Thermodynamical cost of accuracy and stability of information processing

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Fields Institute, Toronto , August 2013

Talk based on arXiv:1305.4910,

see also video by Lidia del Rio and Philipp Kammerlander

http://www.youtube.com/watch?v=gtcPp7FY0gU

Quantum measurement and information processing

The common issues:

1) Recognition of pointer (information carrier) states

2) Stability of pointer states with respect to joint thermal and quantum noise

3) Thermodynamical cost of encoding, readout and altering

of measurement result (information)

So-called Measurement Problem

The observation of the pointer requires another measuring instrument, which in turn requires yet another instrument, and so on, in such a way that the whole process involves an infinite regression ending up in the observers brain.

Cutting the Gordian knot

There exist stable pointer states which can be distinguished with an error probability ϵ given by their overlap (quantum transition probability). The life-time of the pointer states scales like $\frac{1}{\epsilon}$ and the thermodynamical cost of pointer states recognition vanishes with $\epsilon \to 0$.

A quantum model of a single-bit memory

Harmonic oscillator (pointer) + spin-$1/2$ (interface) (spin-oscillator system, SOS) weakly interacting with a heat bath.
SOS Hamiltonian (renormalized) $\hbar = k_B = 1$

$$\hat{H} = \omega_0(\hat{a}^\dagger - D\hat{\sigma}^3)(\hat{a} - D\hat{\sigma}^3), \quad \omega_0, D > 0, \tag{1}$$

SOS-bath interactions and bath's spectral densities

$$\hat{H}_{int}^{(o)} = (\hat{a} + \hat{a}^\dagger)\hat{F}_o, \quad \hat{H}_{int}^{(3)} = \hat{\sigma}^3\hat{F}_3, \tag{2}$$

$$\hat{H}_{int}^{(1)} = \hat{\sigma}^1\hat{F}_1 \tag{3}$$

$$G_j(\omega) = \int_{-\infty}^{+\infty} e^{i\omega t}\langle\hat{F}_j(t)\hat{F}_j\rangle\, dt = G_j(-\omega)e^{\omega/T}). \tag{4}$$

Markovian master equation ($\hat{H}_{int}^{(1)}$-neglected)

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H},\hat{\rho}] + \frac{1}{2}\gamma\Big([\hat{b},\hat{\rho}\hat{b}^\dagger] + [\hat{b}\hat{\rho},\hat{b}^\dagger]\Big) + \frac{1}{2}\gamma e^{-\omega_0/T}\Big([\hat{b}^\dagger,\hat{\rho}\hat{b}] + [\hat{b}^\dagger\hat{\rho},\hat{b}]\Big) - \frac{1}{2}\Gamma[\hat{\sigma}^3,[\hat{\sigma}^3,\hat{\rho}]].$$

where $\hat{b} = \hat{a} - D\sigma^3$, the dissipation rate $\gamma = G_o(\omega_0)$, the pure decoherence rate $\Gamma = 4D^2G_o(0) + G_3(0)$.
Biased SOS Gibbs states (stationary with respect to master eq.)

$$\hat{\rho}^{(\pm)} = \big(1 - e^{-\omega_0/T}\big)|\pm\rangle\langle\pm|\, e^{-\frac{\omega_0}{T}(\hat{a}^\dagger \mp D)(\hat{a}\mp D)} \tag{6}$$

and the corresponding pointer (oscillator) states

$$\hat{\rho}_{\mathcal{P}}^{(\pm)} = \big(1 - e^{-\omega_0/T}\big)e^{-\frac{\omega_0}{T}(\hat{a}^\dagger \mp D)(\hat{a}\mp D)}. \tag{7}$$

The overlap (probability of error)[Paraoanu and Scutaru, PRA 58, 869 (1998)]

$$\epsilon = \mathrm{Tr}\left(\sqrt{\sqrt{\hat{\rho}_{\mathcal{P}}^{(+)}}\hat{\rho}_{\mathcal{P}}^{(-)}\sqrt{\hat{\rho}_{\mathcal{P}}^{(+)}}}\right) = \exp\left\{-\frac{\bar{W}}{\Theta}\right\} \tag{8}$$

$$\bar{W} = \frac{1}{2}\omega_0(2D)^2, \quad \Theta = \frac{\omega_0}{e^{\omega_0/T} - 1} + \frac{\omega_0}{2}. \tag{9}$$

