Title: The multinorm principle

Abstract: A finite extension L/K of global fields is said to satisfy the Hasse norm principle if $K^\times \cap N_{L/K}(J_L) = N_{L/K}(L^\times)$, where $N_{L/K}\colon J_L \to J_K$ denotes the natural extension of the norm map associated with L/K to the corresponding groups of ideles. The obstruction for the Hasse norm principle, which is often nontrivial, was computed by Tate[1] in the Galois case and by Drakokhrust[3] in the general case. Similarly, a pair of finite extensions L_1, L_2 of K is said to satisfy the multinorm principle if

$$K^\times \cap N_{L_1/K}(J_{L_1})N_{L_2/K}(J_{L_2}) = N_{L_1/K}(L_1^\times)N_{L_2/K}(L_2^\times).$$

Some sufficient conditions for the multinorm principle were given by Hürlimann[4], Colliot-Thélène–Sansuc[2], Platonov–Rapinchuk[5], and Prasad–Rapinchuk[7]. These results assert the validity of the multinorm principle if the extensions are disjoint (or their Galois closures are disjoint) and one of the extensions satisfies the usual Hasse norm principle. In my joint work with Rapinchuk[6], we show that the multinorm principle always holds for a pair of linearly disjoint Galois extensions (even if both extensions fail to satisfy the Hasse norm principle). I will outline the proof of this theorem and gives some additional results and examples. In particular, I will discuss the situation for extensions that are not Galois or disjoint, and talk about the generalization of the multinorm principle for three or more extensions.

References

[1] J.W.S. Cassels, A. Frölich (Eds.), *Algebraic Number Theory*, Thompson Book Company Inc., Washington D.C., 1967.

[2] Colliot-Thélène, Sansuc, Private Communication.

[3] Yu. A. Drakokhrust, *On the complete obstruction to the Hasse principle*, Amer. Math. Soc. Transl.(2) **143** (1989), 29-34.

[4] Hürlimann, *On algebraic tori of norm type*, Comment. math. Helv. **59**(1984), 539-549.

[5] V.P. Platonov, A.S. Rapinchuk, *Algebraic Groups and Number Theory*, Academic Press, 1994.

[6] T. Pollio, A.S. Rapinchuk, *The Multinorm Principle for linearly disjoint Galois extensions*, arXiv:1203.0359v1.

[7] G. Prasad, A.S. Rapinchuk, *Local-global principles for embedding of fields with involution into simple algebras with involution*, Comment. math. Helv. **85**(2010), 583-645.