Rational approximation of degree n in Hardy spaces of the disk is a nonlinear optimization problem whose criterion can naturally be defined on Blaschke products of degree n. The latter have the topology of a projective space of dimension n, and the structure of a stratified manifold. When the function to be approximated is analytic in a slightly larger disk, the criterion is differentiable and Morse inequalities can be used to derive constraints on the critical points of the criterion. These can in turn be used to approach the issue of uniqueness of a best approximant. We shall present and illustrate criteria for uniqueness to hold.