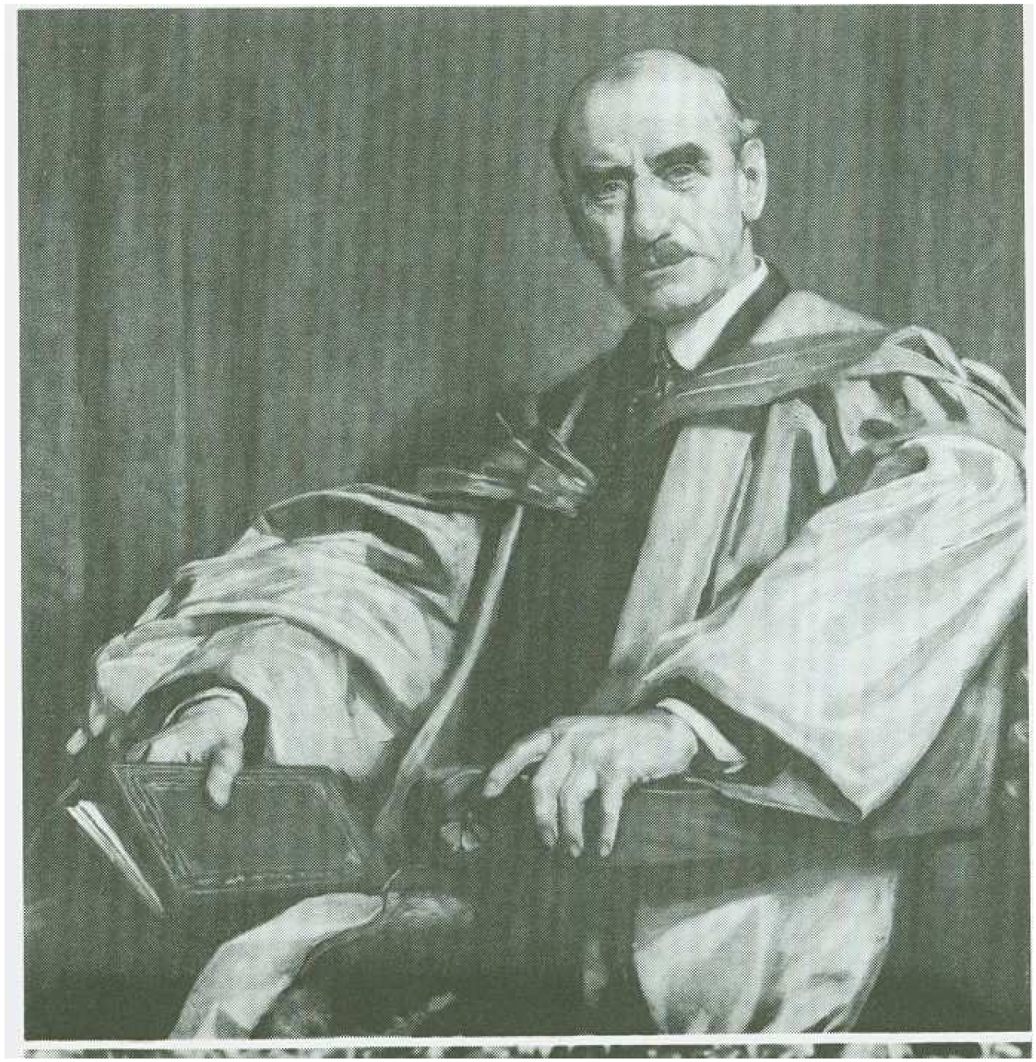
John Charles Fields (1863-1932)

University College, University of Toronto
Home of Department of Mathematics (1900)

Alfred Baker. Mathematics Dept. Chairman University of Toronto (1887 – 1919)



In the chapter on Baker in his history of the Department Gilbert Robinson writes

“Essentially, the question we should ask at this point is ‘what was different in the general spirit or attitude of the members of the Department in Baker’s day as compared with what had gone before?’ **The conclusion is inescapable - the role of Mathematics – its meaning in the University – had become more abstract; it had reverted to the Greek conception as the epitome of the thought process**.”

