

# Quantum Frequency Conversion and Temporal-Mode Multiplexing of States of Light

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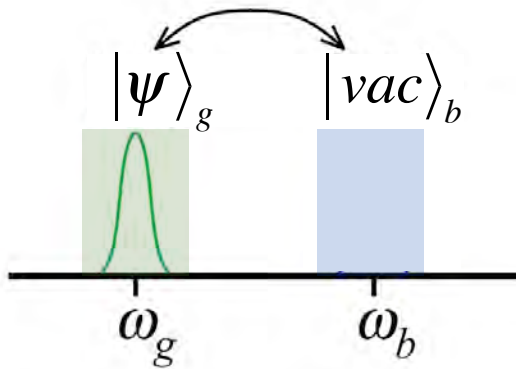
Lasse Mejling and Karsten Rottwitt  
Technical University of Denmark

Conference on Quantum Information and  
Quantum Control (CQIQC-V) Toronto 2013

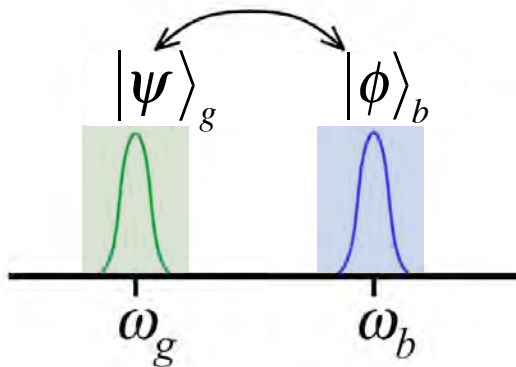


## Quantum Frequency Conversion (QFC):

The complete or partial exchange of quantum states between two spectral bands.



$$|\psi\rangle_g |vac\rangle_b \mapsto |vac\rangle_g |\psi\rangle_b$$



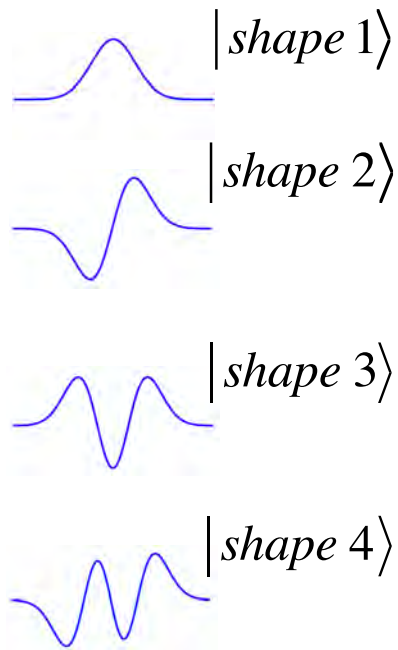
$$|\psi\rangle_g |\phi\rangle_b \mapsto \alpha |\psi\rangle_g |\phi\rangle_b + \beta |\phi\rangle_g |\psi\rangle_b$$

note: need phase coherence for the latter

# Potential Uses of Single-Photon States

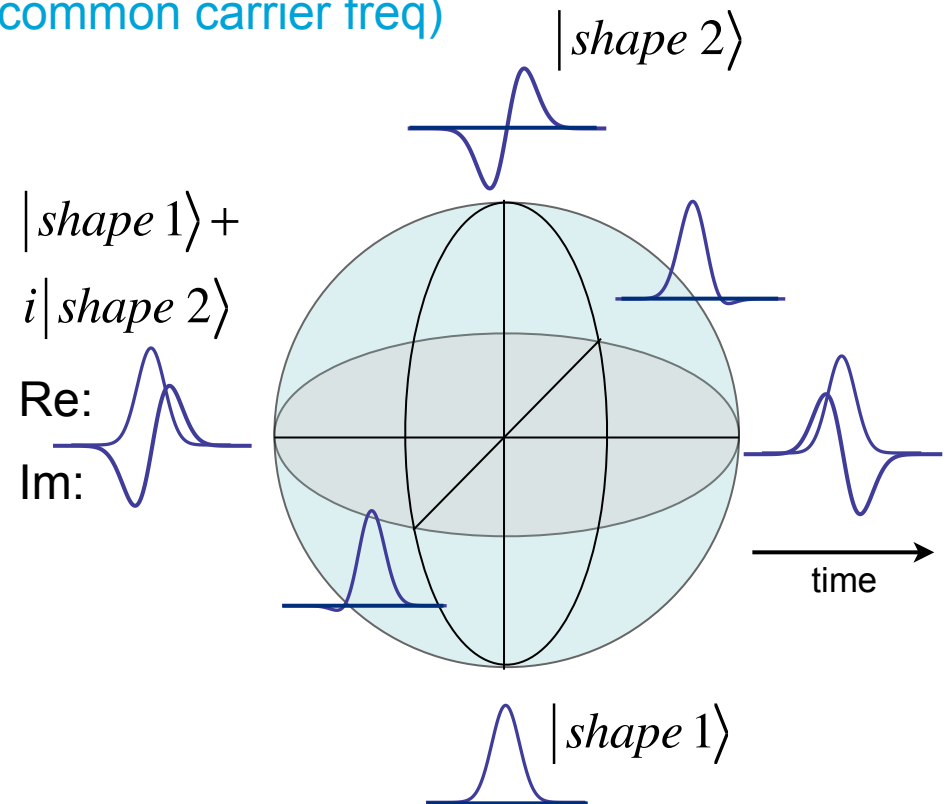
## A. Many Classical Bits in Single Photon

