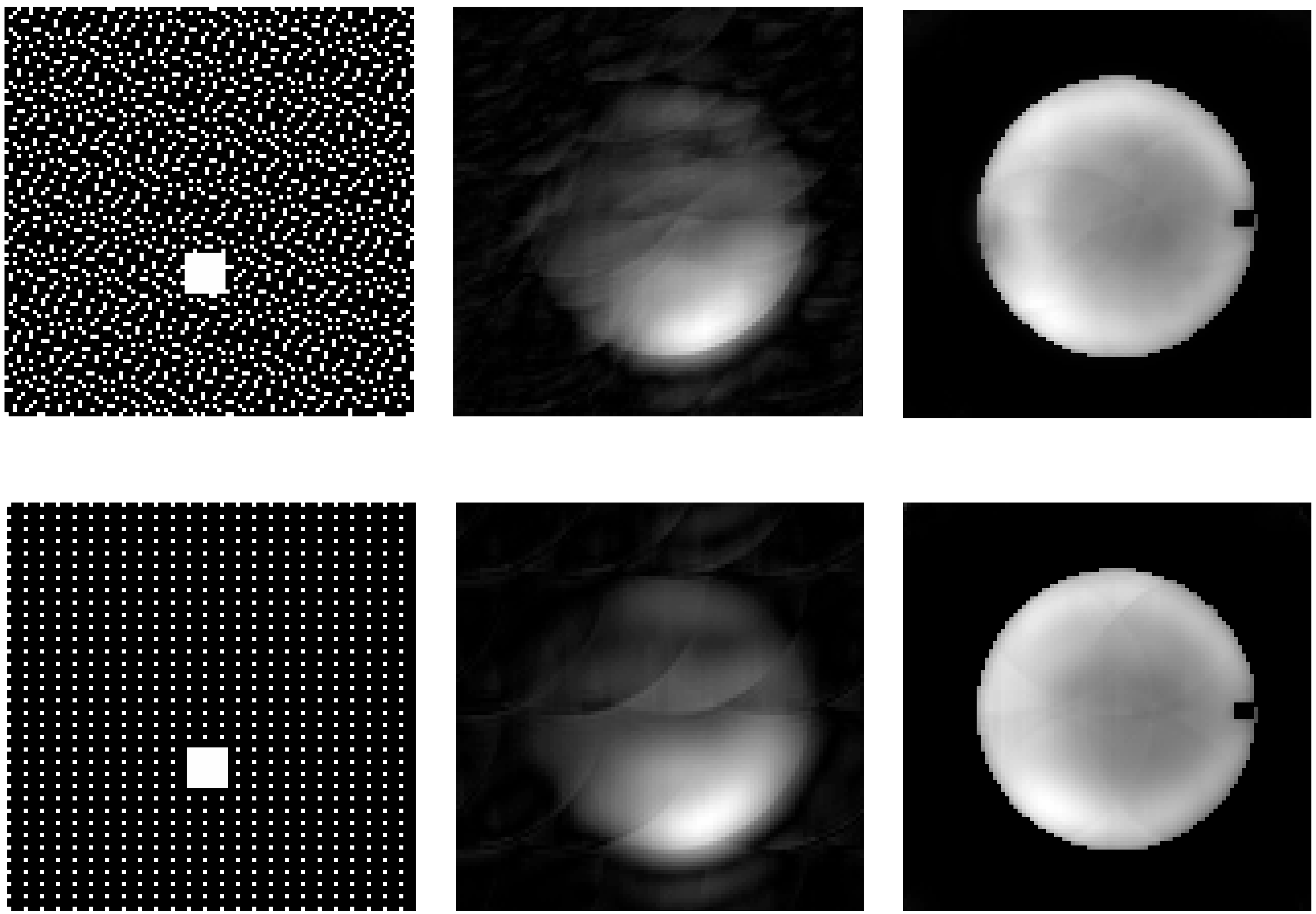




- Condition number improvement: 12.95 v.s. 8.5563
- Sample data: 11.7%

Results & Discussions

- Comparison with Regular Sampling



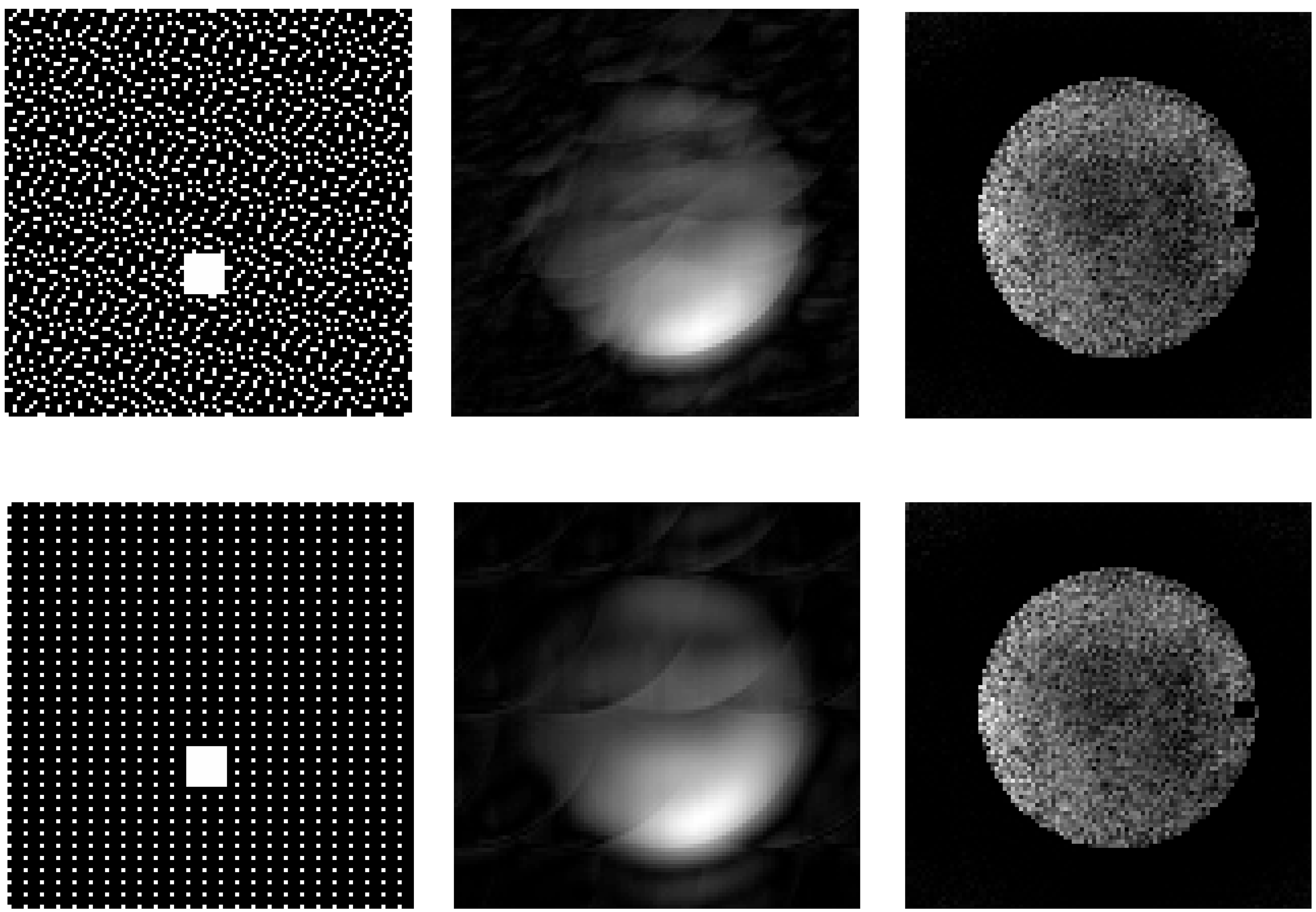
8.5%

V.S

9.3%

Results & Discussions

- SNR Enhancement



3.52×10^6

V.S

3.78×10^6

Conclusion

- Sensitivity polynomial fitting provides an effective approach to estimate sensitivity on arbitrary plane in 3D
- K-space under-sampling model provide significant theoretical improvement
- Further sampling k-space data is needed to investigate for better reconstruction results

Acknowledgement

THANKS FOR YOUR ATTENTION!