partial hyperbolicity and topology of 3-manifolds

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celebrating Mike's work
May 8, 2012

setting

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- M^3 closed Riemannian 3-manifold

setting

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- M^3 closed Riemannian 3-manifold
- $f : M \to M$ partially hyperbolic diffeomorphism

setting

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- M^3 closed Riemannian 3-manifold
- $f : M \to M$ partially hyperbolic diffeomorphism
- f conservative (not always, but most of the time)

partial hyperbolicity

partial hyperbolicity

$f : M^3 \to M^3$ is partially hyperbolic

partial hyperbolicity

partial hyperbolicity

$f : M^3 \to M^3$ is partially hyperbolic

$$TM = \underset{\substack{\uparrow \\ \text{contracting}}}{E^s} \oplus \underset{\substack{\uparrow \\ \text{intermediate}}}{E^c} \oplus \underset{\substack{\uparrow \\ \text{expanding}}}{E^u}$$

partial hyperbolicity

partial hyperbolicity

$f : M^3 \to M^3$ is partially hyperbolic

$$\begin{array}{ccccccc} TM & = & E^s & \oplus & E^c & \oplus & E^u \\ & & \uparrow & & \uparrow & & \uparrow \\ & & \text{1-dim} & & \text{1-dim} & & \text{1-dim} \end{array}$$

example

example (conservative)

$$f : \mathbb{T}^2 \times \mathbb{T}^1 \to \mathbb{T}^2 \times \mathbb{T}^1$$

example

example (conservative)

$$f : \mathbb{T}^2 \times \mathbb{T}^1 \to \mathbb{T}^2 \times \mathbb{T}^1$$

such that

$$f = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \times id$$

example

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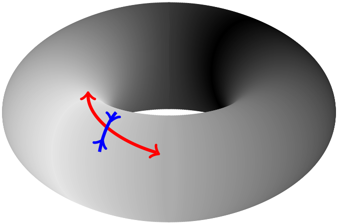
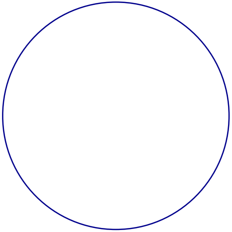


$\times$



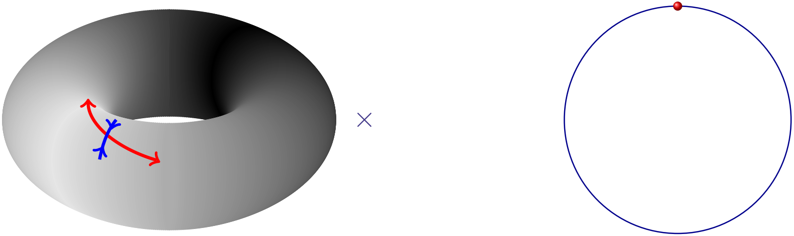
example

example (conservative)

$$f : \mathbb{T}^2 \times \mathbb{T}^1 \to \mathbb{T}^2 \times \mathbb{T}^1$$

such that

$$f = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \times R_\theta$$



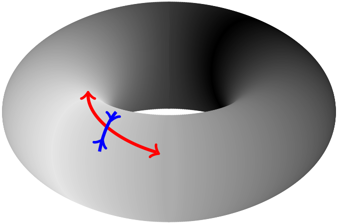
example

example (non-conservative)

$$f : \mathbb{T}^2 \times \mathbb{T}^1 \to \mathbb{T}^2 \times \mathbb{T}^1$$

such that

$$f = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \times NPSP$$



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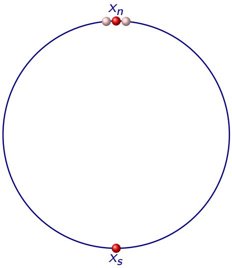


open problems

problems

- ergodicity

open problems

problems

- ergodicity
- dynamical coherence

open problems

problems

- ergodicity
- dynamical coherence
- classification

most ph are ergodic

conjecture (pugh-shub)