(common carrier freq)



## B. Spectral-Temporal Photonic Qubit

Bloch sphere  
(common carrier freq)



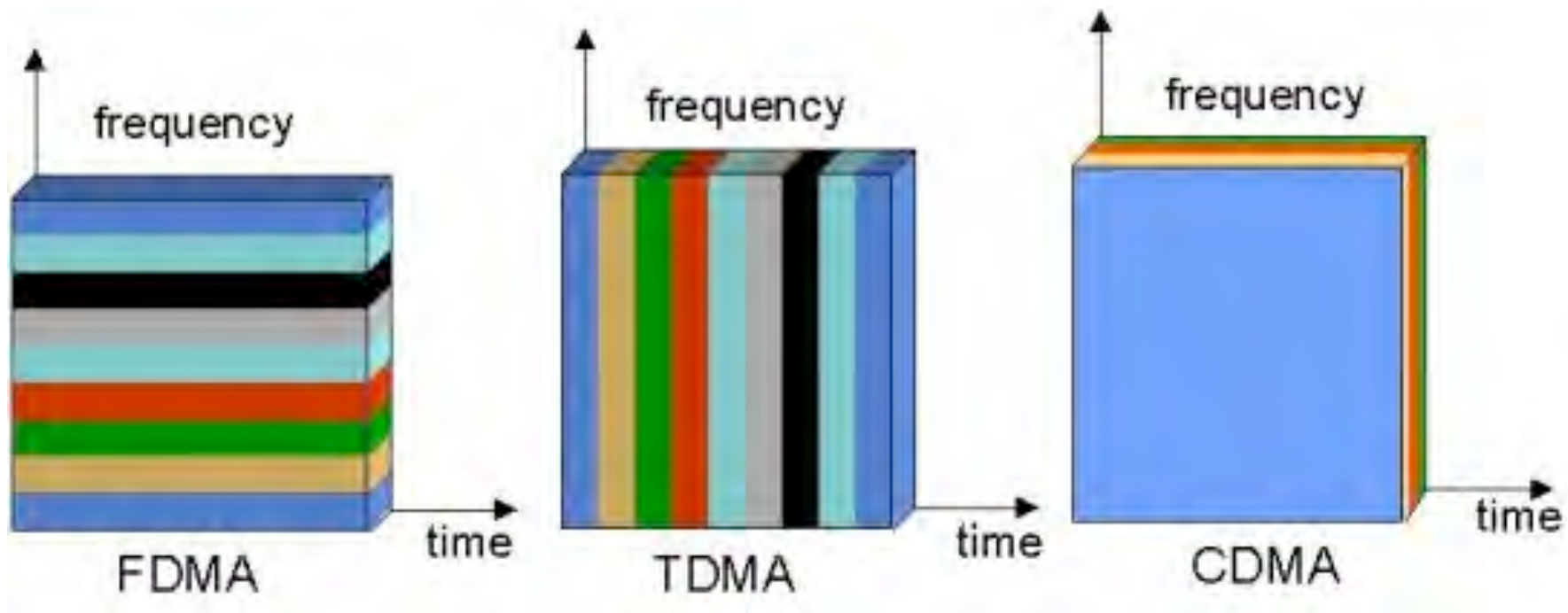
Need Pulse-Shape Multiplexing

# Commonly used multiplexing