Workshop on Positivity

Positivity in the classification of operator algebras and dynamical systems, in finite- and infinite-dimensional linear algebra, and its outgrowths

Fields Institute
Toronto, Canada

August 2–4, 2011

Contents

Workshop schedule

	9:45 - 10:30	*Registration + Coffee*
	10.30 - 11.30	George Elliott
Tuesday	11.30 - 12.30	Bruce Reznick
August 2	14.00 - 15.00	Andrew Toms
	15.00 - 15.30	Aaron Tikuisis
	15.30 - 16.00	*Coffee*
	16.00 – 17.00	Bruce Blackadar
	17.00 - 17.30	Wadii Hajji
	17.30 -	*Reception*
	9.30 - 10.30	Mike Boyle
	10.30 - 11.00	*Coffee*
	11.00 - 12.00	Vadim Kaimanovich
Wednesday	12.00 - 12.30	Charles Starling
August 3	14.00 - 15.00	Brian Markus
	15.00 - 15.30	Termeh Kousha
	15.30 - 16.00	*Coffee*
	16.00 - 17.00	Klaus Schmidt
	19.00 -	*Banquet*[‡]
	9.30 - 10.30	Jean Renault
	10.30 - 11.00	*Coffee*
	11.00 - 12.00	David Kerr
Thursday	12.00 - 12.30	Catalin Rada
August 4	14.00 - 15.00	Man-Duen Choi
	15.00 - 15.30	Ion Nechita
	15.30 - 16.00	*Coffee*
	16.00 - 17.00	Christian Skau

All talks, as well as registration, are taking place in the Bahen Room 1200.
[‡] *Banquet* location: to be announced on the first day of the conference.

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Workshop organizers: Thierry Giordano and Vladimir Pestov.

Recent results on semiprojectivity

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We will discuss some recent results about semiprojectivity of $C*$-algebras, including equivariant semiprojectivity.

The Generalized Spectral Conjecture, and the Williams Conjecture for positive matrices over unital subrings of the reals

Mike Boyle
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The Generalized Spectral Conjecture of David Handelman and myself is that under certain simple necessary condtions, a matrix over a unital subring S of the reals is strong shift equivalent over S to a primitive (i.e. square irreducible nonnegative aperiodic) matrix over S. In the first part of this talk, I'll discuss the meaning and current status of the conjecture, and a related result of Tom Laffey.

A "Williams Conjecture" means here a conjecture that shift equivalence of two matrices (in some class, over some unital semiring) implies their strong shift equivalence. I'll discuss the path method of Kim and Roush, and their forthcoming paper which extends to a larger class of subrings of the reals their verification of the Williams Conjecture for strictly positive matrices with just one nonzero eigenvalue, and the relevance to the Williams Conjecture for primitive matrices over $\mathbb{Z}$. The path method setup is one in which can attack the Williams Conjecture for positive matrices in general – despite the failure to date of anyone to find the next big insight forward.

Positive linear maps through quantum computers

Man-Duen Choi
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With pride and prejudice in operator theory, we seek sense and sensibility of POSITIVITY associated with the incredible quantum computers. There are many new problems of unknown depth in connections with positive linear maps on matrix algebras, even in very low dimensional cases.

Not every functor is manifestly covariant

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Most functors, i.e., maps from one category to another which are covariant, in the sense that they not only take objects into objects but also arrows into arrows, and, furthermore, preserve composition of arrows, are manifestly covariant – it is not necessary to prove that they are functors. (Of course, the same situation obtains for what are referred to as contravariant functors – but these never need to be considered as one can pass to the dual, or opposite, of one of the categories.)

Some functors which are obviously functors come up in C*-algebra classification theory – notably, the Murray-von Neumann semigroup, and hence the K-groups.