partially hyperbolic diffeomorphisms

$\cup$

C^1-open and C^r-dense set of ergodic diffeomorphisms

most ph are ergodic

hertz-hertz-ures08

partially hyperbolic diffeomorphisms

$\cup$

C^1-open and C^∞-dense set of ergodic diffeomorphisms

open problem

open problem

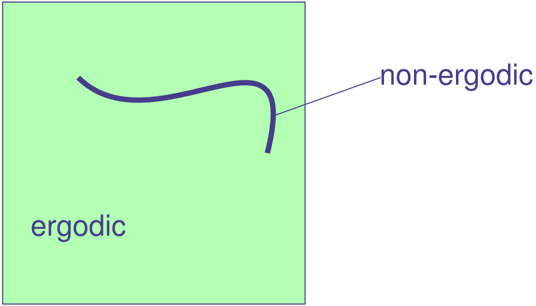
describe non-ergodic partially hyperbolic diffeomorphisms

open problem

open problem

describe non-ergodic partially hyperbolic diffeomorphisms

non-ergodicity



open problem

open problem

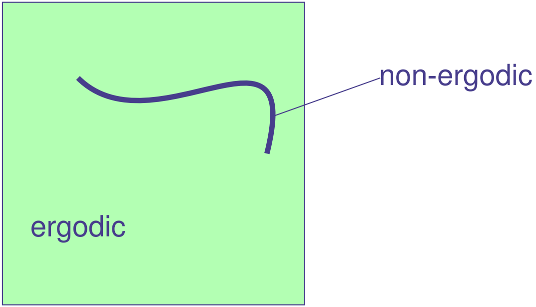
describe 3-manifolds
supporting non-ergodic partially hyperbolic diffeomorphisms

open problem

open problem

describe 3-manifolds
supporting non-ergodic partially hyperbolic diffeomorphisms

3-manifolds

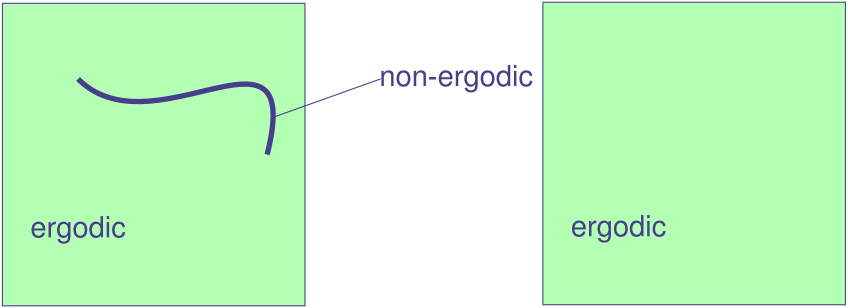


open problem

open problem

describe 3-manifolds
supporting non-ergodic partially hyperbolic diffeomorphisms

3-manifolds



conjecture

subliminal conjecture

most 3-manifolds

conjecture

subliminal conjecture

most 3-manifolds do not support
non-ergodic partially hyperbolic diffeomorphisms

evidence

hertz-hertz-ures08

N 3-nilmanifold,

evidence

hertz-hertz-ures08

N 3-nilmanifold, then either

evidence

hertz-hertz-ures08

N 3-nilmanifold, then either

- $N = \mathbb{T}^3$,

evidence

hertz-hertz-ures08

N 3-nilmanifold, then either

- $N = \mathbb{T}^3$, or
- {partially hyperbolic } $\subset$ { ergodic }

evidence

hertz-hertz-ures08

N 3-nilmanifold, then either

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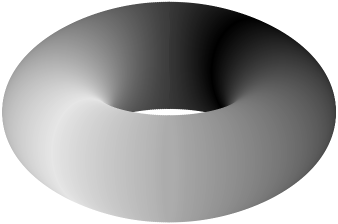
nilmanifolds