In Robinson’s history:

Fields is termed an “abstractionist” and contrasted with M. A. Mackenzie, who was a pioneer in the field of actuarial science.

A former student from the period would comment that Fields was the only research mathematician in the department.

“The conclusion is inescapable - the role of Mathematics – its meaning in the University – had become more abstract; it had reverted to the Greek conception as the epitome of the thought process.”

THEORY OF THE

ALGEBRAIC FUNCTIONS

OF A

COMPLEX VARIABLE

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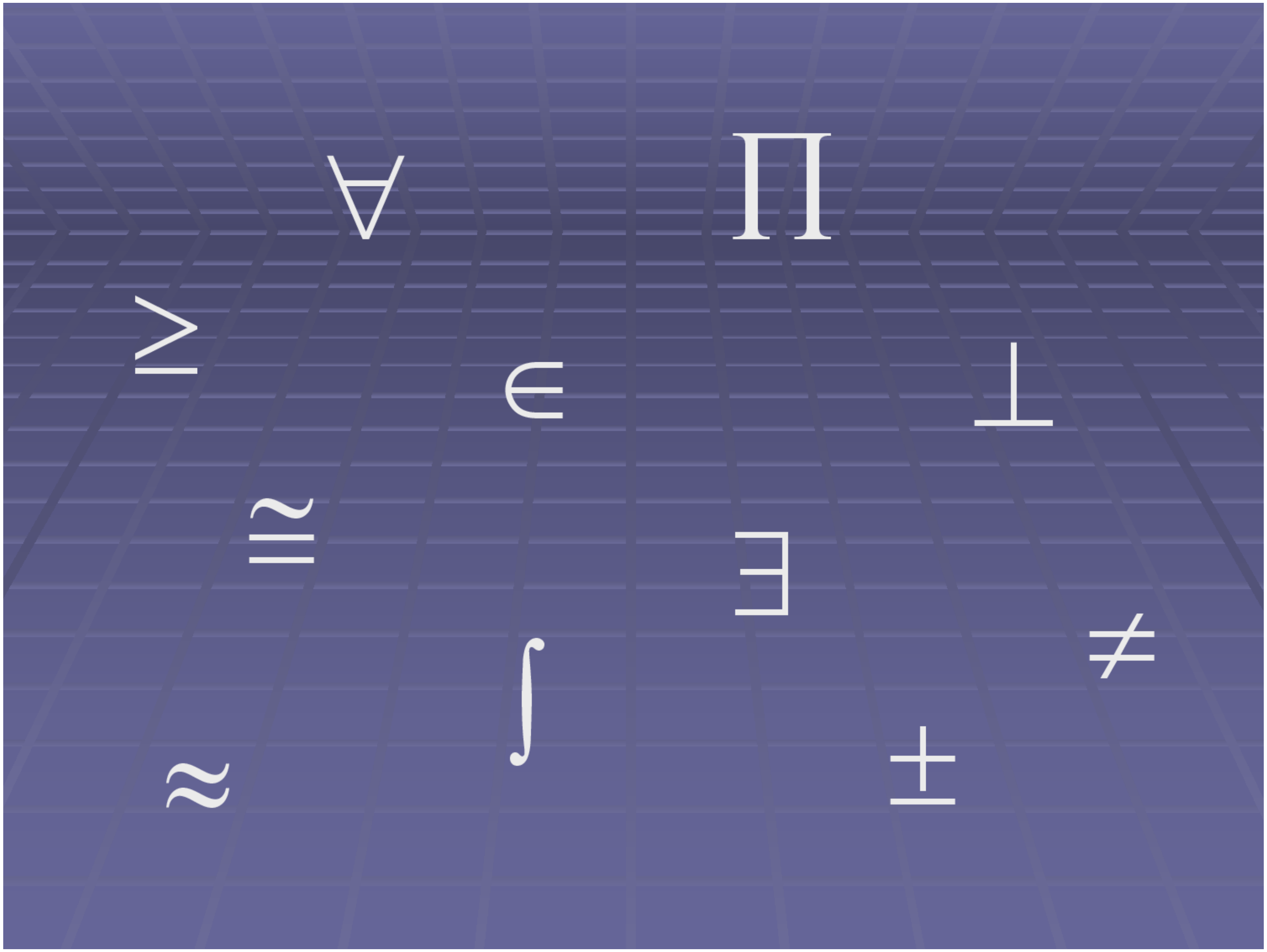
ASSOCIATE PROFESSOR OF MATHEMATICS IN THE UNIVERSITY OF TORONTO