Interestingly, some functors which are not obviously functors also come up in C*-algebra classification theory. (The most notable example of this is perhaps the Bratteli diagram, which is not even usually thought of as a functor. Glimm's isomorphism theorem for UHF algebras might be most simply interpreted as a proof that, for these algebras, the Bratteli diagram – corresponding to a given expression of the algebra as an inductive limit of full matrix algebras – is in fact a functor. In Bratteli's setting, the diagram is still a functor, but this is no longer the end of the story. Starting from the assumption that the diagrams were the same – after passing to a common refinement – Bratteli had to prove that the algebras were isomorphic – an implication that was trivial in Glimm's case.)

(Another example arises in the setting of dimension drop interval algebras – joint work with Z. Li. In this case, looking at non-simple but real rank zero inductive limits, one finds that the invariant required is the total K-theory, with an order structure which is finer than that of Dadarlat and Loring and only obviously exists at the building block level; the functoriality – not to mention the existence! – of this positivity structure for the inductive limit category has to be proved, in much the same way as does the functoriality of the Bratteli diagram.)

Compact inverse semigroup representation

Wadii Hajji
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In this talk we will construction a Haar system in a compact groupoid with finitely many identities which allows us to prove the following: A finite dimensional representation of a compact inverse semigroup is equivalent to a *-representation. The second part is the main result in this talk, we will give a parameterization of all finite dimensional irreducible representations of a compact inverse semigroup.

Rate of escape on groupoids

Vadim Kaimanovich
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Markov chains on groupoids provide a unified approach to numerous classes of Markov chains characterized by presence of a certain stochastic homogeneity (e.g., ordinary and matrix-valued random walks on groups, in random environment, along classes of equivalence relations, etc.). We extend to this generality earlier results of Karlsson and Ledrappier (obtained in the framework of classical random walks) on horospheric approximation of sample paths (law of large numbers) and on equivalence of trivial boundary behaviour (the Liouville property) and vanishing of the rate of escape. The talk is based on a joint work with M. Anderegg.

Topological dynamics and crossed products: finite approximation and paradoxicality

David Kerr
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For discrete groups Tarski showed that there is a dichotomy between amenability and paradoxical decomposability by analyzing the existence of states on an associated ordered semigroup. I will discuss some recent results and open problems concerning the extent to which this kind of dichotomy applies to group actions on compact Hausdorff spaces and their C*-crossed products.

Asymptotic behavior and moderate deviation principle for the maximum of a Dyck path

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By using a new representation of Catalan number, relying on the spectral properties of an associated adjacency matrix, we find the distribution of the maximum of Dyck path for the case where the length of the Dyck path is proportional to the square root of the height. We also consider other cases and find moderate and large deviation principles for the law of the maximum of random Dyck path for those cases.

A complex Hilbert metric with application to entropy of Hidden Markov Chains

Brian Marcus
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Coauthors: Guangyue Han (University of Hong Kong)
Yuval Peres (Microsoft Research)

A positive linear map is contractive with respect to the Hilbert metric. We show that the metric can be extended to be contractive for complex perturbations of a positive linear map. We apply this result to show that the entropy of the observation process of a Markov Chain transmitted over a noisy channel is analytic with respect to the parameters of the underlying Markov chain. We also discuss related metrics of Dubois and Rugh.

Positivity as a tool in quantum entanglement theory

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Deciding if a quantum state is separable or entangled is a notroiously difficult, NP-hard problem. We show how positivity can be used to detect and quantify entanglement in finite dimensional quantum systems. Such criteria are applied to study entanglement for random quantum states in large dimensions. We compare our results with other methods for detecting entanglement. Our tools come from random matrix theory and free probability.

