Fundamental laws for quantum physics: How inequality violations originate from weak value statistics

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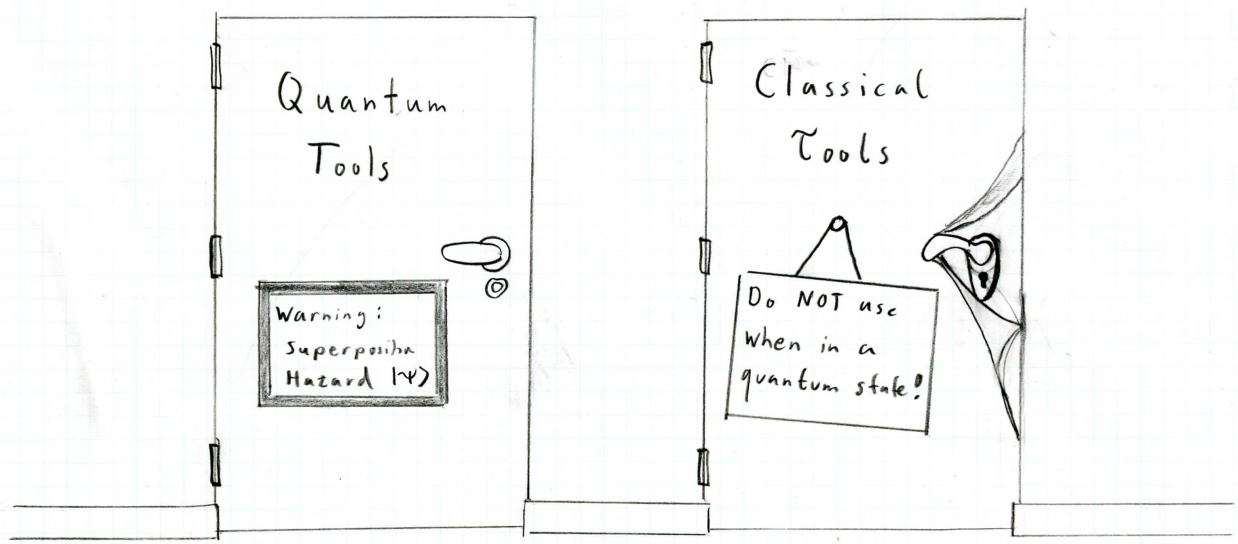
- Joint statistics without joint reality
- Complex probabilities as universal laws of physics
- Half-periodic transformations as origin of quantum paradoxes

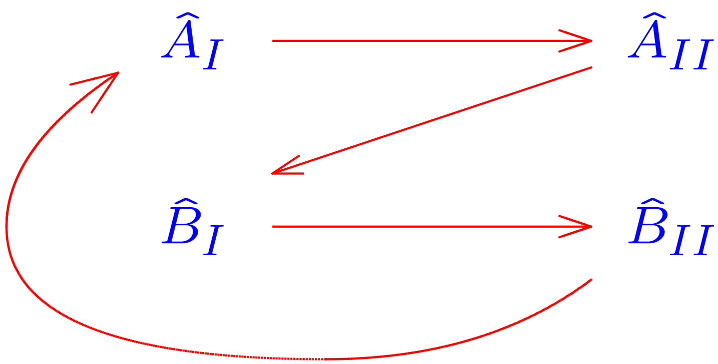
Quantum effects are observed with “classical” tools!



There is no paradox in the experimental data ...

How to construct a paradox:
empirical reality versus dogmatic realism

Correlated properties



Separately observed correlations cannot be explained by positive joint probabilities.

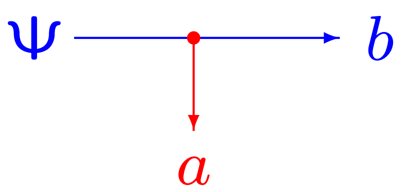
$\Downarrow$

Paradoxes result from the artificial assumption of an unobserved joint reality.

Quantum mechanics should provide the correct description of the relation between alternative empirical realities!

Weak measurement statistics: joint probability without joint reality

Weak measurements of $\mid a\rangle\langle a \mid$ conditioned by a final measurement of $\mid b\rangle$ result in a complex joint probability $\rho(a,b)$,



$$p(a|\psi,b) = \frac{\langle b \mid a\rangle\langle a \mid \psi\rangle}{\langle b \mid \psi\rangle}$$

$$\begin{aligned} \rho(a,b) &= p(a|\psi,b)\, p(b|\psi) \\ &= \langle b \mid a\rangle\langle a \underbrace{\mid \psi\rangle\langle\psi \mid}_{=\hat{\rho}} b\rangle \end{aligned}$$

The complex joint probability $\rho(a,b)$ completely determines the quantum statistics of the density operator $\hat{\rho}$.