evidence

hertz-hertz-ures08

N 3-nilmanifold, then either

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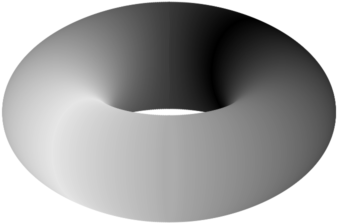
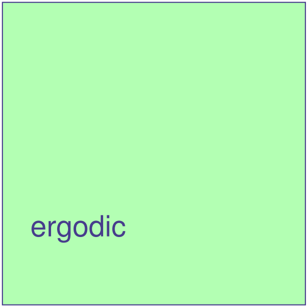
nilmanifolds





non-ergodic conjecture

non-ergodic conjecture (hertz-hertz-ures)

the only 3-manifolds supporting
non-ergodic PH diffeomorphisms are:

non-ergodic conjecture

non-ergodic conjecture (hertz-hertz-ures)

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non-ergodic PH diffeomorphisms are:

1. the 3-torus,

non-ergodic conjecture

non-ergodic conjecture (hertz-hertz-ures)

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1. the 3-torus,
2. the mapping torus of $-id : \mathbb{T}^2 \to \mathbb{T}^2$

non-ergodic conjecture

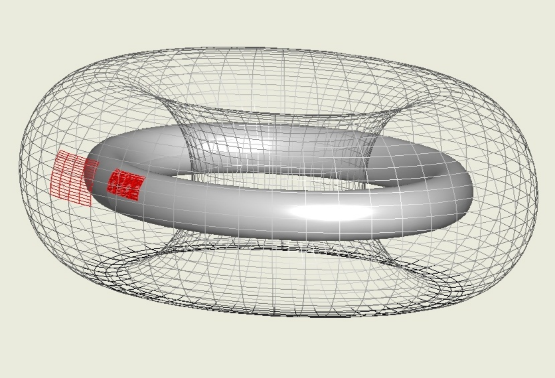
non-ergodic conjecture (hertz-hertz-ures)

the only 3-manifolds supporting
non-ergodic PH diffeomorphisms are:

1. the 3-torus,
2. the mapping torus of $-id : \mathbb{T}^2 \to \mathbb{T}^2$
3. the mapping tori of hyperbolic automorphisms of $\mathbb{T}^2$

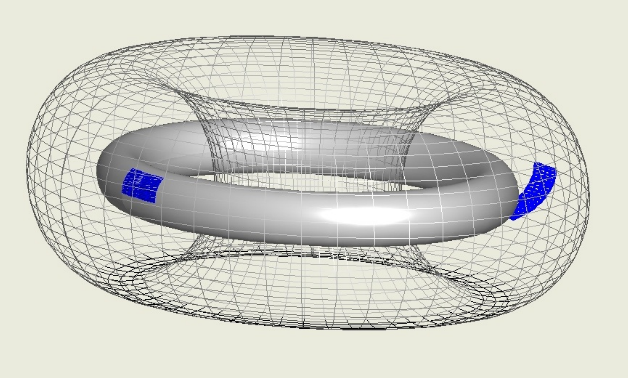
non-ergodic conjecture

① the 3-torus



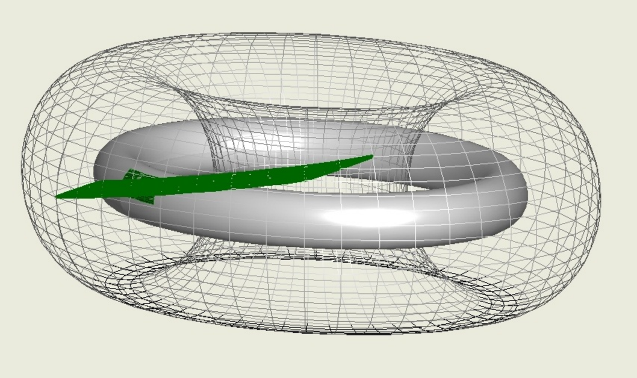
non-ergodic conjecture

② the mapping torus of $-id$



non-ergodic conjecture

3 the mapping torus of a hyperbolic automorphism



stronger non-ergodic conjecture

stronger non-ergodic conjecture

$f : M \to M$ non-ergodic partially hyperbolic diffeomorphism, then

stronger non-ergodic conjecture

stronger non-ergodic conjecture

$f : M \to M$ non-ergodic partially hyperbolic diffeomorphism, then

- $\exists$ torus tangent to $E^s \oplus E^u$

integrability

integrability

$f : M^3 \to M^3$ is partially hyperbolic

$$TM = E^s \oplus E^c \oplus E^u$$

integrability

integrability

$f : M^3 \to M^3$ is partially hyperbolic

$$\begin{array}{ccccccc} TM & = & E^s & \oplus & E^c & \oplus & E^u \\ & & \uparrow & & & & \uparrow \\ & & \mathcal{F}^s & & & & \mathcal{F}^u \end{array}$$

