The Liouville Theorem on Conformal Mappings

Zhuomin Liu, University of Pittsburgh

Abstract: The celebrated Liouville Theorem from 1850 states that in dimension n~I3, the only conformal maps are M\"obius transforms. Liouville's proof, as well as many subsequent proofs, required the mappings to be diffeomorphisms of class C3. However, since C1 regularity is sufficient to define conformal maps, one may inquire whether the Liouville theorem remains true under that, or even weaker conditions, e.g. Sobolev functions. The reduction from C3 regularity turned out to be very difficult. It this talk we will discuss the development of the Liouville Theorem under weaker and weaker regularity assumptions, including results of Gerhing, Reshnetyak, Bojarski and Iwaniec, Iwaniec and Martin on W1,nloc conformal mappings. Furthermore, Iwanice and Martin proved that in even dimensions n~I4, W1,n/2loc conformal mappings are M\"{o}bious transforms and they conjectured that it should also be true in odd dimensions. We also discuss a proof of the Liouville Theorem f~H~HW1,1loc in dimension n~I3 under one additional assumption on the norm of the of the first order derivative.