schemes in radio technology

Frequency-division multiple access

Time-division multiple access

Code-division multiple access

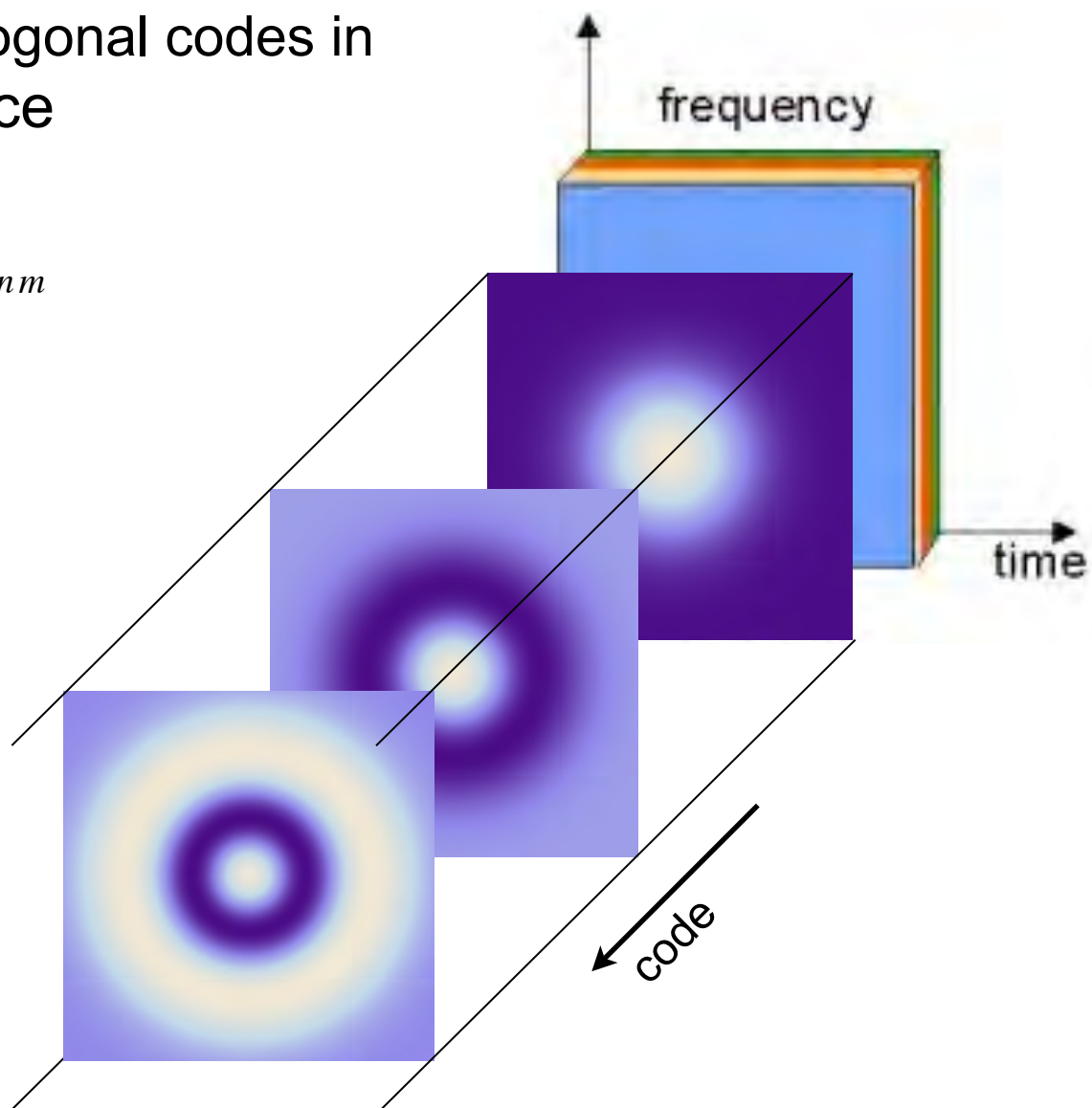
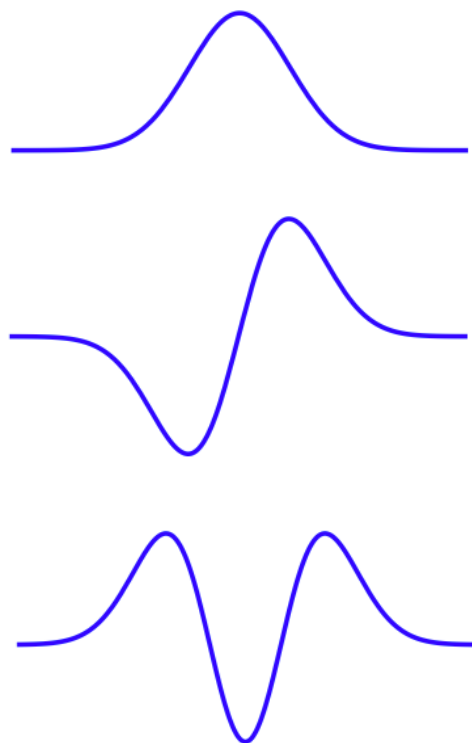