integrability

integrability

$f : M^3 \to M^3$ is partially hyperbolic

$$
\begin{array}{ccccccc}
TM & = & E^s & \oplus & E^c & \oplus & E^u \\
 & & \uparrow & & \uparrow & & \uparrow \\
 & & \mathcal{F}^s & & ? & & \mathcal{F}^u
\end{array}
$$

dynamical coherence

dynamical coherence

dynamical coherence

dynamical coherence

1. $\exists$ invariant $\mathcal{F}^{cs}$ tangent to $E^s \oplus E^c$

dynamical coherence

dynamical coherence

1. $\exists$ invariant $\mathcal{F}^{cs}$ tangent to $E^s \oplus E^c$
2. $\exists$ invariant $\mathcal{F}^{cu}$ tangent to $E^c \oplus E^u$

dynamical coherence

dynamical coherence

1. $\exists$ invariant $\mathcal{F}^{cs}$ tangent to $E^s \oplus E^c$
2. $\exists$ invariant $\mathcal{F}^{cu}$ tangent to $E^c \oplus E^u$

remark

$\Rightarrow \exists$ invariant $\mathcal{F}^c$ tangent to E^c

open question

longstanding open question

$f : M^3 \to M^3$ partially hyperbolic $\overset{?}{\Rightarrow}$ f dynamically coherent

open question

longstanding open question

$f : M^3 \to M^3$ partially hyperbolic $\overset{?}{\Rightarrow}$ f dynamically coherent

hertz-hertz-ures10

NO

counterexample

hertz-hertz-ures10

$\exists f : \mathbb{T}^3 \to \mathbb{T}^3$ partially hyperbolic

counterexample

hertz-hertz-ures10

$\exists f : \mathbb{T}^3 \to \mathbb{T}^3$ partially hyperbolic

- non-dynamically coherent

counterexample

hertz-hertz-ures10

$\exists f : \mathbb{T}^3 \to \mathbb{T}^3$ partially hyperbolic

- non-dynamically coherent
- non-conservative

counterexample

hertz-hertz-ures10

$\exists f : \mathbb{T}^3 \to \mathbb{T}^3$ partially hyperbolic

- non-dynamically coherent
- non-conservative
- “robust”

open problem

open problem

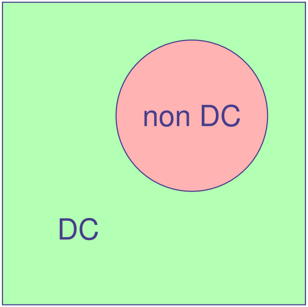
describe 3-manifolds supporting non-dynamically coherent examples

open problem

open problem

describe 3-manifolds supporting non-dynamically coherent examples

3-manifolds

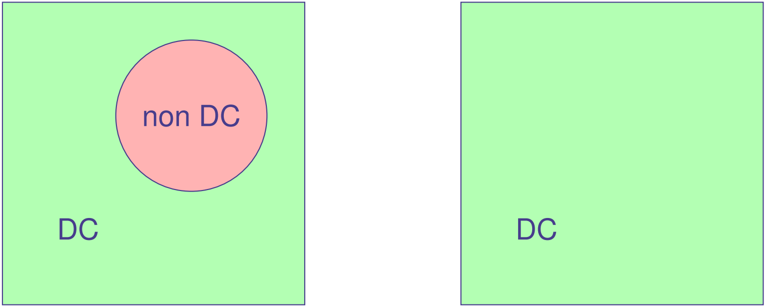


open problem

open problem

describe 3-manifolds supporting non-dynamically coherent examples

3-manifolds



non-dynamically coherent conjecture

non-dynamically coherent conjecture (hertz-hertz-ures)