On crossed products by Z of algebras arising from reduced free group C*-algebras

Catalin Rada
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If Γ is a free group on n generators $g_1, g_2, \ldots, g_n$, where $2 \leq n < \infty$, let $\Gamma_+^{'}$ be the subset $\{g_{i_1}^{m_1} g_{i_2}^{m_2} \ldots g_{i_k}^{m_k} | k, m_1, \ldots, m_k \in \mathbf{N}\} \bigcup \{e\}$, and let $P_+^{'}$ be the projection onto the subspace $l^2(\Gamma_+^{'})$ of $l^2(\Gamma)$. Every character χ of Γ induces an automorphism on C^*-algebra $C_r^*(\Gamma, P_+^{'})$ generated by the reduced group C^*-algebra $C_r^*(\Gamma)$ and the projection $P_+^{'}$. We talk about the classification of the C^*-algebra crossed products $C_r^*(\Gamma, P_+^{'}) \rtimes_{\alpha_\chi} \mathbf{Z}$, where α_χ is the automorphism associated to χ.

Conditionally negative type functions on groupoids

Jean Renault
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I will present the notion of a conditionally negative type function on a locally compact groupoid as an illustration of Sauvageot's theory of non-commutative Dirichlet forms. I will give some examples and applications.

Positive polynomials, from Hilbert's 17th Problem to today

Bruce Reznick
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We will talk about some of the main developments in the study of positive real polynomials, including Hilbert's 17th problem and its solution by Artin, Polya's Theorem and its modern quantitative versions, the work of Motzkin, Robinson, Choi, Lam and others starting in the 1960s, and the current state of the art. Some of David Handelman's theorems will be discussed. The speaker will orient his presentation to the prevalence of analysts in the audience.

Entropy and periodic points of algebraic group actions

Klaus Schmidt
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Abstract: If G is a discrete amenable group, every element f in the integral group ring ZG defines a measure-preserving action α_f of G by automorphisms of a compact abelian group X_f. This action is automatically ergodic whenever G is not essentially equal to the group of integers.

The entropy of this 'algebraic' action α_f is an interesting quantity associated with the element f in ZG. If $G = Z^d$, then it is the logarithmic Mahler measure of f (viewed as a Laurent polynomial in d variables with integer coefficients). If G is residually finite and the action α_f is expansive, it is the logarithmic growth rate of the number of periodic points of α_f, which in turn coincides with the logarithm of the Fuglede-Kadison determinant associated with f (viewed as an element of the group von Neumann algebra of G).

If α_f is nonexpansive, the connection between entropy, the logarithmic growth rate of periodic points, and Fuglede-Kadison determinants holds many mysteries, even for $G = Z^d$. Some of these problems will be discussed in this lecture.

This talk is based on joint work with C. Deninger, D. Lind and E. Verbitskiy.

Toeplitz flows and their K-theory

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Toeplitz flows are a family of Cantor minimal systems that have been extensively studied in topological dynamics. In a certain sense they are the simplest Cantor minimal systems beyond the odometer systems,to which they are related in an explicit way. By looking at the associated C*-crossed products and their K-theory a new and fruitful approach was introduced in the study of Toeplitz flows. Besides giving new proofs of known results about these systems,the K-theoretic approach brought about an entirely new perspective,which in turn led to new results. We will illustrate this by presenting recent results and some open problems.

Finite Group Actions on C*-algebras Associated to Tilings

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Putnam, Kellendonk, and other have studied the C*-algebras arising from aperiodic tilings such as the Penrose tiling. Such tilings frequently exhibit some rotational and dihedral symmetries. We examine the actions of finite symmetry groups on these algebras and obtain results about the related crossed products, including computation of the K-groups.

Representing dimension groups in real vector spaces

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Many important insights into dimension groups have come from considering positive functionals from such a group to the real line. These considerations have lead, for example, to an explicit description of simple dimension groups in the classic paper of Effros, Handelman and Shen. I will discuss attempts at gaining a similar understanding of nonsimple dimension groups, chiefly by looking at positive functionals on ideals of the group.

This has lead to an explicit description of finite rank dimension groups, in terms of certain embeddings into ordered real vector spaces with Riesz interpolation. Our results also explicitly describe such ordered vector spaces in the finite dimensional case, and show in particular that there are only finitely many n-dimensional ordered real vector spaces with Riesz interpolation for any finite n.