Θ - effective noise temperature

$$\Theta \simeq T \quad \text{for} \quad \frac{T}{\omega_0} >> 1, \quad \Theta \simeq \frac{\omega_0}{2} \quad \text{for} \quad \frac{T}{\omega_0} << 1 \tag{10}$$

$\Delta E = \omega_0(2D)^2$ averaged energy splitting between stationary SOS states and "excited" ones

$$\hat{\rho}_*^{(\pm)} = \left(1 - e^{-\omega_0/T}\right)|\mp\rangle\langle\mp|\, e^{-\frac{\omega_0}{T}(\hat{a}^\dagger \mp D)(\hat{a} \mp D)} \tag{11}$$

Dissipative tunneling and life-time of memory

The tunneling process is slow due to the energy barrier $\Delta E = \omega_0(2D)^2$.The memory life-time is characterized by the inverse of the initial tunneling rate

$$\Gamma_{tun} = \frac{1}{2}\mathrm{Tr}\Big(\hat{\sigma}^3\frac{d\hat{\rho}}{dt}|_{t=0}\Big) = \frac{1}{2}\mathrm{Tr}\Big(\hat{\sigma}^3\mathcal{L}^{(1)}\hat{\rho}^{(+)}\Big) \simeq \frac{1}{2}G_1(0)e^{-\frac{\bar{W}}{\Theta}} \tag{12}$$

$\frac{1}{2}G_1(0)$ - pure decoherence rate for uncoupled spin

$e^{-\frac{\bar{W}}{\Theta}}$ - Boltzmann-like suppressing factor (compare with Kramers formula)