$f : M^3 \to M^3$ non-dynamically coherent,

non-dynamically coherent conjecture

non-dynamically coherent conjecture (hertz-hertz-ures)

$f : M^3 \to M^3$ non-dynamically coherent,
then M is either:

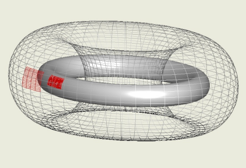
non-dynamically coherent conjecture

non-dynamically coherent conjecture (hertz-hertz-ures)

$f : M^3 \to M^3$ non-dynamically coherent,
then M is either:

1. $\mathbb{T}^3$

3-manifolds



non-dynamically coherent conjecture

non-dynamically coherent conjecture (hertz-hertz-ures)

$f : M^3 \to M^3$ non-dynamically coherent,
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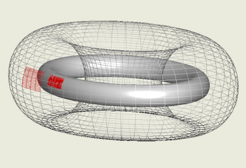
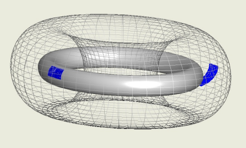
1. $\mathbb{T}^3$
2. the mapping torus of $-id$

3-manifolds





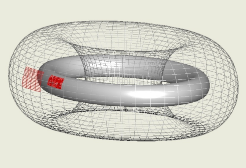
non-dynamically coherent conjecture

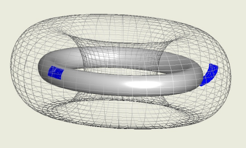
non-dynamically coherent conjecture (hertz-hertz-ures)

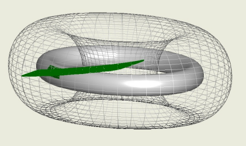
$f : M^3 \to M^3$ non-dynamically coherent,
then M is either:

1. $\mathbb{T}^3$
2. the mapping torus of $-id$
3. the mapping torus of a hyperbolic automorphism

3-manifolds







stronger conjecture

stronger non-dynamically coherent conjecture

$f : M^3 \to M^3$ non-dynamically coherent, then either

stronger conjecture

stronger non-dynamically coherent conjecture

$f : M^3 \to M^3$ non-dynamically coherent, then either

- $\exists$ torus tangent to $E^c \oplus E^u$, or

stronger conjecture

stronger non-dynamically coherent conjecture

$f : M^3 \to M^3$ non-dynamically coherent, then either

- $\exists$ torus tangent to $E^c \oplus E^u$, or
- $\exists$ torus tangent to $E^s \oplus E^c$

intermediate conjecture

intermediate conjecture

f volume preserving $\Rightarrow$ f dynamically coherent

evidence

potrie11

$f : \mathbb{T}^3 \to \mathbb{T}^3$ non-dynamically coherent, then

evidence

potrie11

$f : \mathbb{T}^3 \to \mathbb{T}^3$ non-dynamically coherent, then

- $\exists$ torus tangent to $E^c \oplus E^u$, or

evidence

potrie11

$f : \mathbb{T}^3 \to \mathbb{T}^3$ non-dynamically coherent, then

- $\exists$ torus tangent to $E^c \oplus E^u$, or
- $\exists$ torus tangent to $E^s \oplus E^c$

examples of ph dynamics

known ph dynamics in dimension 3

examples of ph dynamics

known ph dynamics in dimension 3

- perturbations of time-one maps of Anosov flows

examples of ph dynamics

known ph dynamics in dimension 3

- perturbations of time-one maps of Anosov flows
- certain skew-products

examples of ph dynamics

known ph dynamics in dimension 3

- perturbations of time-one maps of Anosov flows
- certain skew-products
- certain DA-maps

examples of ph dynamics

known ph dynamics in dimension 3

- perturbations of time-one maps of Anosov flows
- certain skew-products
- certain DA-maps

new example

- non-dynamically coherent example

question

question

are there more examples?