FDMA and TDMA use only time-frequency space

CDMA uses field-orthogonal codes in code space

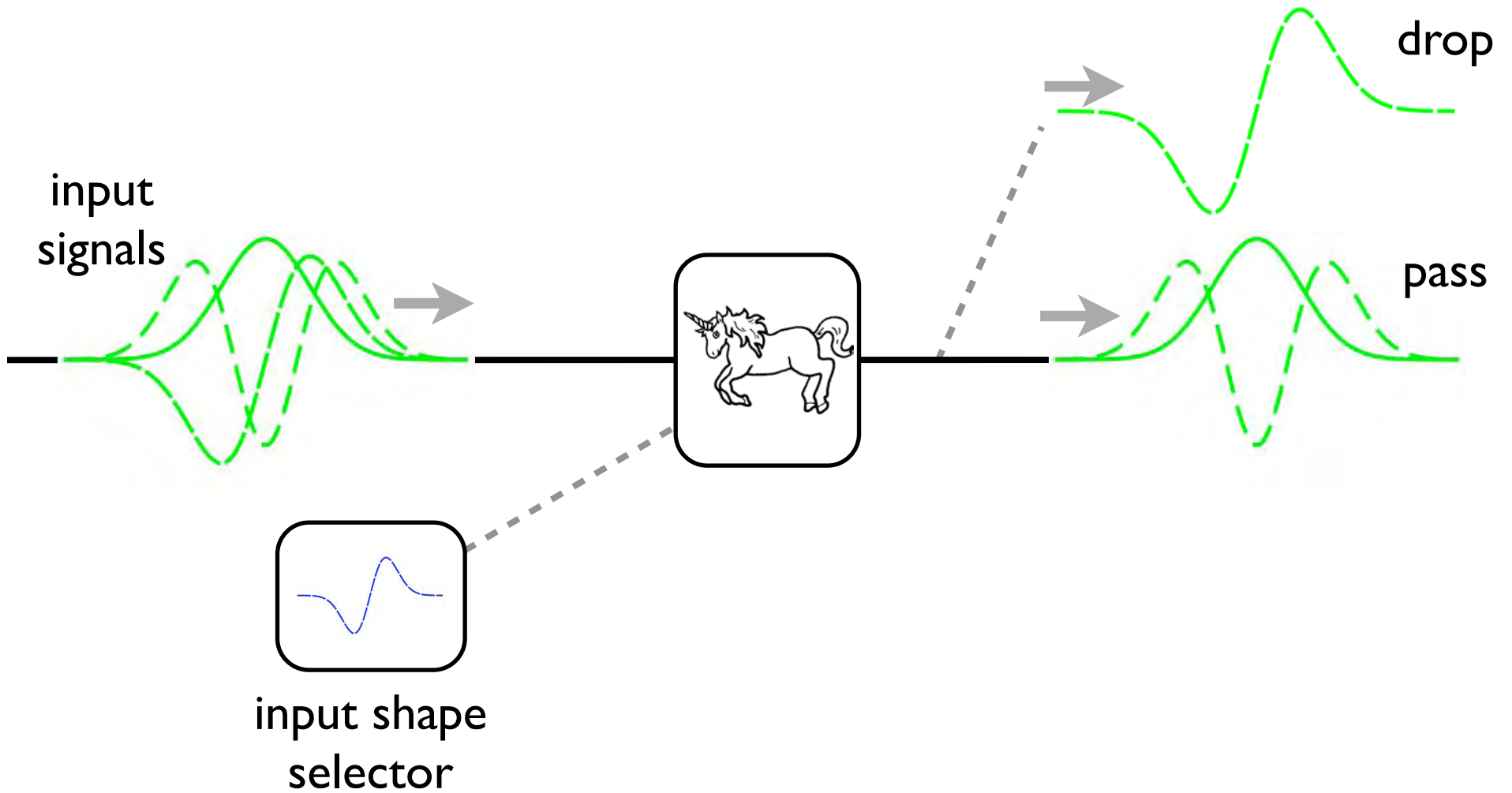
$$\int E_n^*(t)E_m(t)dt = \delta_{nm}$$

temporal modes

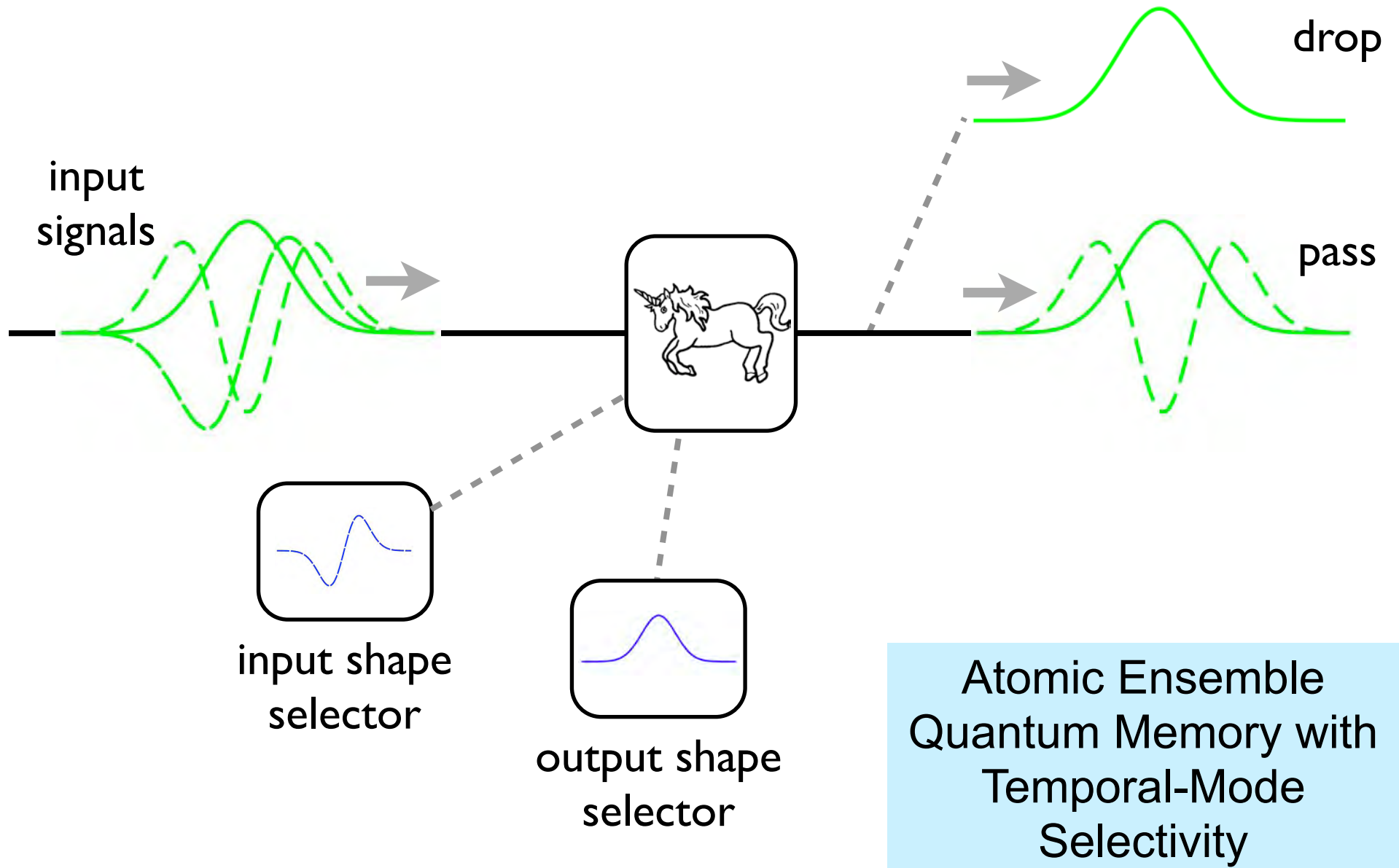


In radio, field-orthogonal codes are easily demultiplexed.  
In optical, there is **NO** known method to demultiplex field-orthogonal codes.  
efficiently

