PALINDROMES IN PURE MORPHIC WORDS

Michelangelo Bucci
with
Elise Vaslet

$a \longrightarrow ab$

$b \longrightarrow a$

a

ab

aba

abaab

abaababa

...

abaababaabaababaababaaba...

$$\text{beeblebrox}^{\sim} = \text{xorbelbeeb}$$

$$\text{Pal} = \{w \mid w^{\sim} = w\}$$

$$\text{Pal}(w) = \{ v \in \text{Fact}(w) \text{ s. t. } v \in \text{Pal} \}$$

$$P(n) = \#\{ v \in \text{Pal}(w) \text{ s. t. } |v| = n \}$$

$$\#\mathrm{Pal}(v) \leq |v| + 1$$

$$D(w) = |w| + 1 - \#Pal(w)$$

abca

aababbaa

PwP s.t. $w^{\sim} \neq w$

Palindromic defect conjecture (Blondin Massé, Brlek, Garon, Labbé): Let u be the fixed point of a primitive morphism. If $0<D(u)<\infty$ then u is periodic.

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BUT...

$$a \longmapsto aabcacba$$

$$b \longmapsto aa$$

$$c \longmapsto a$$

$$u = aabcacbaaabcacbaaaaaabcacbaaa\ldots$$

$$a \longmapsto aabcacba = aP$$

$$b \longmapsto aa$$

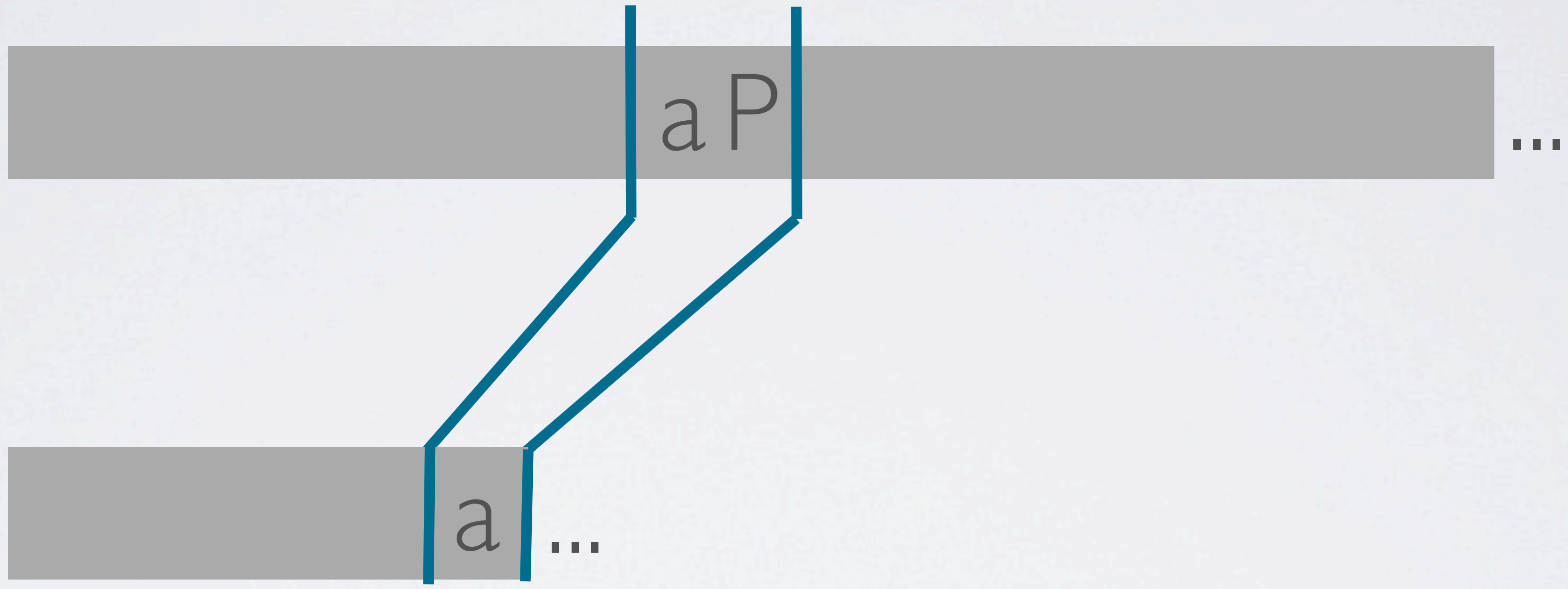
$$c \longmapsto a$$

$$u = aPaPaaaaPaaaaP\ldots$$

P ...

a P ...


a P
...