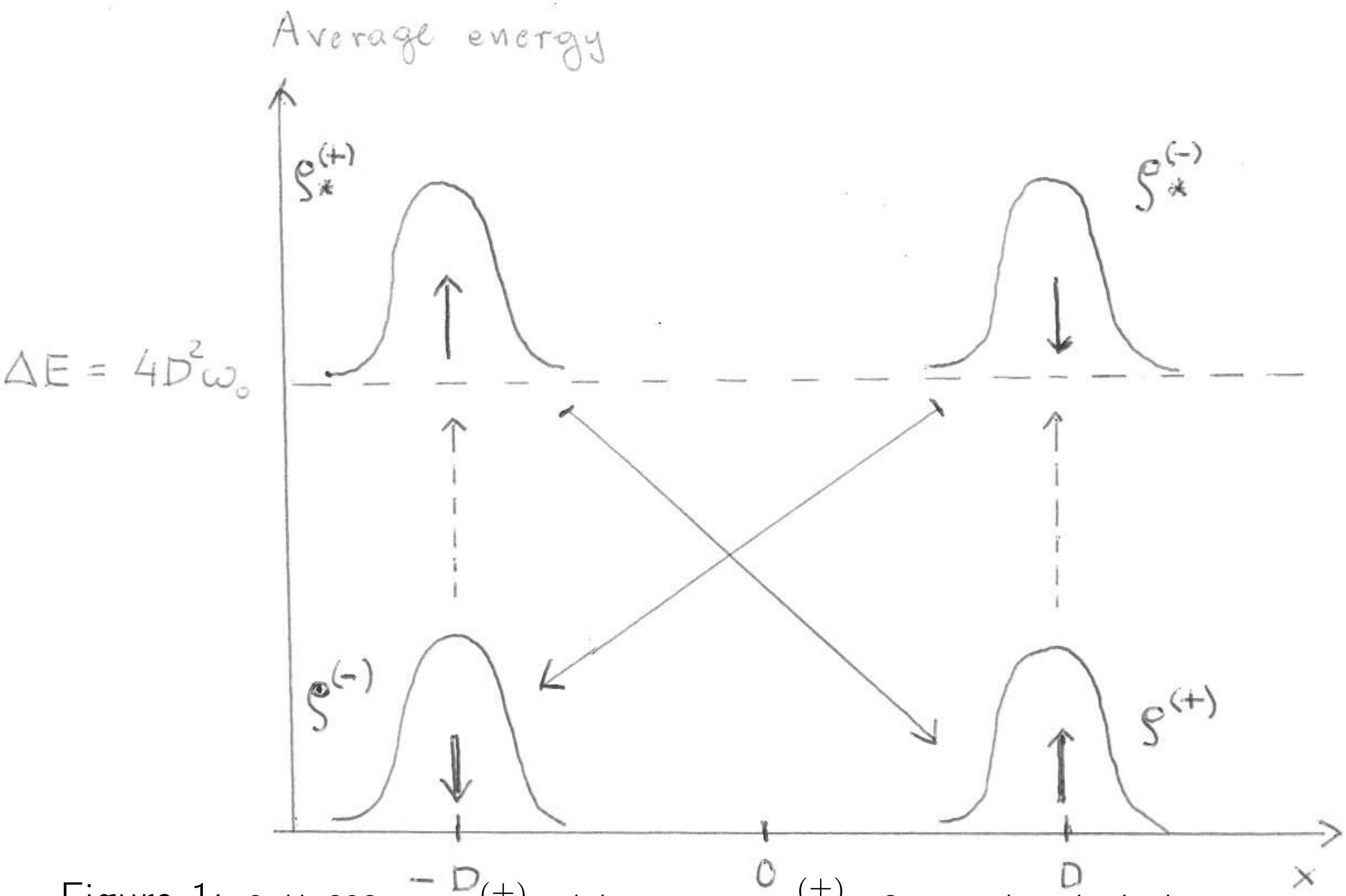

Figure 1: Stable SOS states $\rho^{(\pm)}$ and their excitations $\rho_*^{(\pm)}$. Gaussians depict localized pointer states with arrows inside corresponding to spin states. The solid arrows – dissipation routes, the dashed ones – tunneling process.