(Dirac 1945, Johansen 2007, Hofmann 2012, Lundeen and Bamber 2012)

Prediction of other experimental probabilities

The complex joint probability $\rho(a, b)$ determines all possible measurement probabilities in accordance with Bayes' law,

$$\langle m \mid \hat{\rho} \mid m \rangle \; = \; \sum_{a,b} p(m|a,b) \, \rho(a,b)$$

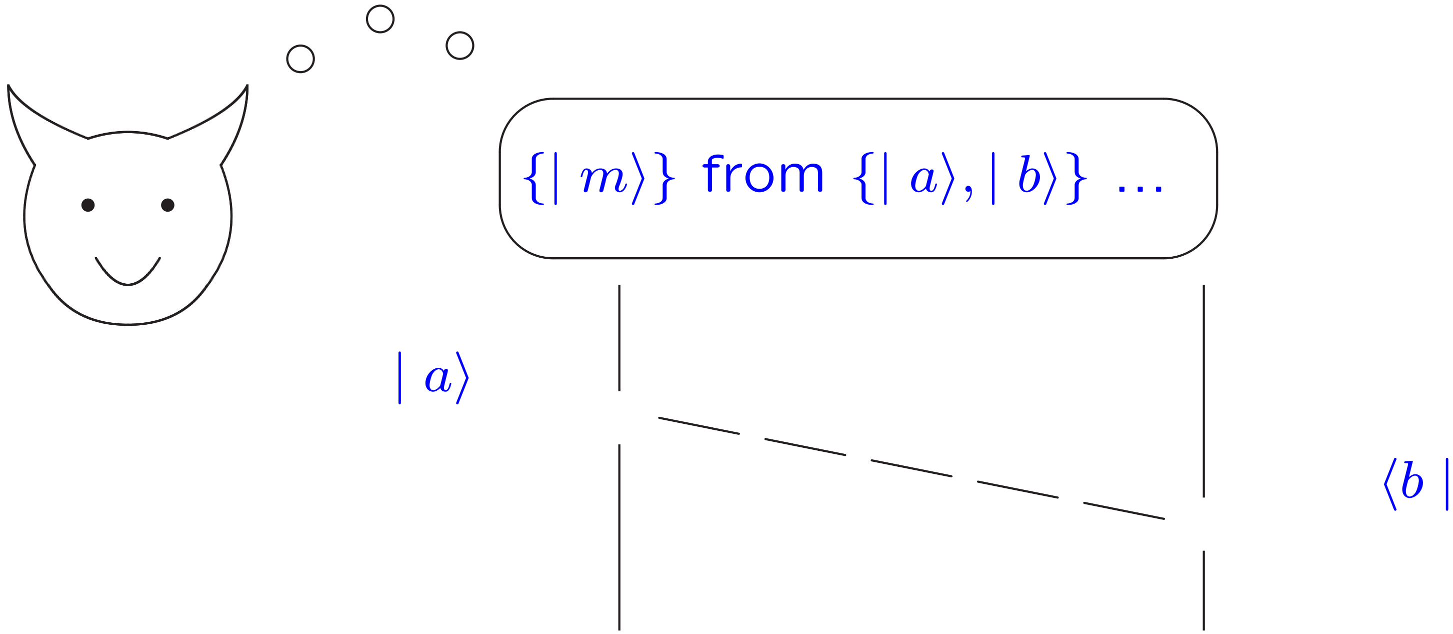
The complex conditional probability is given by the weak value

$$p(m|a,b) = \frac{\langle b \mid m \rangle \langle m \mid a \rangle}{\langle b \mid a \rangle}$$

This relation is a universal and state independent description of how (a, b) determines m.

Laplacian determinism in weak measurements

Preparation of a and measurement of b completely determine the accessible reality of each individual system.