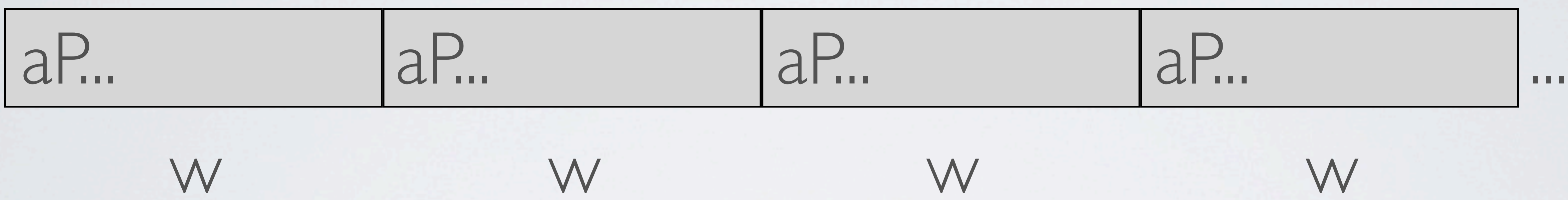

a P
...
a
...

P	P	P

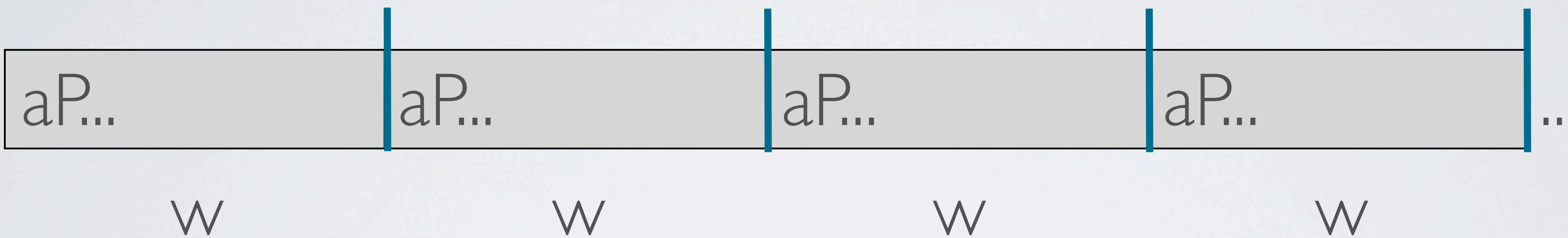

a P P P
a

u = aPaPaaaaPaaaaP...

