# Differential games and Zubov's Method

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Zubov's method is a classical method for computing Lyapunov functions and domains of attraction for differential equations

$$\dot{x} = f(x), \quad x \in \mathbb{R}^N$$

with a locally asymptotically stable equilibrium $x^* \in \mathbb{R}^N$. Zubov's main result states that under appropriate conditions and for a suitable function $g : \mathbb{R}^N \to \mathbb{R}$ the Zubov equation

$$\nabla W(x)f(x) = -g(x)(1 - W(x))\sqrt{1 + \|f(x)\|^2}, \quad (1)$$

a first order partial differential equation, has a unique differentiable solution $W : \mathbb{R}^N \to [0, 1]$ with $W(x^*) = 0$, which characterizes the domain of attraction $\mathcal{D}$ of $x^*$ via $\mathcal{D} = \{x \in \mathbb{R}^N \mid W(x) < 1\}$ and which is a Lyapunov function on $\mathcal{D}$. We provide generalizations of Zubov's equation to differential games without Isaacs' condition. We show that both generalizations of Zubov's equation (which we call min-max and max-min Zubov equation, respectively) possess unique viscosity solutions which characterize the respective controllability domains. As a consequence, we show that under the usual Isaacs condition the respective controllability domains as well as the local controllability assumptions coincide.

Keywords: asymptotic null controllability, differential games, Lyapunov functions, Hamilton-Jacobi-Bellman equation, viscosity solutions, Zubov's method

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