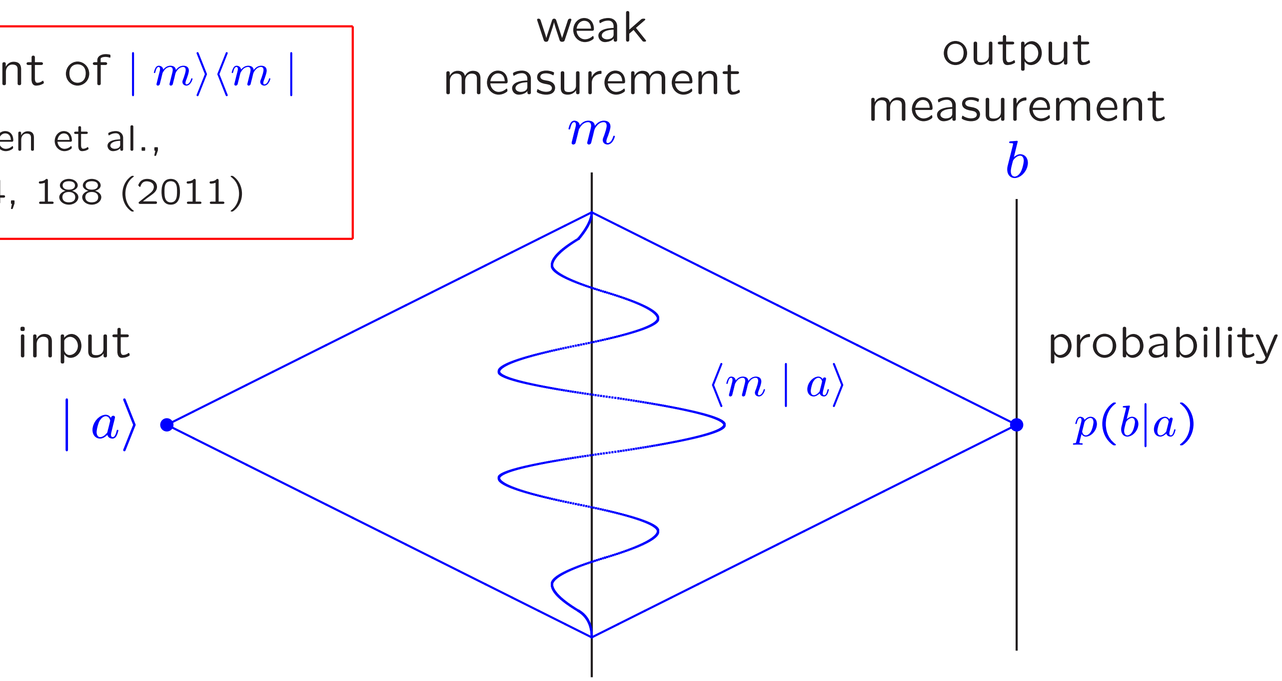


Weak measurements identify the universal relation between the actual reality (a, b) and the potential reality m.

Quantum coherence as complex probability

Measurement of $|\, m\rangle\langle m \,|$

Lundeen et al.,
Nature 474, 188 (2011)



a,b, and m all represent potential realities of the quantum system.

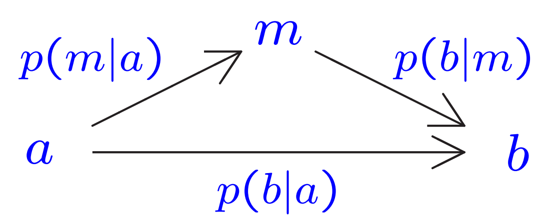
The universal relation of a,b, and m is $p(m|a,b) = \sqrt{\frac{p(b|m)}{p(b|a)}}\langle m \mid a\rangle$

Beyond dualism: the physics of complex probabilities

The state vector is a redundant concept - all relations can be expressed by complex conditional probabilities!

$|\langle m \mid a\rangle|^2 = p(m|a)$ is replaced by

$$|p(m|a,b)|^2 \, p(b|a) = p(b|m) \, p(m|a)$$



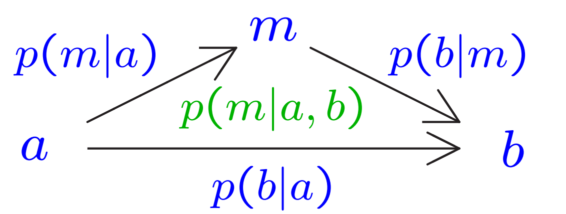
This rule relates the conditional probabilities to the back-action dynamics of m-measurements (quantum ergodicity).

The law of quantum ergodicity

The ergodic probability $p(m|a)$ is defined as the probability of m averaged over transformations along constant a.