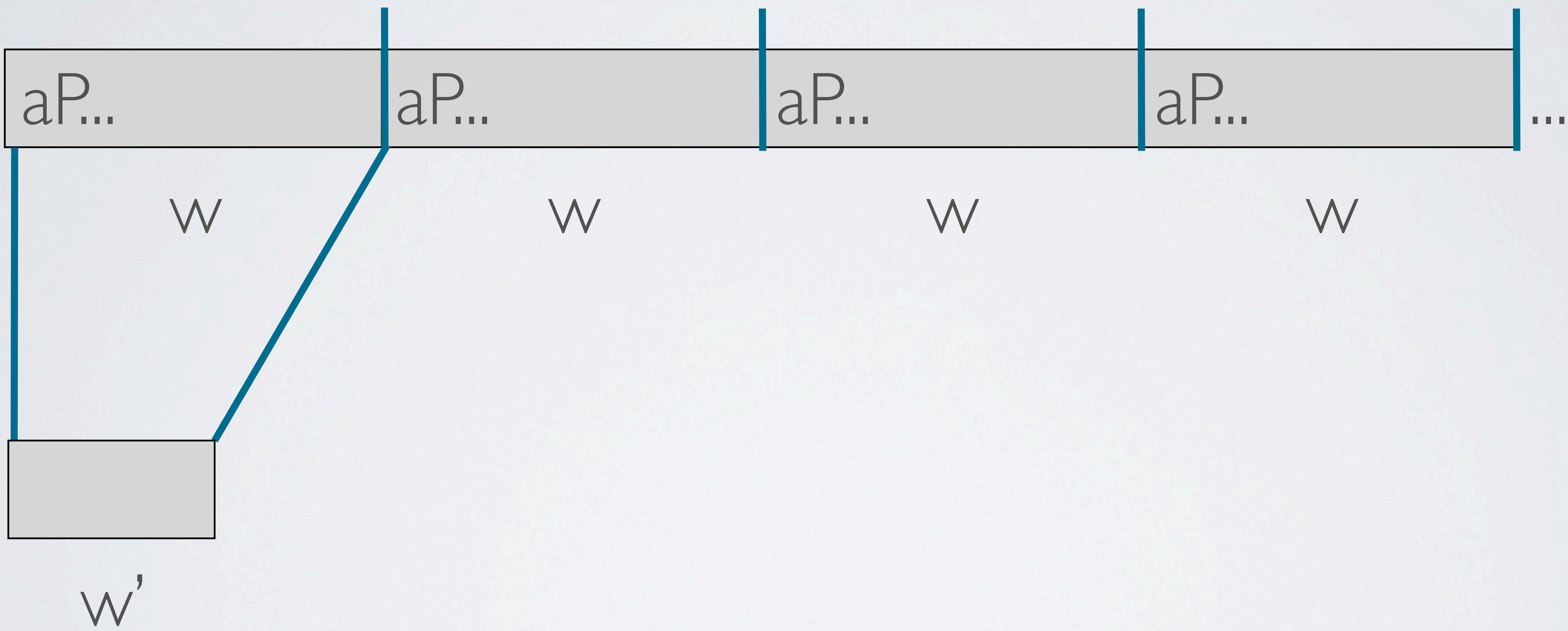
u = aPaPaaaaPaaaaP...



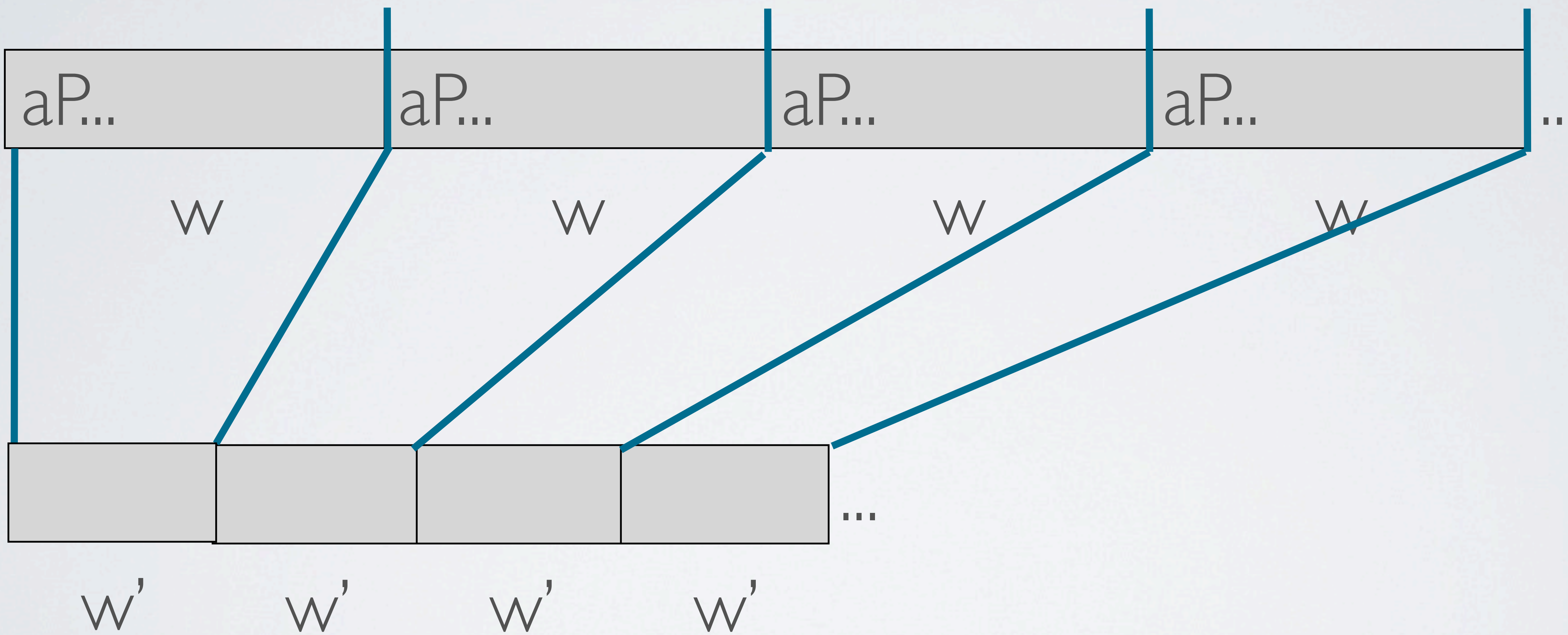
u = aPaPaaaaPaaaaP...



