Dynamics of varroa-mites infested honey bee colonies model

Kazeem O. OKOSUN

Vaal University of Technology, Vanderbijlparrk, South Africa.

BIOMAT 2013: November 7, 2013

Talk Outline

- Introduction
- Model Formulation
- Optimal Control Analysis
- Numerical Results

- The global treat to insect pollinators of crops and wild plants are becoming alarming and their exstintion may have high impact on economic and environmental consequencies
- Not less than 15% of human food production relies on animal pollinators, most especially the bees.
- For instance, US fruit and vegetable growers requires the honey bees pollination services to generate $8-10 billion as farm income annually
- In addition to pollination, honey bees also play significant role as producer of honey and wax which in turn result in various nutritional and industrial uses.

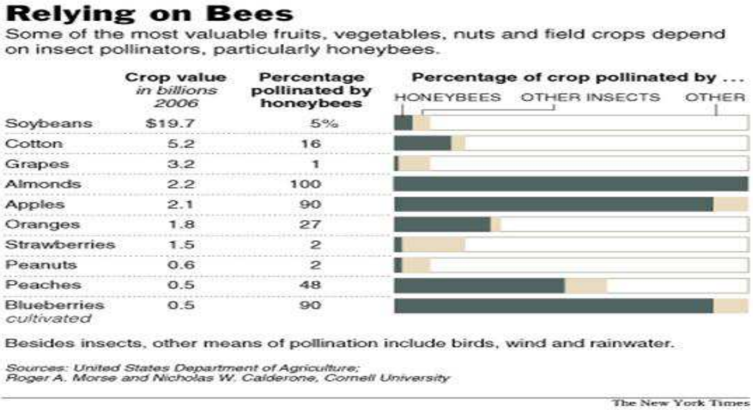
- Varroa mites are parasitic mites that survive by sucking the blood from honey bees. The mites target the pupal bees that are sealed inside a wax cell as the bees develop from a larva to an adult bee.
- Honey bee colonies with large mite infestations will be so weakened that the entire colony will eventually die out.
- Colony collapse disorder (CCD), also known as honey bee colony depopulation syndrome, is essentially the sudden disappearance of honey bees from their colony. As more and more bees disappear, the colony fails and ultimately dies.
- Varroa mites have a huge economic impact on the beekeeping industry, they do not only feed on bees but also transmit deadly viruses (e.g. Acute Bee Paralysis Virus (ABPV)) to the bees
- Much has not been studied regarding the disease dynamics of honeybee and varroa mite population in the literature

Figure: Varroa mites on a honey bee pupal and a varroa mite up close. Photos: Zachary Huang and USDA-ARS

(a) (b)

Figure: Frames taken from colonies suffering from colony collapse disorder have few bees, but large numbers of developing larval and pupal bees. Compare with a frame full of bees taken from a healthy colony. Photos: Keith Delaplane from Oldroyd, 2007 and Reed Johnson

Why is this study important? Honeybees are responsible for pollinating roughly 1/3 of the ingredients found in food consumed today.



Relying on Bees

Some of the most valuable fruits, vegetables, nuts and field crops depend on insect pollinators, particularly honeybees.

	Crop value *in billions 2006*	Percentage pollinated by honeybees
Soybeans	$19.7	5%
Cotton	5.2	16
Grapes	3.2	1
Almonds	2.2	100
Apples	2.1	90
Oranges	1.8	27
Strawberries	1.5	2
Peanuts	0.6	2
Peaches	0.5	48
Blueberries *cultivated*	0.5	90

Besides insects, other means of pollination include birds, wind and rainwater.

Sources: United States Department of Agriculture; Roger A. Morse and Nicholas W. Calderone, Cornell University

The New York Times

Parameter	Definition
β	rate at which uninfected bees become infected
λ	rates at which mites acquire virus
σ	unheathy rate at which hives become foragers
δ	rates at which mites kills bees
μ_i	bees mortality rate $(i = 1, 2, 3)$
μ_4	mites mortality rate
α	maximum rate that hive bees become foragers
ρ	number of mites that can be sustained per bee on average
ω	saturation constant
r	mites birth rate

