Superimposed codes

Zoltán Füredi
Department of Mathematics, University of Illinois at Urbana-Champaign,
Urbana, IL 61801, USA and
Rényi Institute of Mathematics of the Hungarian Academy of Sciences,
Budapest, P. O. Box 127, Hungary-1364
e-mail: `z-furedi@illinois.edu` and `furedi.zoltan@renyi.mta.hu`

There are many instances in Coding Theory when codewords must be restored from partial information, like defected data (error correcting codes), or some superposition of the strings. These lead to superimposed codes, a close relative of group testing problems.

There are lots of versions and related problems, like Sidon sets, sum-free sets, union-free families, locally thin families, cover-free codes and families, etc. We discuss two cases *cancellative* and *union-free* codes.

A family of sets $\mathcal{F}$ (and the corresponding code of 0-1 vectors) is called **union-free** if $A \cup B \neq C \cup D$ and $A, B, C, D \in \mathcal{F}$ imply $\{A, B\} = \{C, D\}$. $\mathcal{F}$ is called t-**cancellative** if for all distict $t + 2$ members $A_1, \ldots, A_t$ and $B, C \in \mathcal{F}$

$$A_1 \cup \ldots \cup A_t \cup B \neq A_1 \cup \ldots A_t \cup C.$$

Let $c_t(n)$ be the size of the largest t-cancellative code on n elements. We significantly improve the previous upper bounds of Körner and Sinaimeri, e.g., we show $c_2(n) \leq 2^{0.322n}$ (for $n > n_0$).