

We consider the nonlinear Riemann-Hilbert problem with circular target curves: Given Hölder continuous functions c and r on the unit circle \mathbb{T} , with r strictly positive on \mathbb{T} , we seek functions w in the disk algebra $H^\infty \cap C$ of bounded holomorphic functions continuous on the closed unit disk, satisfying

$$|w(t) - c(t)| = r(t), \quad \forall t \in \mathbb{T}.$$

This is also called the generalized modulus problem, see [3]. The complete set of solutions w can be described using an approach based on the interplay with the Nehari problem of best approximation by bounded holomorphic functions. The problems fall into three classes (regular, singular, and void), and emphasizing constructive aspects of the matter the full solution set of regular problems can be parametrized in a Nevanlinna parametrization with the aid of the set of all finite Blaschke products.

When c and r^2 are rational functions all solutions of the problem are rational and can be constructed by solving interpolation problems of generalized Nevanlinna-Pick type for finite Blaschke products. In the regular cases these interpolation problems are reduced to linear systems, which leads to efficient numerical methods.

The results have been obtained in collaboration with E. Wegert.

References:

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- [2] C. Glader, E. Wegert: Nonlinear Rational Riemann-Hilbert Problems with Circular Target Curves, *Computational Methods and Function Theory*, Vol. 9 (2009), No. 2, 653–678.
- [3] V.V. Mityushev, S.V. Rogosin: Constructive methods for linear and nonlinear boundary value problems for analytic functions: theory and applications, Chapman & Hall/CRC 2002.