Table: (1)

$$
\begin{aligned}
\frac{d}{dt}H &= L\left(\frac{H+F}{\omega+H+F}\right) - H\left(\alpha - \sigma\frac{F}{F+H}\right) - \mu_1 H \\
\frac{d}{dt}F &= H\left(\alpha - \sigma\frac{F}{F+H}\right) - \mu_2 F - \frac{\beta F m}{F+I} - \delta M F \\
\frac{d}{dt}I &= \frac{\beta F m}{F+I} - \mu_3 I - \delta M I \\
\\
\frac{d}{dt}m &= \beta_2 (M-m)\frac{I}{F+I} + \frac{\lambda I M}{F+I} - \mu_4 m \\
\frac{d}{dt}M &= rM\left(1 - \frac{M}{\rho(F+I)}\right) - \frac{\lambda I M}{F+I} - \mu_4 M
\end{aligned} \tag{1}
$$

Stability of the disease-free equilibrium (DFE)

$$E_0 = (H^*, F^*, I^*, M^*, m^*) = \left(H^*, F^*, 0, 0, M^*\right) \tag{2}$$

The linear stability of E_0 can be established using the next generation operator method. It follows that the reproduction number of the model is given by

$$\mathcal{R}_0 = \sqrt{\frac{\beta\rho\beta_2(r-\mu_4)}{(r\mu_3 + F^*\delta\rho(r-\mu_4))(\beta+\mu_4)}}, \tag{3}$$

Existence of endemic equilibrium: The endemic equilibrium satisfies the following two polynomial scenario. We consider the bee endemic population expressed in terms of the varrao mites virus population in the first case, that is,

$$P(I^*) = I^*[A{I^*}^2 + BI^* + C] = 0 \tag{4}$$

where,

$$\begin{aligned} A &= \delta\rho(r - \mu_4)(\beta_2 + \mu_4), \\ B &= [\Omega^*\delta\rho[r(\beta + \beta_2) + 2\mu_4] + r\mu_3(\beta_2 + \mu_4)][1 - R_k^2], \\ C &= \Omega^*(r\mu_3 + \Omega^*\delta\rho(r - \mu_4))(\beta + \mu_4)[1 - \mathcal{R}_0^2], \end{aligned} \tag{5}$$

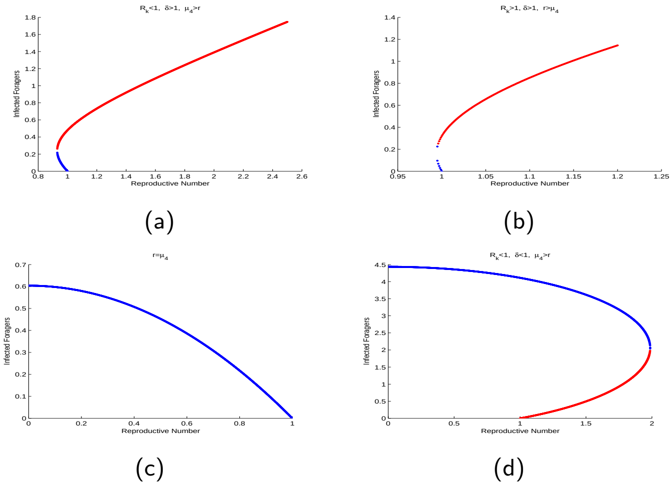
$$R_k = \frac{\Omega^*\delta\rho\mu_4(\beta_2 + \beta\delta + \mu_4)}{\Omega^*\delta\rho[r(\beta + \beta_2) + 2\mu_4] + r\mu_3(\beta_2 + \mu_4)}$$

Proposition

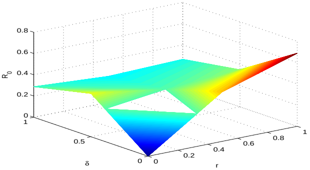
1. *If $r > \mu_4$, $R_k \leq 1$ then system (1) exhibits a transcritical bifurcation.*
2. *If $r > \mu_4$, $R_k \geq 1$ then system (1) exhibits a backward bifurcation.*
3. *If $r < \mu_4$, $R_k \leq 1$ then system (1) exhibits a backward bifurcation.*
4. *If $r < \mu_4$, $R_k \geq 1$ then system (1) exhibits a transcritical bifurcation.*

Proof:

1. For $R_k \leq 1$ and $r > \mu_4$, $A > 0$ we obtain when $R_0 > 1$ that $C < 0$. This implies that system (1) has a unique endemic steady state. If $R_0 \leq 1$, then $C \geq 0$ and $B \geq 0$. In this case system (1) has no endemic steady states.
ii. If $R_0 \leq 1$, then $C \geq 0$ and $r < \mu_4$, then $A < 0$, the discriminant of (4), $\Delta(R_0) := B^2 - 4AC > 0$, then there exists R_{0c} such that $\Delta(R_{0c}) = 0$, $\Delta(R_0) < 0$ for $1 < R_0 < R_{0c}$ and $\Delta(R_0) > 0$ for $R_{0c} > R_0$. One endemic steady state when $R_0 = R_{0c}$ and two endemic steady states when $1 < R_0 < R_{0c}$.
iii. If $R_0 \leq 1$ and $r = \mu_4$, the $A = 0$, $C > 0$ and $B > 0$ the system has no endemic equilibrium. If $R_0 > 1$, there exist a unique equilibrium.
2. For $R_k \geq 1$ we discuss the following cases:
i. If $R_0 > 1$ and $r > \mu_4$ in this case $C < 0$ and system (1) has a unique endemic steady state. If $r < \mu_4$, so $A < 0$ then the system has no endemic state
ii. If $R_0 \leq 1$ and $r > \mu_4$ in this case $C > 0$ and $B < 0$, while the discriminant of (4), $\Delta(R_0) := B^2 - 4AC$, can be either positive or negative. We have $\Delta(1) = B^2 > 0$ and $\Delta(1) = -4AC < 0$, then there exists R_{0c} such that $\Delta(R_{0c}) = 0$, $\Delta(R_0) < 0$ for $R_{0c} < R_0 < 1$ and $\Delta(R_0) > 0$ for $R_{0c} < R_0$. One endemic steady state when $R_0 = R_{0c}$ and two endemic steady states when $R_{0c} < R_0 < 1$.
iii. If $R_0 \leq 1$ and $r = \mu_4$, the $A = 0$, $C > 0$ and $B < 0$ there exists has a unique endemic equilibrium. If $R_0 > 1$, the system has no endemic equilibrium.


R_k<1, δ>1, μ_4>r
Infected Foragers
Reproductive Number
R_k>1, δ>1, r>μ_4
Infected Foragers
Reproductive Number
r=μ_4
Infected Foragers
Reproductive Number
R_k<1, δ<1, μ_4>r
Infected Foragers
Reproductive Number

(a) (b) (c) (d)



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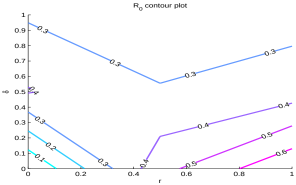

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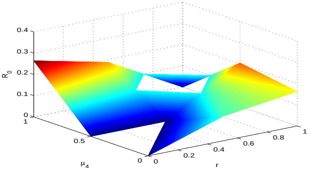

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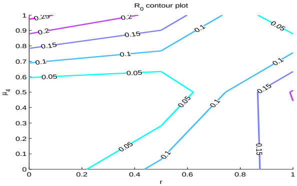

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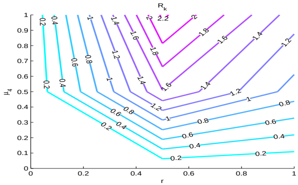
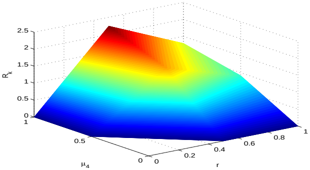


(i)



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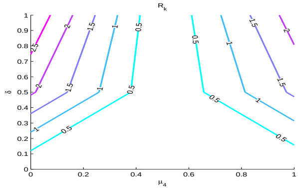

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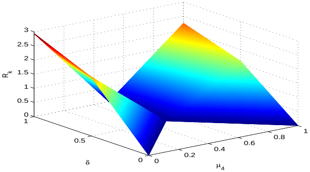


(l)

Summary of Results		
$R_0 < 0,$	$R_k < 1$	50% of μ_4, 10% of r
$R_0 < 0,$	$R_k < 1$	5% of μ_4, 50% of r
$R_0 < 0,$	$R_k < 1$	10% of δ, $0 < \mu_4 < 40\%$
$R_0 < 0,$	$R_k < 1$	10% of δ, $60\% < \mu_4 < 100\%$
$R_0 < 0,$	$R_k > 1$	50% of μ_4, 30% of r
$R_0 < 0,$	$R_k > 1$	33% of μ_4, 50% of r
$R_0 < 0,$	$R_k > 1$	25% of δ, $0 < \mu_4 < 30\%$
$R_0 < 0,$	$R_k > 1$	25% of δ, $70\% < \mu_4 < 100\%$