conjecture pujals

classification conjecture (pujals01)

If $f : M^3 \to M^3$ is a transitive partially hyperbolic diffeomorphism, then f is (finitely covered by) either

conjecture pujals

classification conjecture (pujals01)

If $f : M^3 \to M^3$ is a transitive partially hyperbolic diffeomorphism, then f is (finitely covered by) either

1. a perturbation of a time-one map of an Anosov flow

conjecture pujals

classification conjecture (pujals01)

If $f : M^3 \to M^3$ is a transitive partially hyperbolic diffeomorphism, then f is (finitely covered by) either

1. a perturbation of a time-one map of an Anosov flow
2. a skew-product

conjecture pujals

classification conjecture (pujals01)

If $f : M^3 \to M^3$ is a transitive partially hyperbolic diffeomorphism, then f is (finitely covered by) either

1. a perturbation of a time-one map of an Anosov flow
2. a skew-product
3. a DA-map

conjecture

classification conjecture (hhu)

If $f : M^3 \to M^3$ is partially hyperbolic and dynamically coherent, then f is

conjecture

classification conjecture (hhu)

If $f : M^3 \to M^3$ is partially hyperbolic and dynamically coherent, then f is

1. a perturbation of a time-one map of an Anosov flow,

conjecture

classification conjecture (hhu)

If $f : M^3 \to M^3$ is partially hyperbolic and dynamically coherent, then f is

1. a perturbation of a time-one map of an Anosov flow,
2. a skew-product, or

conjecture

classification conjecture (hhu)

If $f : M^3 \to M^3$ is partially hyperbolic and dynamically coherent, then f is

1. a perturbation of a time-one map of an Anosov flow,
2. a skew-product, or
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conjecture

classification conjecture (hhu)

If $f : M^3 \to M^3$ is partially hyperbolic and dynamically coherent, then f is

1. leafwise conjugate to an Anosov flow
2. a skew-product, or
3. a DA-map.

conjecture

classification conjecture (hhu)

If $f : M^3 \to M^3$ is partially hyperbolic and dynamically coherent, then f is

1. leafwise conjugate to an Anosov flow
2. leafwise conjugate to a skew-product with linear base
3. a DA-map.

conjecture

classification conjecture (hhu)

If $f : M^3 \to M^3$ is partially hyperbolic and dynamically coherent, then f is

1. leafwise conjugate to an Anosov flow
2. leafwise conjugate to a skew-product with linear base
3. leafwise conjugate to an Anosov map in $\mathbb{T}^3$.

problems

- ergodicity

problems

- ergodicity
- dynamical coherence

problems

- ergodicity
- dynamical coherence
- classification

problems

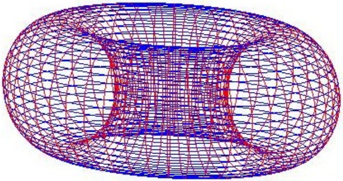
- ergodicity
- dynamical coherence
- classification

$\Bigg\} \rightarrow$ Anosov torus

Anosov torus

Anosov torus

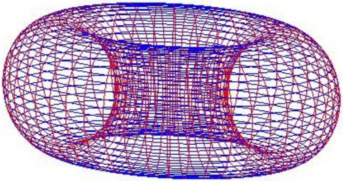
- T embedded 2-torus



Anosov torus

Anosov torus

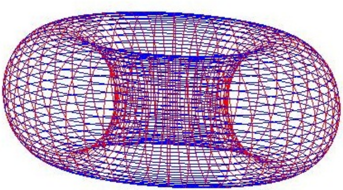
- T embedded 2-torus
- $\exists f : M \to M$ s.t.