u = aPaPaaaaPaaaaP...



u = aPaPaaaaPaaaaP...



$D(u) = \#\{\ v$ s.t. $v \in Pref(u)$ and v is end-lacunary $\}$

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$u = aabca\ldots$

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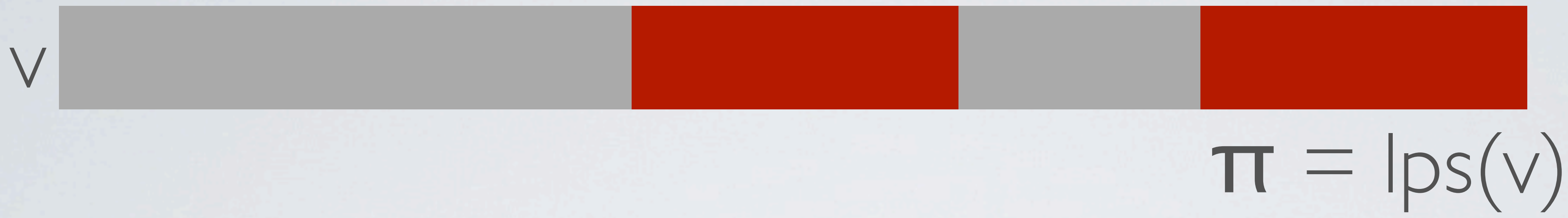
$u = aabca...$

$D(u) \geq 1$


v
π = lps(v)


v
π = lps(v)
v’

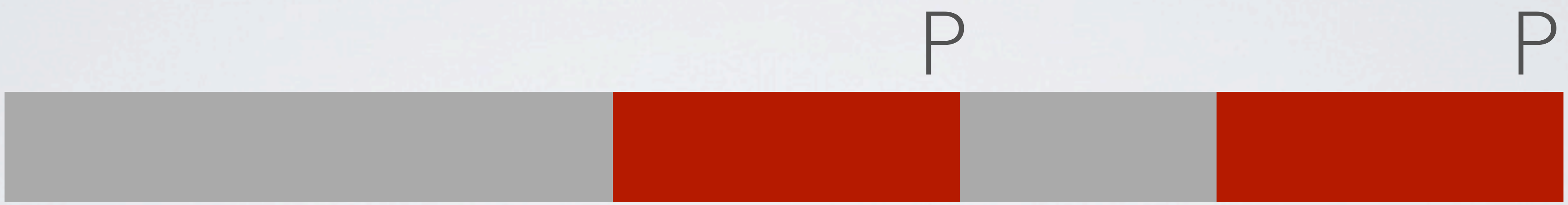

v
π = lps(v)
v'
v''