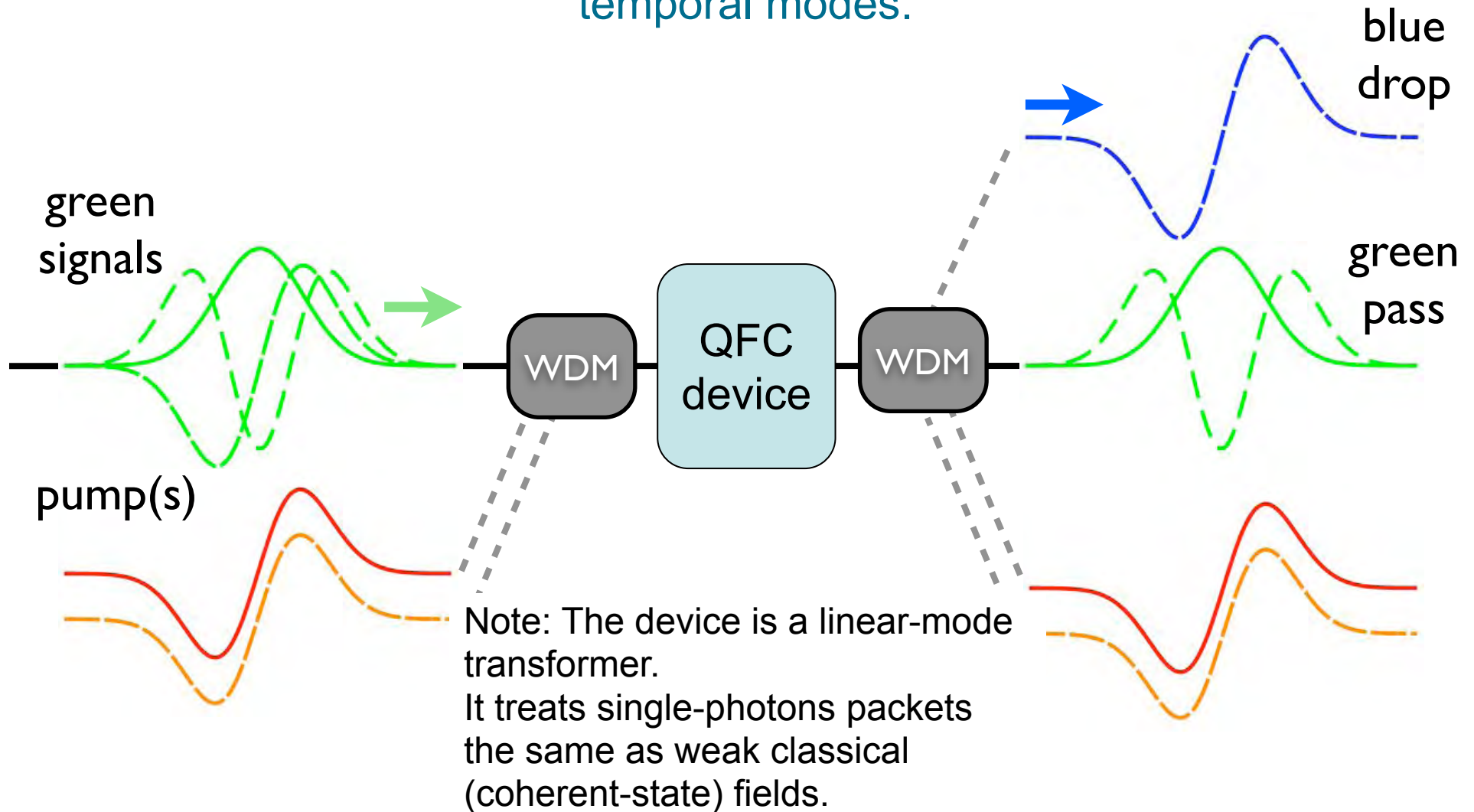
# The Mythical Device



# The Mythical Device with Optional Output Shape Control



# Nonlinear Optical Frequency Conversion: a potential method to spatially separate field-orthogonal temporal modes.



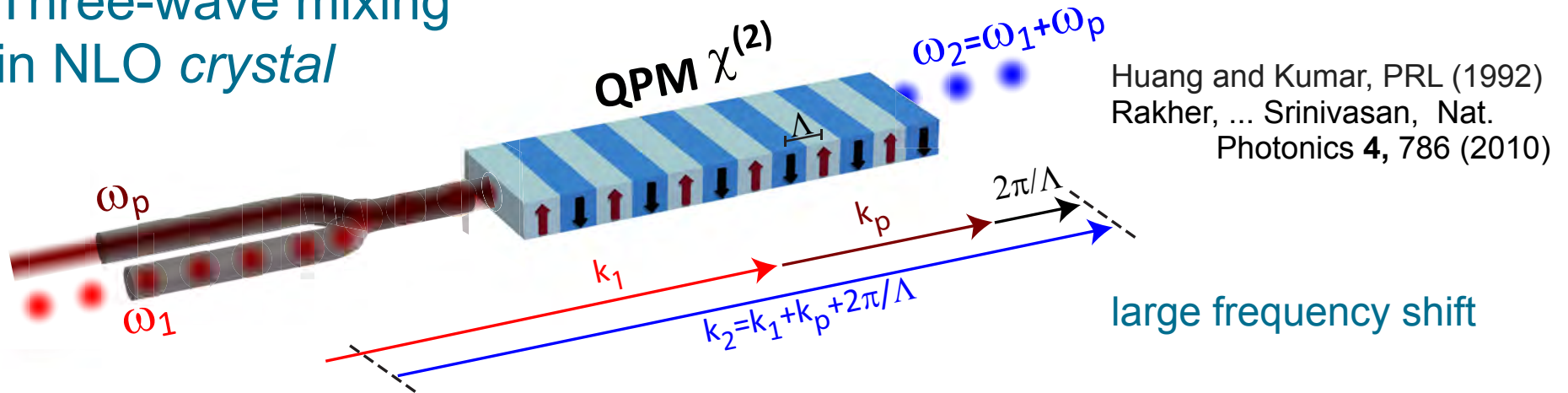
