

Assume that a sequence of complex numbers $\{\lambda_k\}$ ($k = 1, 2, \dots$) satisfies the conditions : $\Re(\lambda_k) > 0$, $\lambda_k \neq \lambda_j$ for $k \neq j$ and $\sum_{k=1}^{\infty} \frac{\Re(\lambda_k)}{1+|\lambda_k|^2} < +\infty$. It is known that under the above conditions, the Blaschke product $W(\xi) = \prod_{k=1}^{\infty} \left[\frac{\xi - \lambda_k}{\xi + \bar{\lambda}_k} \cdot \frac{|1 - \lambda_k^2|}{1 - \lambda_k^2} \right]$ converges to an analytic function $W(\xi)$ in the right half-plane $\Re(\xi) > 0$, and that the exponential system

$$\{e^{-\lambda_k x}\} \quad (k = 1, 2, \dots) \quad (1)$$

is incomplete in $L^2(0, \infty)$. V. Kh. Musoyan [1] showed that if

$$\psi_k(x) = -\frac{1}{\overline{W'(\lambda_k)}} \cdot \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{e^{-i\tau x}}{W(i\tau)(i\tau + \bar{\lambda}_k)} d\tau \quad (k = 1, 2, \dots), \quad (2)$$

then the systems (1) and (2) are bi-orthogonal in $L^2(0, +\infty)$. Using the Fourier transform and corresponding results in the Hardy space H_+^2 for the upper half-plane, the bi-orthogonal expansions with respect to the systems (1) and (2) will be obtained.

Keywords : bi-orthogonal expansion, Hardy space, exponential system.

[1] V. Kh. Musoyan, Summation of biorthogonal expansions in incomplete systems of exponentials and rational functions, English translation : *Soviet J. Contemporary Math. Anal.*, 21, no. 2 (1986), 59–83.