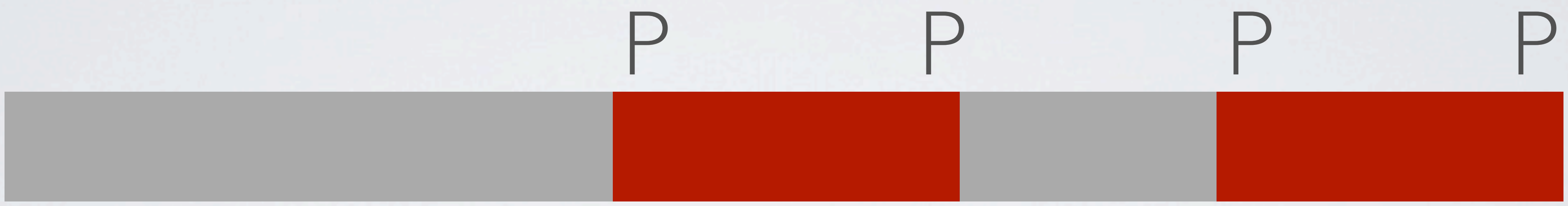
aabca ⟼ aabcabcaaabcacbaaaaaabcacba

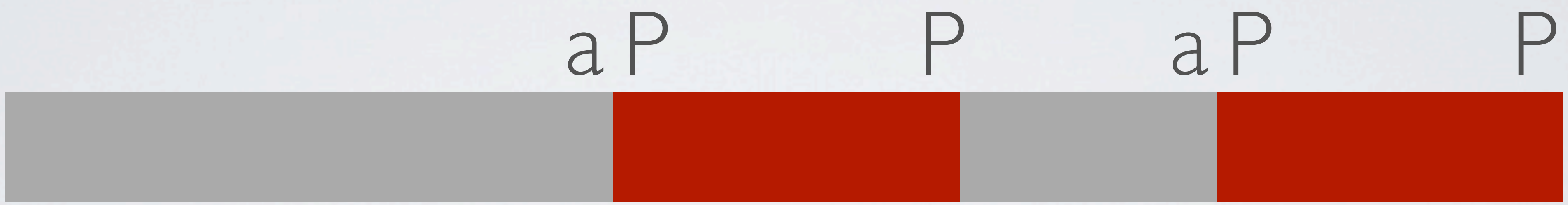
$$u \in a\{Pa, Paaaa\}^*$$

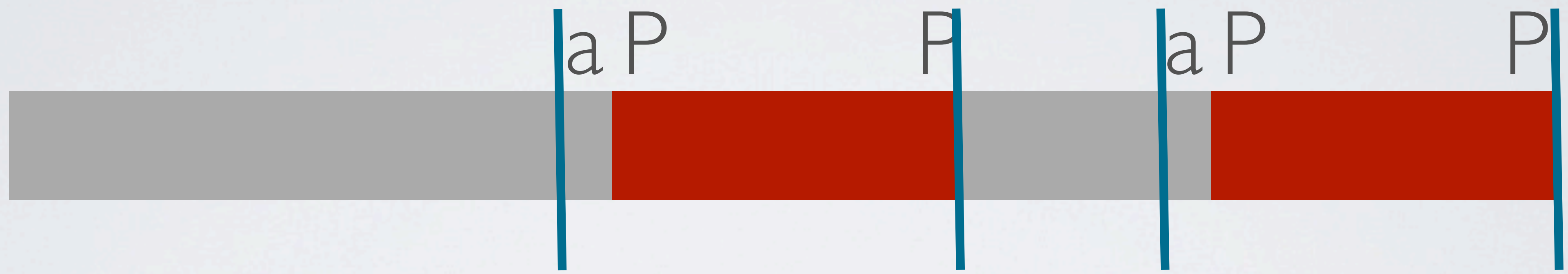
$$|v| > K \;\Rightarrow\; P \in \mathrm{Fact}(\mathrm{lps}(v))$$

