We study the connected components of certain natural open sets of the space of bounded analytic functions in the unit disc, which arise when one considers continuous versions of several open problems on approximation by interpolating Blaschke products. The new results in the talk are joint work with Daniel Suarez.