Table: (2)

Given the objective function J, as:

$$J(u_1, u_2) = \int_0^{t_f} [AI + Bm + CM + Du_1^2 + Eu_2^2]dt \tag{6}$$

where $\mathcal{U} = \{(u_1, u_2)$ such that u_1 and u_2 are measurable with $0 \leq u_1 \leq 1$ and $0 \leq u_2 \leq 1$, for $t \in [0, t_f]\}$ is the control set.

Subject to equations (1), we apply Pontryagin's Maximum Principle

The co-state variable associated with the system is represented by $G(t)$, the current value Hamiltonian is then written as

$$
\begin{aligned}
H_a &= AI + Bm + CM + Du_1^2 + Eu_2^2 \\
&+ G_H\left\{L\left(\frac{H+F}{\omega+H+F}\right) - H\left(\alpha - \sigma\frac{F}{F+H}\right) - \mu_1 H\right\} \\
&+ G_F\left\{H\left(\alpha - \sigma\frac{F}{F+H}\right) - \mu_2 F - \frac{\beta F m}{F+I}(1-u_2) - u_1\delta MF\right\} \\
&+ G_I\left\{\frac{\beta F m}{F+I}(1-u_2) - \mu_3 I - u_1\delta MI\right\} \\
&+ G_m\left\{\beta_2(M-m)\frac{I}{F+I} + \frac{\lambda IM}{F+I}(1-u_2) - \mu_4 m\right\} \\
&+ G_M\left\{rM\left(1 - \frac{M}{\rho(F+I)}\right) - \frac{\lambda IM}{F+I}(1-u_2) - \mu_4 M\right\}
\end{aligned}
$$

Theorem

Given optimal controls u_1^*, u_2^* and solutions H, F, I, M, m of the corresponding state system (1) and (6) that minimize $J(u_1, u_2)$ over U. Then there exists adjoint variables G_H, G_F, G_I, G_M, G_m satisfying

$$\frac{-dG_i}{dt} = \frac{\partial H_a}{\partial i} \tag{8}$$

where $i = H, F, I, M, m$ and with transversality conditions

$$G_H(t_f) = G_F(t_f) = G_I(t_f) = G_M(t_f) = G_m(t_f) = 0 \tag{9}$$

$$u_1^* = \min\left\{1, \max\left(0, \left(\frac{\beta Fm(G_F - G_I) + \lambda IM(G_M - G_m)}{2D(F+I)}\right)\right)\right\}, \tag{10}$$

$$u_2^* = \min\left\{1, \max\left(0, \left(\frac{\delta M(FG_F - IG_I)}{2E}\right)\right)\right\}, \tag{11}$$

By standard control arguments involving the bounds on the controls, we conclude

$$u_1^* = \begin{cases} 0 & \text{If } \xi_1^* \le 0 \\ \xi_1^* & \text{If } 0 < \xi_1^* < 1 \\ 1 & \text{If } \xi_1^* \ge 1 \end{cases} \quad ; \quad u_2^* = \begin{cases} 0 & \text{If } \xi_2^* \le 0 \\ \xi_2^* & \text{If } 0 < \xi_2^* < 1 \\ 1 & \text{If } \xi_2^* \ge 1 \end{cases}$$

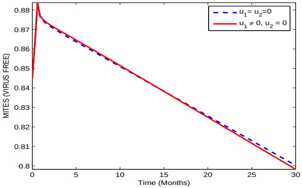
$\xi_1^* = \frac{\beta Fm(G_F - G_I) + \lambda IM(G_M - G_m)}{2D(F+I)}$ and $\xi_2^* = \frac{\delta M(FG_F - IG_I)}{2E}$

Proof.