Three-wave mixing: Eckstein, Brecht, Silberhorn, *Opt. Express* 19, 13770 (2010)

Four-wave mixing: McKinstrie, Mejling, Raymer, Rottwitz, *Phys. Rev. A* 85, 053829 (2012)



# Methods for Quantum Frequency Conversion

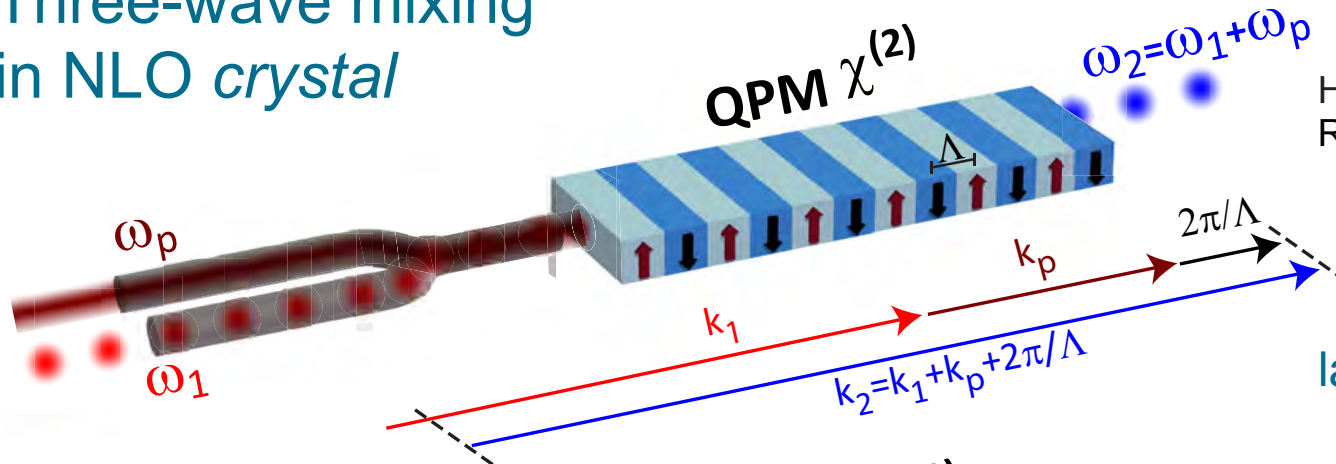
## Three-wave mixing in NLO crystal



from: MR and KS, Physics Today, **65**, 32 (2012)

