

Pick's generalization of Schwarz's lemma has shown that holomorphic functions from the unit disc D to itself are best studied from the point of view of hyperbolic geometry. This philosophy led to the recent introduction of hyperbolic divided differences (L. B., P. Rivard and E. Wegert, CMFT **9**, 2009) If z, w are two points in the unit circle, set $[z, w] := (w - z)/(1 - \bar{w}z)$. If $f: D \rightarrow D$, and $z_1 \in D$ is fixed, then the hyperbolic divided difference is defined as $\Delta_{z_1} f(z) := [f(z), f(z_1)]/[z, z_1]$. These can be used to characterize finite Blaschke products and to solve the Nevanlinna-Pick problem. In this talk I will discuss some properties of hyperbolic divided differences and present some recent results of my student Patrice Rivard.