Anosov torus

Anosov torus

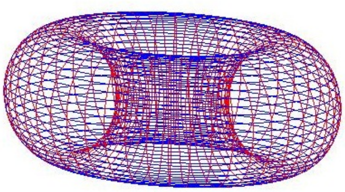
- T embedded 2-torus
- $\exists f : M \to M$ s.t.
  1. $f(T) = T$



Anosov torus

Anosov torus

- T embedded 2-torus
- $\exists f : M \to M$ s.t.
  1. $f(T) = T$
  2. $f|_T$ isotopic to Anosov



invariant tori in ph dynamics

invariant tori in PH dynamics

T invariant torus tangent to

invariant tori in ph dynamics

invariant tori in PH dynamics

T invariant torus tangent to

- $E^s \oplus E^u$

invariant tori in ph dynamics

invariant tori in PH dynamics

T invariant torus tangent to

- $E^s \oplus E^u$
- $E^c \oplus E^u$

invariant tori in ph dynamics

invariant tori in PH dynamics

T invariant torus tangent to

- $E^s \oplus E^u$
- $E^c \oplus E^u$
- $E^s \oplus E^u$

invariant tori in ph dynamics

invariant tori in PH dynamics

T invariant torus tangent to

- $E^s \oplus E^u$
- $E^c \oplus E^u$
- $E^s \oplus E^u$

$\Rightarrow$ T Anosov torus

Anosov torus

conjectures

conjectures

non-ergodic conjecture

$f : M \to M$ non-ergodic partially hyperbolic

conjectures

non-ergodic conjecture

$f : M \to M$ non-ergodic partially hyperbolic

non-dyn. coh. conjecture

$f : M \to M$ non-dyn. coherent partially hyperbolic

conjectures

non-ergodic conjecture

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then M is either

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conjectures

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then M is either

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conjectures

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$f : M \to M$ non-ergodic partially hyperbolic

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then M is either

1. $\mathbb{T}^3$
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3. the mapping torus of a hyperbolic map of $\mathbb{T}^2$

conjectures

non-ergodic conjecture

$f : M \to M$ non-ergodic partially hyperbolic

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3. the mapping torus of a hyperbolic map of $\mathbb{T}^2$

stronger conjecture

$\exists$ Anosov torus tangent to $E^s \oplus E^u$

conjectures

non-ergodic conjecture

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non-dyn. coh. conjecture

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2. the mapping torus of $-id : \mathbb{T}^2 \to \mathbb{T}^2$
3. the mapping torus of a hyperbolic map of $\mathbb{T}^2$

stronger conjecture

$\exists$ Anosov torus tangent to $E^s \oplus E^u$

stronger conjecture

$\exists$ Anosov torus tangent to $E^c \oplus E^u$ or $E^s \oplus E^c$

why stronger conjectures?

hertz-hertz-ures11

M irreducible contains an Anosov torus,

why stronger conjectures?

hertz-hertz-ures11

M irreducible contains an Anosov torus, then M is either

why stronger conjectures?

hertz-hertz-ures11

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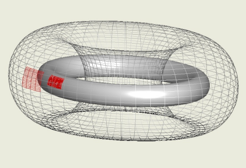
1. $\mathbb{T}^3$

3-manifolds



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hertz-hertz-ures11

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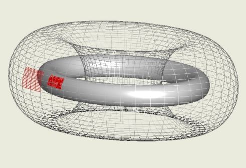

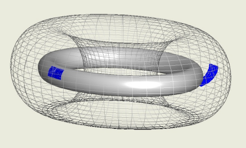

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M irreducible contains an Anosov torus, then *M* is either

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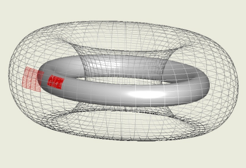

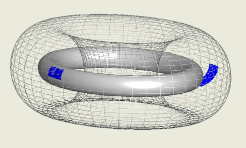

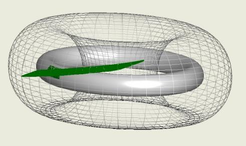

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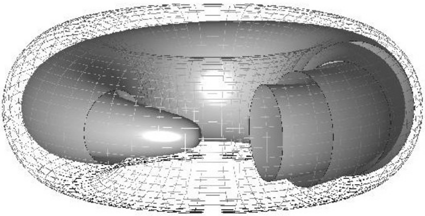