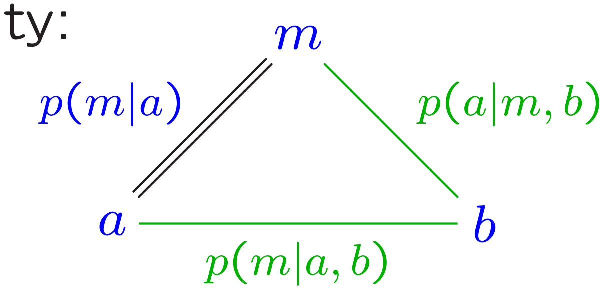
Back-action form of quantum ergodicity:

$$|p(m|a,b)|^2\, p(b|a) = p(b|m)\, p(m|a)$$



Gauge reference form of quantum ergodicity:

$$p(a|m,b)\, p(m|a,b) = p(m|a)$$



The origins of Hilbert space

Complex conditional probabilities are deterministic, because (a, b) and (a, m) are related by

$$\sum_m p(a'|m, b)\, p(m|a, b) = \delta_{a,a'}$$

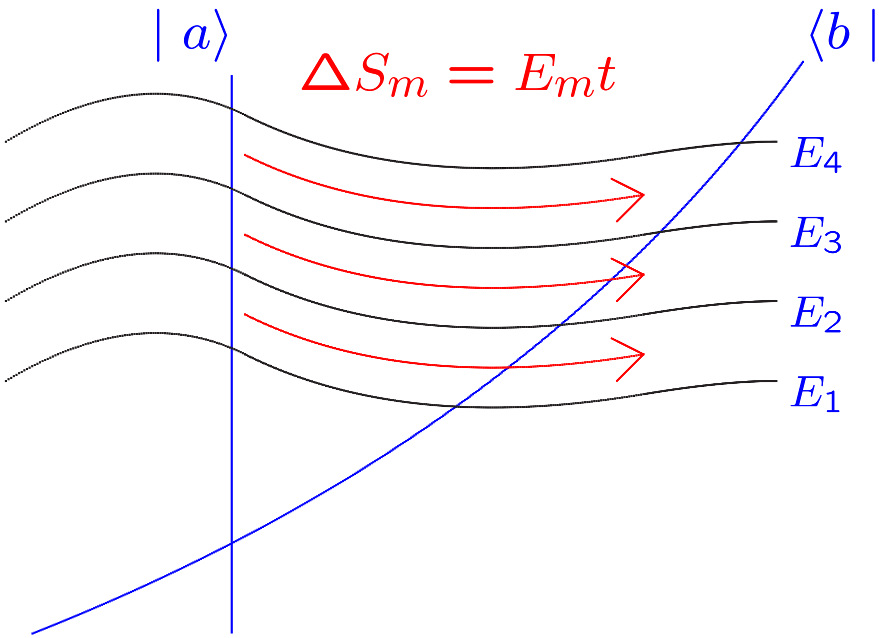
According to quantum ergodicity, the contributions of (m, b) to (a, b) in the sum for $a' = a$ are independent of b,

$$p(a|m, b)\, p(m|a, b) = p(m|a)$$

Mathematically, these two relations define an inner product of a and m, where b only contributes to the complex phases.

Complex phases in action

Complex probability unifies static probability and the physcis of motion and force, with $\hbar$ as fundamental action-phase ratio:



$$p(b;t) = \left| \sum_m p(m|a,b)\, \mathrm{e}^{-i\frac{E_m t}{\hbar}} \right|^2 \times |\langle b \mid a\rangle|^2$$

There is no instantaneous reality $b(t)$ - there are only complex probabilities describing the relations between potential measurements.

Transformation distance as logical tension

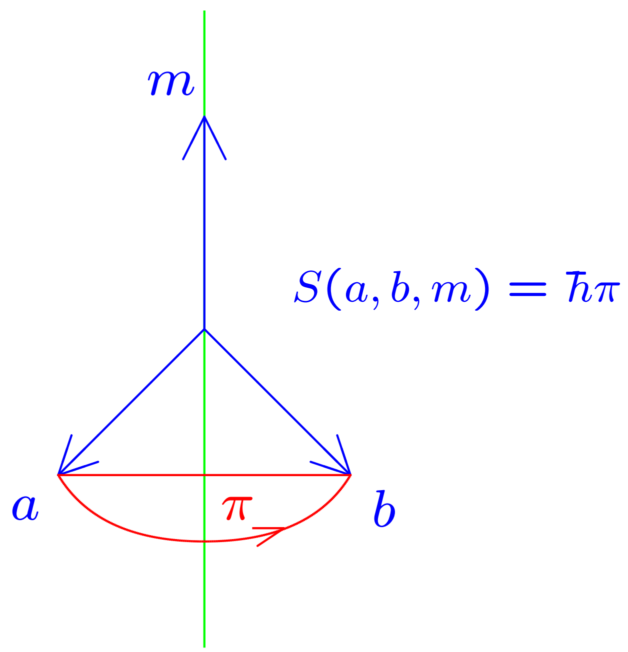
$p(m|a,b)$ determines the transformation dynamics between a and b for transformations conserving m,