BERLIN
MAYER & MÜLLER

Royal Canadian Institute

Fields was actively involved with the Royal Canadian Institute for 27 years. Not just someone living in the ivory tower of pure mathematics, Fields was also a man who could work with leaders of government and industry in securing support for scientific research.

The Ideal of Pure Mathematics



G. H. Hardy: *A Course of Pure Mathematics* (1908)

At German universities in the nineteenth century, the view emerged that

“the sciences, as well as the humanities, trained the intellect (Geist) and led to the refinement (Bildung) of the individual. Younger scientists emphasized "pure" science and rejected any utilitarian approach, which they, like their colleagues in the humanities, condemned as "bread-study" (Brotstudium).” – Turner (1971)

Franz Neumann wrote in 1844:

"True, its [science’s] heritage is not old; true, it was born in the modern state, in the service of those arts and trades which attend only to the requirements and conveniences of the external life; that it does not deny. But through great, unceasing rigors it has emancipated itself and made itself a free man, has created for itself a realm in which reigns only the free force of intellect, and where independent thought and research alone obtain." (Quoted in Turner (1971), p.

A belief in pure mathematics became well established at German universities by the end of the nineteenth century. Writing about mathematics professors of the period at the University of Berlin, historian R. Haubrich has observed:

"They all felt deeply obliged to carry on the Prussian neo-humanistic tradition of university research and teaching as they themselves had experienced it as students. This is especially true of [Ferdinand] Frobenius. He considered himself to be a scholar whose duty it was to contribute to the knowledge of pure mathematics. Applied mathematics, in his opinion, belonged to the technical colleges." (Haubrich, 1998)

Abstraction

Generality

Specialization

Ernest Brown in his presidential address to the AMS (1917):

"As time went on, mathematics was allowed to progress in its own way, unhindered by any necessity for present or future applications. This freedom from a somewhat entangling alliance has resulted in a marvellous structure of thought. The fundamental bases of the subject have been clarified and organized, the modes of reasoning have become subject to careful scrutiny and every effort has been directed to make sure that the structure built on these lines shall be without blemish."

Brown continued:

"Yet one cannot help asking whether it is for the best interests of the subject that it should continue in this isolation. An external stimulus seems to be necessary, at least from time to time, to produce the best elements of growth in most forms of human activity. Is the continued development of pure mathematics an exception to the rule? One may argue that isolation produces a pure strain, but is it not also true that an occasional crossing of the breed is necessary to prevent the species from running itself out?"