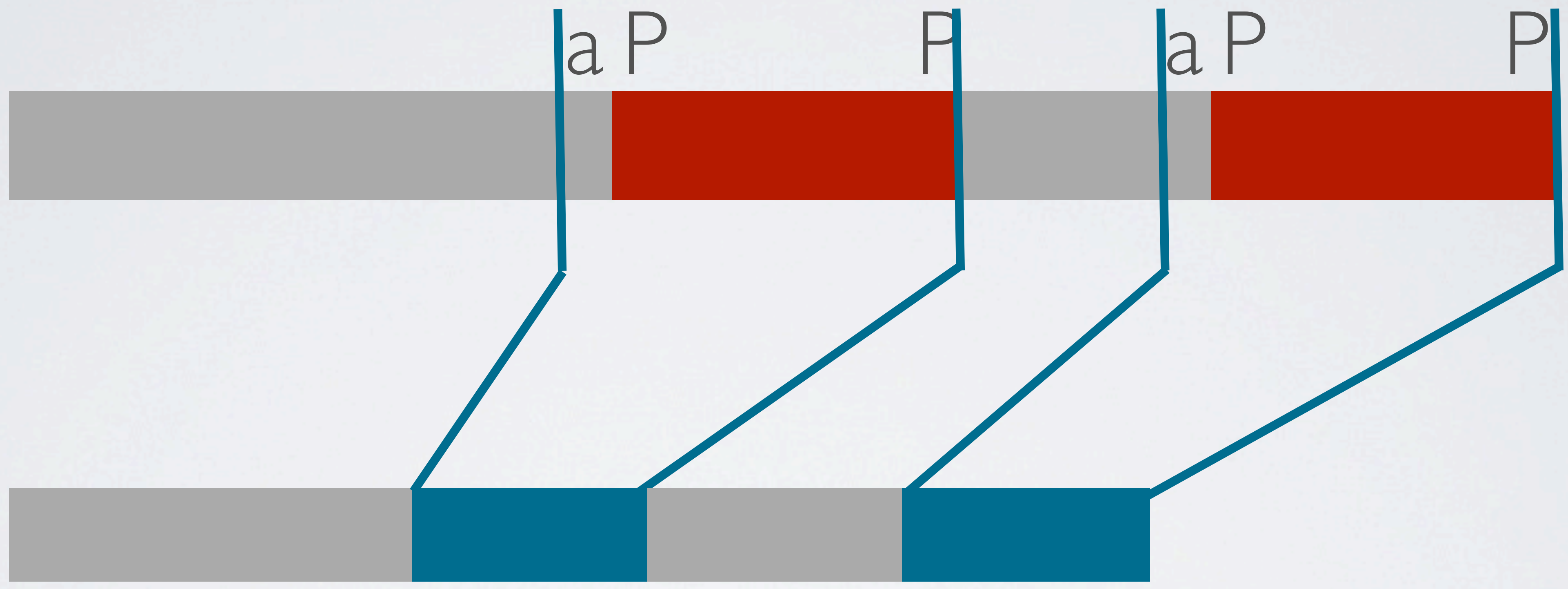
$$w \longmapsto aw',\;\; w^{\sim} \longmapsto aw'' \Rightarrow w' = w''^{\sim}$$

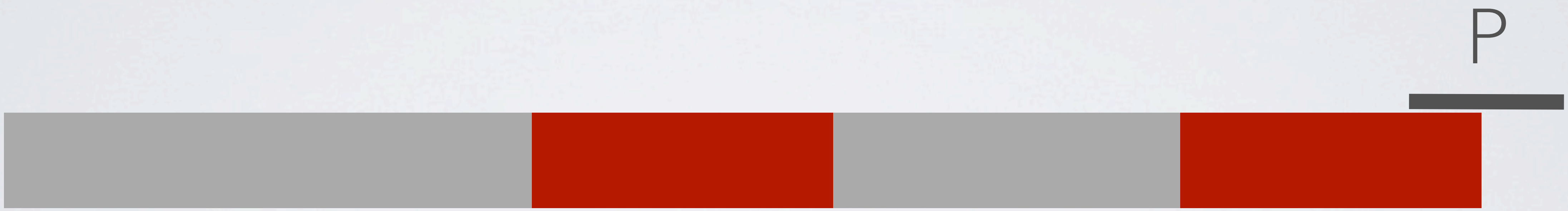

P
P


P
P
P
P

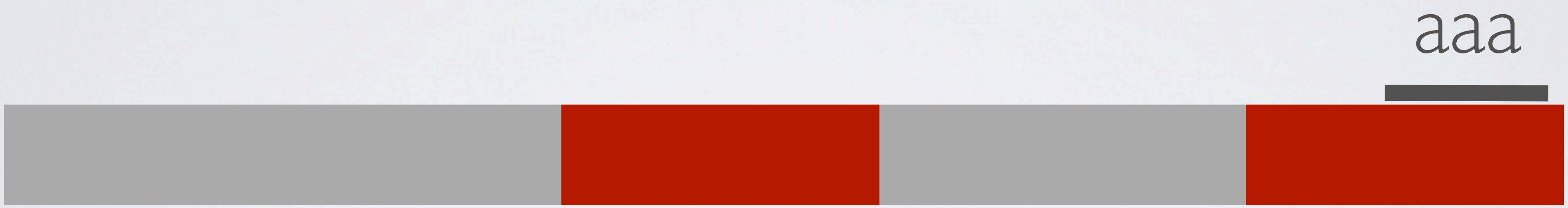

a P
P
a P
P


a P
P
a P
P


a P
P
a P
P


P


P


aaa


aaa
P

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Thank you