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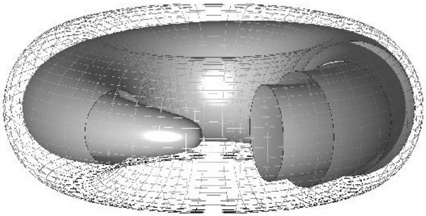

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- Burago-Ivanov08

reduced theorem

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- ∂N consists of Anosov tori

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- $\Rightarrow$

$$N = \mathbb{T}^2 \times [0, 1]$$

brief sketch of the proof

JSJ-decomposition

brief sketch of the proof

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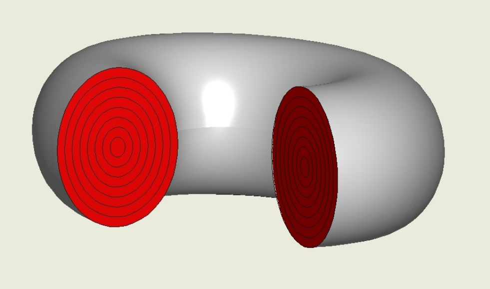
brief sketch of the proof

① N = solid torus



brief sketch of the proof

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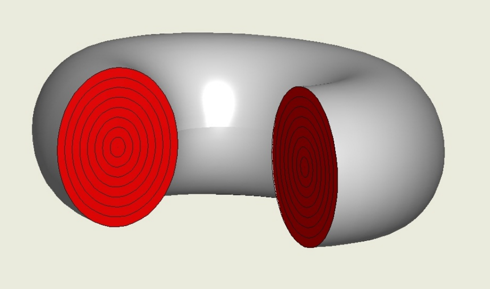

$\partial N = T$

brief sketch of the proof

(2) N_S = twisted I-bundle over K

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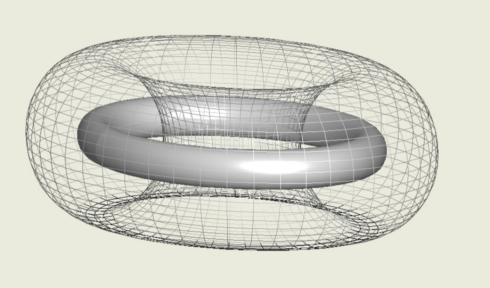

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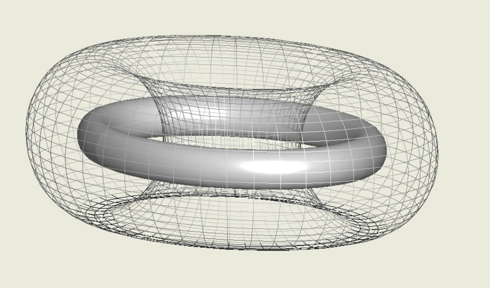

$\partial N = T \sqcup T$

brief sketch of the proof

final lemma

- ∂N consist of Anosov tori

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Anosov torus

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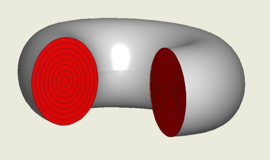

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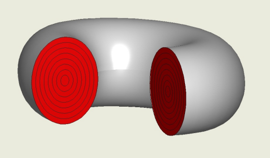

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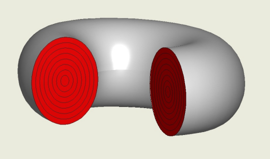

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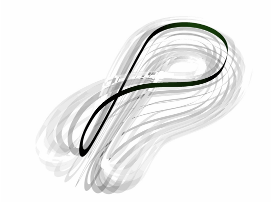

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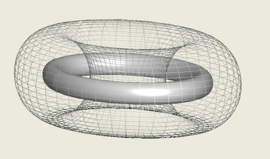

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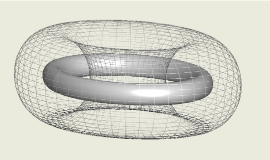

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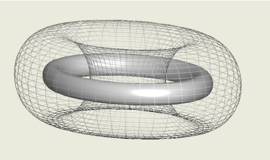

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thank you

thank you

THANK YOU MIKE!