Mathematics 1900-1914: “The Golden Years”

Hilbert's Paris Address of 1900: 23 problems for the new century

"This conviction of the solvability of every mathematical problem is a powerful incentive to the worker. We hear within us the perpetual call: There is the problem. Seek its solution. You can find it by pure reason .... "

Mathematical logic: Russell and Whitehead (1910), Löwenheim (1915)

Set theory: Ernst Zermelo (1908)

Algebraic number theory: Kurt Hensel (1908, 1913)

General topology: Maurice Fréchet (1904-1906) and Felix Hausdorff (1914)

Integral equations and functional analysis: Ivar Fredholm (1903), David Hilbert (1905-1910), Frigyes Riesz (1907-1910)

Theory of integration: Henri Lebesgue (1902, 1904, 1906)

Abstract algebra: Joseph Wedderburn (1905, 1907), Ernst Steinitz (1910)

Number theory and infinite series: G. H. Hardy and Srinivasa Ramanujan begin their collaboration (1913)

Some Notable Mathematics Books of the Period 1900-1914

Hardy: *A Course of Pure Mathematics* (1908)

Russell and Whitehead: *Principia Mathematica* (1910)

Hensel: *Theorie der algebraischen Zahlen* (1908)

Lesbesgue: Intégrale, longueur, aire (1902)

Steinitz: Algebraische Theorie der Körper (1914)

Hadamard: Leçons sur le calcul des variations (1910)

Character of 20th century mathematics firmly in place

- “Modern” mathematics: mathematics as the study of abstract structures developed axiomatically: Noether, van der Waerden, Bourbaki

- Foundations of mathematics: mathematical logic, set theory: Goedel, Turing, Cohen

The “golden age”, the period before 1914, was also a period of international cooperation in mathematics.

The Great War 1914-1918

From Brown's presidential address (1917):

"The possibility of great changes coming rapidly and with little warning must be faced. We cannot therefore be content to sit back and await events. The future of research in pure science is in danger as never before and the less practical the science the greater the danger. New economic and social forces are at work in every direction and may call many of us to activities which may stop temporarily and perhaps permanently the work and thought which have in the past most fully occupied us. Two generations of steady and continuous development have tended to make us settle in grooves of thought and habits of mind in which great upheavals have no place."

Ballistics Research at the Aberdeen Proving Ground

TABLE 1. Some Aberdeen Proving Ground Mathematicians

Name	Location
A. A. Bennett	University of Texas
H. F. Blichfeldt	Stanford University
G. A. Bliss	University of Chicago
F. L. Carmichael	Princeton Graduate Student
C. R. Dines	Dartmouth College
P. A. Fraleigh	Cornell University
Haig Galajikian	Princeton University
H. M. Gehman	
B. P. Gill	College of the City of New York
T. H. Gronwall	New York, NY
J. S. Mikesh	Hibbing Minnesota Junior College
H. H. Mitchell	University of Pennsylvania
W. H. Roever	Washington University
Irwin Roman	Northwestern University
C. A. Shook	Harvard Graduate Student
Oswald Veblen	Princeton
Norbert Wiener	University of Maine
F. E. Wood	University of New Mexico
W. H. Wright	University of California

In a lecture to the American Association for the Advancement of Science in 1919 the physicist Gordon Hull recalled:

"A number of experiments [in ballistics] were carried on at Aberdeen [Proving Ground], chiefly by Major Veblen and Lieutenant Alger . .. It is seen that these experiments added greatly to the effectiveness and therefore to the value of the guns in question. The work belongs to physics, notwithstanding the fact that one of these civilian officers was and is a professor of mathematics of the purest quality. That he was able to bring himself temporarily to neglect the fundamental concepts of geometry, in which realm he is one of our foremost thinkers, to enter into the problems of the war with an eagerness for close observation of actualities and a readiness to tryout new methods, is very greatly to his credit. He is evidently a physicist by intuition and a mathematician by profession.''

Quote given by Jim Ritter in Schlote and Schneider (2011, p. 151)

The experience of World War I led many scientists and university leaders to call for support of scientific research on the grounds of the importance it was said to hold for both national defense and industry.

Sir Robert Falconer, President of the University of Toronto, in a report for the Ontario Commission of Inquiry on University Finances in 1920:

"One of the chief functions of a university is to extend knowledge and to train others who will extend knowledge ... The experience of the Great war and after have rendered unnecessary any extensive advocacy of the value of scientific research … It was the application of the results of scientific research that contributed largely to the successful conduct of the war."