$$|\langle b \mid \hat{U} \mid a\rangle|^2 = \left|\sum_m p(m|a,b)\ \mathrm{e}^{-i\phi_m}\right|^2 |\langle b \mid a\rangle|^2$$

The complex phases ϕ_m represent the action of the transformation in units of $\hbar$. The action that maximizes the outcome b for an initial state a is a measure of distance between a and b along m,

logical tension: $\quad S(a,b,m) = \hbar\ \mathrm{Arg}(p(m|a,b))$

The physics of negative probabilities



If the optimal transformation from a to b along m is half-periodic, action-phases of π correspond to negative conditional probabilities,

$$p(m|a,b) = -\sqrt{\frac{p(b|m)p(m|a)}{p(b|a)}}$$

Negative probabilities of entangling interactions

XX-states in the YY-basis

$$
\begin{aligned}
\mid ++\rangle &= \tfrac{1}{2}(\mid 00\rangle + \mid 01\rangle + \mid 10\rangle + \mid 11\rangle) \\
\mid +-\rangle &= \tfrac{1}{2}(\mid 00\rangle - \mid 01\rangle + \mid 10\rangle - \mid 11\rangle) \\
\mid -+\rangle &= \tfrac{1}{2}(\mid 00\rangle + \mid 01\rangle - \mid 10\rangle - \mid 11\rangle) \\
\mid --\rangle &= \tfrac{1}{2}(\mid 00\rangle - \mid 01\rangle - \mid 10\rangle - \mid 11\rangle)
\end{aligned}
$$

$$
\mid E\rangle = \tfrac{1}{2}(\mid 00\rangle + \mid 01\rangle + \mid 10\rangle - \mid 11\rangle)
$$

The entangled state $\mid E\rangle$ can be transformed into local XX-states by half-periodic transformations in the YY-basis.

Negative probabilities of entangling interactions

Action-Phases of the entangled state E:

	00	01	10	11
$++$	0	0	0	π
$+-$	0	π	0	0
$-+$	0	0	π	0
$--$	π	0	0	0

The entangled state $\mid E\rangle$ can be transformed into local XX-states by half-periodic transformations in the YY-basis.

Negative probabilities in Bell's inequality violations

Joint probabilities for $\langle XY \rangle = \langle YX \rangle = \sin\theta$

$\times\frac{1}{8}$	00	01	10	11
$++$	$\sin\theta$	$1-\cos\theta$	$1-\cos\theta$	$-\sin\theta$
$+-$	$1+\cos\theta$	$-\sin\theta$	$\sin\theta$	$1+\cos\theta$
$-+$	$1+\cos\theta$	$\sin\theta$	$-\sin\theta$	$1+\cos\theta$
$--$	$-\sin\theta$	$1-\cos\theta$	$1-\cos\theta$	$\sin\theta$

$$p(K=+2) = \tfrac{1}{8}(4 + 4\cos\theta + 4\sin\theta) > 1$$

$$p(K=-2) = \tfrac{1}{8}(4 - 4\cos\theta - 4\sin\theta) < 0$$

Resolution of (all) quantum paradoxes

The fundamental relation between physical properties that cannot be measured at the same time is given by complex conditional probabilities, where the complex phase describes the action of transformations between the three properties.

Paradoxes are constructed from separately measured correlations that reveal the effects of negative joint probabilities when added.

Negative probabilities are a result of the dynamical relations between physical properties that cannot be measured jointly.

Conclusions

The laws of physics should be re-formulated in terms of complex conditional probabilities, e.g. for classical laws of motion,

$$p(x(t_3)|x(t_1), x(t_2))$$

This law of motion *replaces* the classical trajectory $x(t)$, since there is no simultaneous reality of $x(t_1)$, $x(t_2)$, and $x(t_3)$ in quantum mechanics.

Complex probabilities relate alternative potential realities through the action-phases of their transformation. Reality cannot be separated from the interaction dynamics by which it is perceived!