Pin merging in planar body frameworks

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We call $G = (V, E)$ a $\frac{3}{2}T$-graph if the edge set E can be partitioned by means of three colours,

$$E = F_R \cup F_Y \cup F_G,$$

such that each pairwise union forms a covering tree of G: $F_R \cup F_Y$, $F_R \cup F_G$ and $F_Y \cup F_G$.
If we specialize the classical result for body and hinge frameworks (Whiteley and Tay, 1988), then we see that G can be realized as a minimally (infinitesimally) rigid planar body framework, associating bodies with the elements of V and pins with the edges of E.

Suppose now that we require that some of these pins physically coincide, causing more than two bodies of this framework to be attached by the same pin. The combinatorial design of such frameworks is given by a hypergraph, clustering certain subsets of E as hyperedges to represent the pins that attach more than 2 bodies. We investigate the conditions under which the result of such clustering can be still realized as a rigid body framework. The $\frac{3}{2}T$-decompositions appear to give crucial information to create a "rigid pin merge".