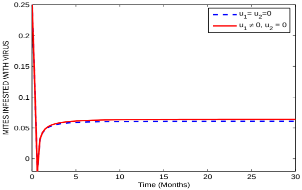
Corollary 4.1 of Fleming and Rishel [8] gives the existence of an optimal control due to the convexity of the integrand of J with respect to optimal pair u_1, u_2, a *priori* boundedness of the state solutions, and the *Lipschitz* property of the state system with respect to the state variables. Then the adjoint equations can be written as

$$
\begin{aligned}
-\frac{dG_H}{dt} &= -\frac{L\omega}{(\omega+H+F)^2}G_H + \alpha(G_H - G_F) + \frac{\sigma F^2}{(F+H)^2}(G_F - G_H) + \mu_1 G_H \\
-\frac{dG_F}{dt} &= -\frac{L\omega}{(\omega+H+F)^2}G_H + \frac{\sigma H^2}{(F+H)^2}(G_F - G_H) + \frac{\beta m I(1-u_2)}{(F+I)^2}(G_F - G_I) \\
&\quad \mu_2 G_F + u_1\delta M G_F + \frac{\beta_2(M-m)I}{(F+I)^2}G_m + \frac{\lambda I M(1-u_2)}{(F+I)^2}(G_m - G_M) - \frac{M^2 r}{\rho(F+I)^2 G_M} \\
-\frac{dG_I}{dt} &= \frac{\beta F m(1-u_2)}{(F+I)^2}(G_I - G_F) + \frac{\beta_2(M-m)F}{(F+I)^2}G_m \\
&\quad +\mu_3 G_I + u_1\delta M G_I + \frac{\lambda F M(1-u_2)}{(F+I)^2}(G_M - G_m) - \frac{rM^2}{\rho(F+I)^2}G_M \\
-\frac{dG_m}{dt} &= \frac{\beta F(1-u_2)}{F+I}(G_F - G_I) + \frac{\beta_2 I}{F+I}G_m + \mu_4 G_m \\
-\frac{dG_M}{dt} &= u_1\delta(FG_F - IG_I) - \frac{\beta_2 I}{F+I}G_m + \frac{\lambda I(1-u_2)}{F+I}(G_M - G_m) + \mu_4 G_M + \frac{2rM}{\rho(F+I)}G_M
\end{aligned} \tag{12}
$$

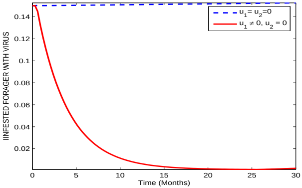
Efforts on prevention of Foragers from infection (u_1) only



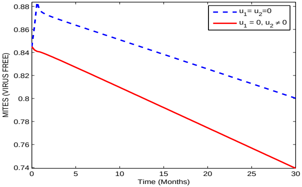
(m)



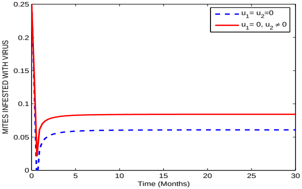
(n)



Efforts on prevention of Foragers from mites attacks (u_2) only



(a)



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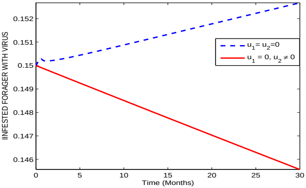
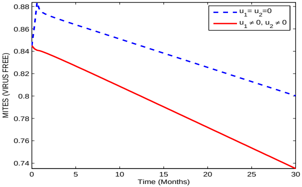


(c)

Prevention of Foragers from infection (u_1) and Protection of Foragers (u_2) from mites attacks



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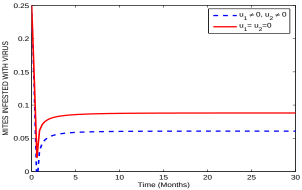

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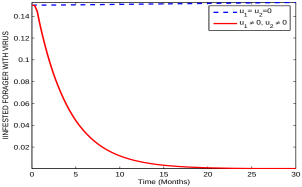

- A mathematical model for the dynamics of varroa-mites infested honey bees colonies is formulated and analyzed.
- It is found that honey bees disease free equilibrium only exists in the absence of varroa mites and the infected bees population increases with increase in the varroa mites population (leading to collony collapse disorder)
- The model is also found that the model exhibit multiple equilibria.
- Incorporating time dependent controls, using Pontryagin's Maximum Principle to derive necessary conditions for the optimal control of the disease, the numerial results suggest that the comibination of prevention control (u_1) on foragers from infection and protection of foragers (u_2) from mites attacks is the most effective strategy to prevent Colony Collapse Disorder in honeybee colony.

Acknowledgement

- Vaal University of Technology, Vanderbijlpark, South Africa
- Organizers of BIOMAT 2013

Thank You

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