# Methods for Quantum Frequency Conversion

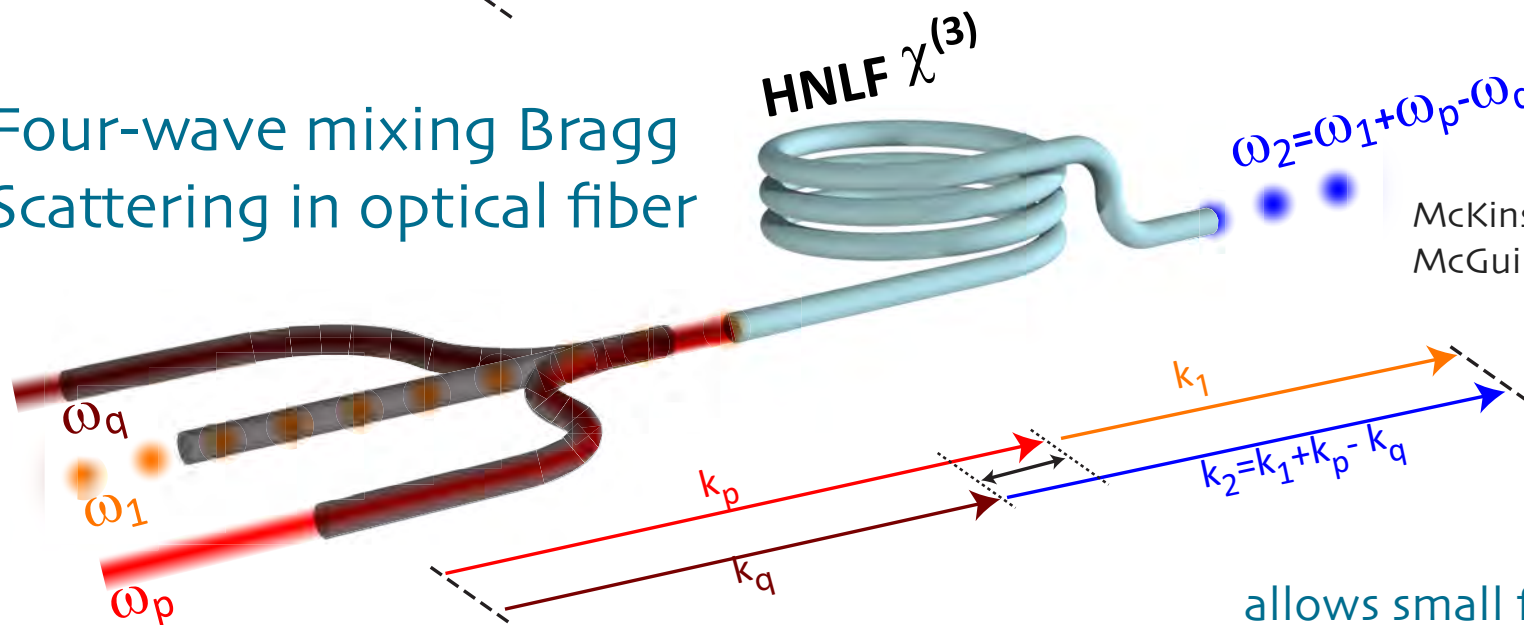
## Three-wave mixing in NLO crystal



Huang and Kumar, PRL (1992)  
Rakher, ... Srinivasan, Nat. Photonics 4, 786 (2010)