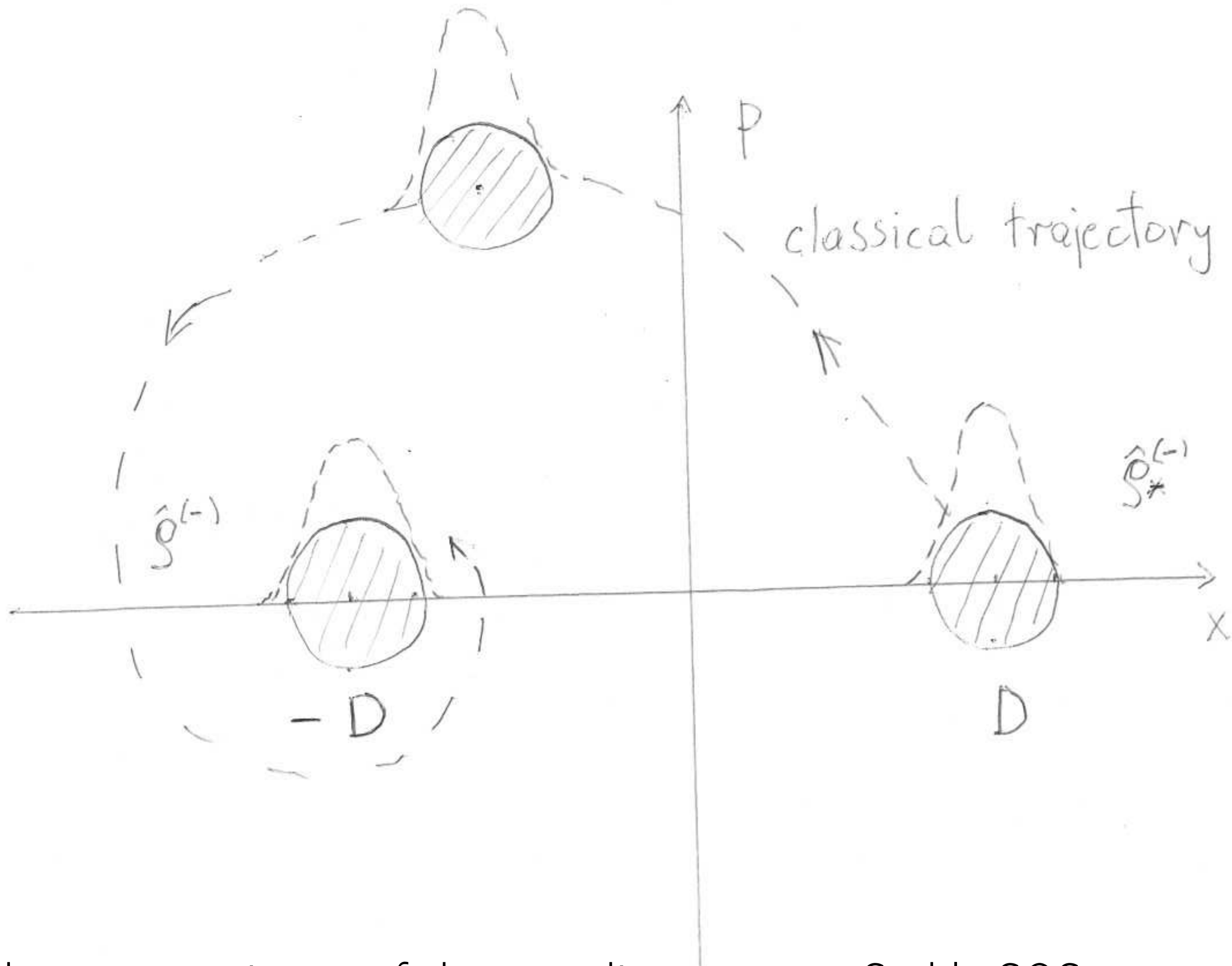

Figure 2: Phase-space picture of the recording process. Stable SOS state $\rho^{(+)}$ is excited to the state $\rho_*^{(-)}$ and then evolves along the damped harmonic oscillator classical trajectory towards the final stable state SOS $\rho^{(-)}$

Quantum measurement model

The dichotomic observable $\hat{X} = \hat{P}_+ - \hat{P}_-$ of the system $\mathcal{O}$

Coupling $\mathcal{O}$ to interface spin - CNOT gate

$$\hat{H}_M = f(t)\hat{\sigma}^1\hat{P}_- \tag{13}$$

where $f(t)$ generates fast spin-flip

On the average measurement costs at least $\bar{W}$ of work.

The steps of measurement process

1) Fast unitary preparation of the entangled state of $\mathcal{O}$ and the interface $c_-|\phi_-\rangle|-\rangle + c_+|\phi_+\rangle|+\rangle$
The post-measurement state of $\mathcal{O}$ - $|c_-|^2|\phi_-\rangle\langle\phi_-| + |c_+|^2|\phi_+\rangle\langle\phi_+|$

2) *Dequantization* irreversible process killing quantum coherences between emerging Schroedinger cat states of SOS. It takes *dequantization time* $t_D \sim \frac{1}{D^2}$.

3) The (conditional) evolution of the pointer Gaussian state along the classical trajectory with relaxation time $t_R = \frac{1}{\gamma}$ - the *recording time*.

4) A very slow *erasure process* of the measurement result on the *memory time* scale $t_E \sim e^{\frac{\bar{W}}{\Theta}} \sim \frac{1}{\epsilon}$.

General conclusions

The minimal work needed to encode or reset a bit of information with an error probability ϵ under the influence of a combined thermal and quantum noise at the noise temperature Θ is given by

$$\bar{W} = \Theta \ln \frac{1}{\epsilon}. \tag{14}$$

Landauer's formula - $\bar{W}_L = T \ln 2$, i.e. $\epsilon = \frac{1}{2}$ and no quantum fluctuations. No need for reseting memory after measurement

The minimal work needed to perform an elementary gate on a protected information carrier is of the order of $\Theta \ln \frac{1}{\epsilon}$, where ϵ is the probability of readout error and Θ is the effective noise temperature. Moreover, the life-time of protected information scales like $\frac{1}{\epsilon}$.

Concluding remarks

1) A cost of a long computation (N logical gates)

$$\bar{W}_N \simeq \Theta N\big(\ln N + \ln\frac{1}{\delta} + \ln\frac{1}{\kappa}\big). \tag{15}$$

δ- error probability, $\kappa =$ (dissipation rate)/(decoherence rate)
Supercomputer with 10^{16} gates/sec working for a day - $N \simeq 10^{21}$ and $\bar{W}_N \simeq 10^2 J$ (actually 10^{10} J)

2) The conflict between reversibility and stability of information processing. The more stable are information carriers the more work must be invested in a logical gate. This work is subsequently dissipated making the gates strongly irreversible. The irreversibility (nonunitarity) does not harm classical computations but can puts limits on quantum ones.