As quoted in Yves Gingras, *The Rise of Scientific Research in Canada* (2002), p. 58

At the Royal Canadian Institute Fields became an advocate for government support of scientific research. In an address that he delivered in March of 1918 to the Toronto Board of Trade he used the experience of the war to justify this support:

> "Whatever other lessons the peoples have learned from the war, it has brought home to them the power of science. This power, too, it is realized, may be used for good or for evil." Riehm and Hoffman (2011), p. 71

In calling for support for scientific research, Fields appealed to very general claims about the value of this research to the day-to-day lives of people in society:

> “The difference between the conditions under which our forefathers lived three hundred years ago and those under which we live to-day is due to research workers. The layman little realizes what an influence individual men among these research workers exercise upon his daily life. In ordinary conversation the name of Shakespeare is heard more frequently than that of Newton, and students of the great dramatist will be surprised to be told that Shakespeare as a factor in determining their live is a bagatelle compared to Newton.” (Riehm and Hoffman, 2011, pp. 72-73)

PROCEEDINGS

OF THE

INTERNATIONAL MATHEMATICAL CONGRESS

HELD IN

TORONTO, AUGUST 11-16, 1924

EDITED BY

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WITH THE COLLABORATION OF AN EDITORIAL COMMITTEE

VOL. I

REPORT OF THE CONGRESS
LECTURES
COMMUNICATIONS TO SECTIONS I AND II

TORONTO:
THE UNIVERSITY OF TORONTO PRESS
1928

A FOUNDATION FOR THE THEORY OF IDEALS

By Professor J. C. Fields,
University of Toronto, Toronto, Canada.

I

The theory developed in this paper was presented in outline at the Australian meeting of the British Association for the Advancement of Science in 1914. During the two following years details were filled in and modifications introduced here and there. The text had been typed and most of the formulae transcribed from the written manuscript to the typewritten copy when, early in 1917, the work was definitely laid aside for activities which seemed nearer the needs of the time. Throughout the intervening years up to the presentation of the paper to the Mathematical Congress the author failed to find the small amount of time which would have sufficed to complete the transcription of the formulae and make some changes in their lettering. These final touches have only been given to the copy at the last moment before handing it over to the printer and doubtless further changes in the terminology could be made with advantage*.

In his address to the 1924 International Congress of Mathematicians Fields asserted that the Congress brought together

“the mathematician whose occupation it is to spin fine webs and elaborate beautiful configurations in the realm of the subjective and the applied man who takes all the risk of assuming that over against the subjective network presented by the mathematician there is something corresponding to the external universe.”

Vannevar Bush (1946)

"… basic research leads to new knowledge. It provides scientific capital. It creates the fund
from which the practical applications of knowledge must be drawn. New products and new processes do not appear full-grown. They are founded on new principles and new conceptions, which in turn are painstakingly developed by research in the purest realms of science … today, it is truer than ever that basic research is the pacemaker of technological progress."

Science The Endless Frontier (quoted in Layton "Mirror-Image Twins," *Tech. & Culture* (1971, p. 563)

Between 1945 and 1960 the American Department of Defense allocated over two and a half billion dollars in support of basic or undirected research.

Project Hindsight (1966)

Major study, examined the contributions of funded research by the Department of Defense to main weapons systems of the nation.

Finding: funding for basic research contributed very little (.3%) to technological innovations involved in defense installations.

Fields arrival at the University of Toronto occurred at a time when the University was becoming a centre in Canada for research, and when it was beginning to be accepted internationally for its contributions to science. Fields' career in mathematics was consistent with the wider development of mathematics at the end of the nineteenth century and the first part of the twentieth century. The tension between Fields' academic specialization in pure mathematics and his advocacy of support for basic scientific research was indicative of larger issues concerning the relationship of science and technological progress that played out in the remainder of the 20th century and are still with us today.