large frequency shift

## Four-wave mixing Bragg Scattering in optical fiber

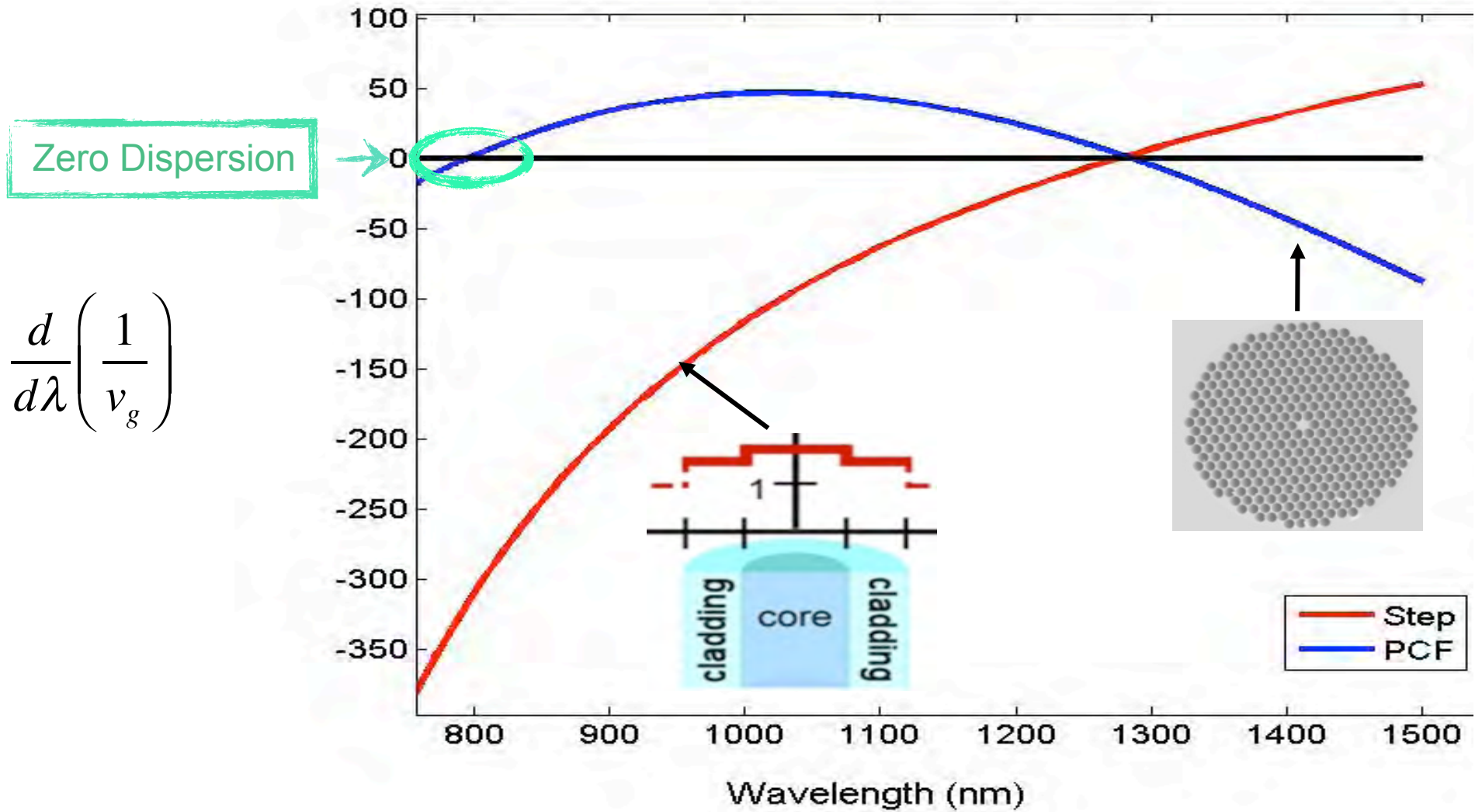


McKinstrie, Opt Ex (2005)  
McGuinness PRL (2010)

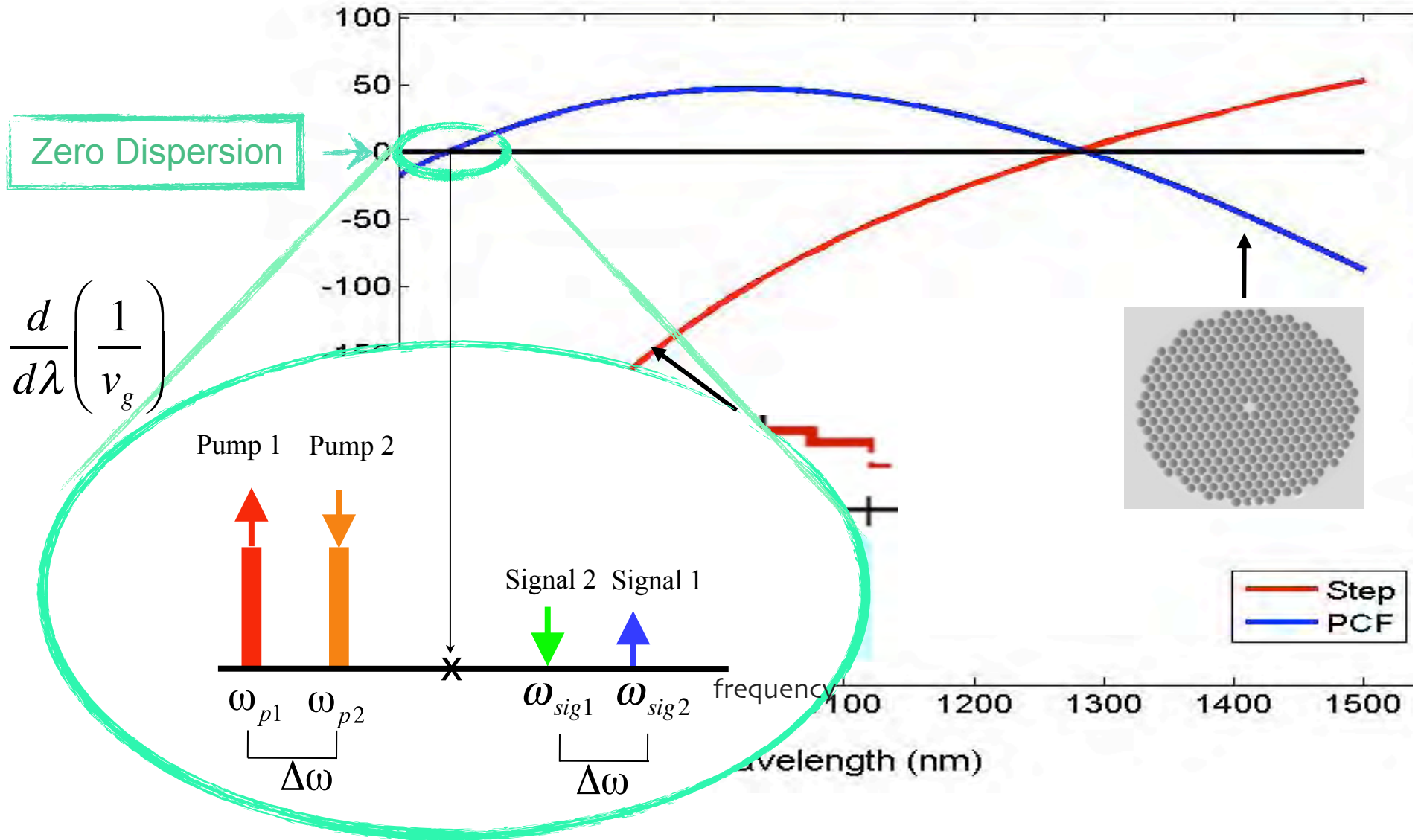
allows small frequency shift

from: MR and KS, Physics Today, **65**, 32 (2012)

# EXPERIMENTS: Dispersion of Step-Index Fiber and Photonic-Crystal Fiber

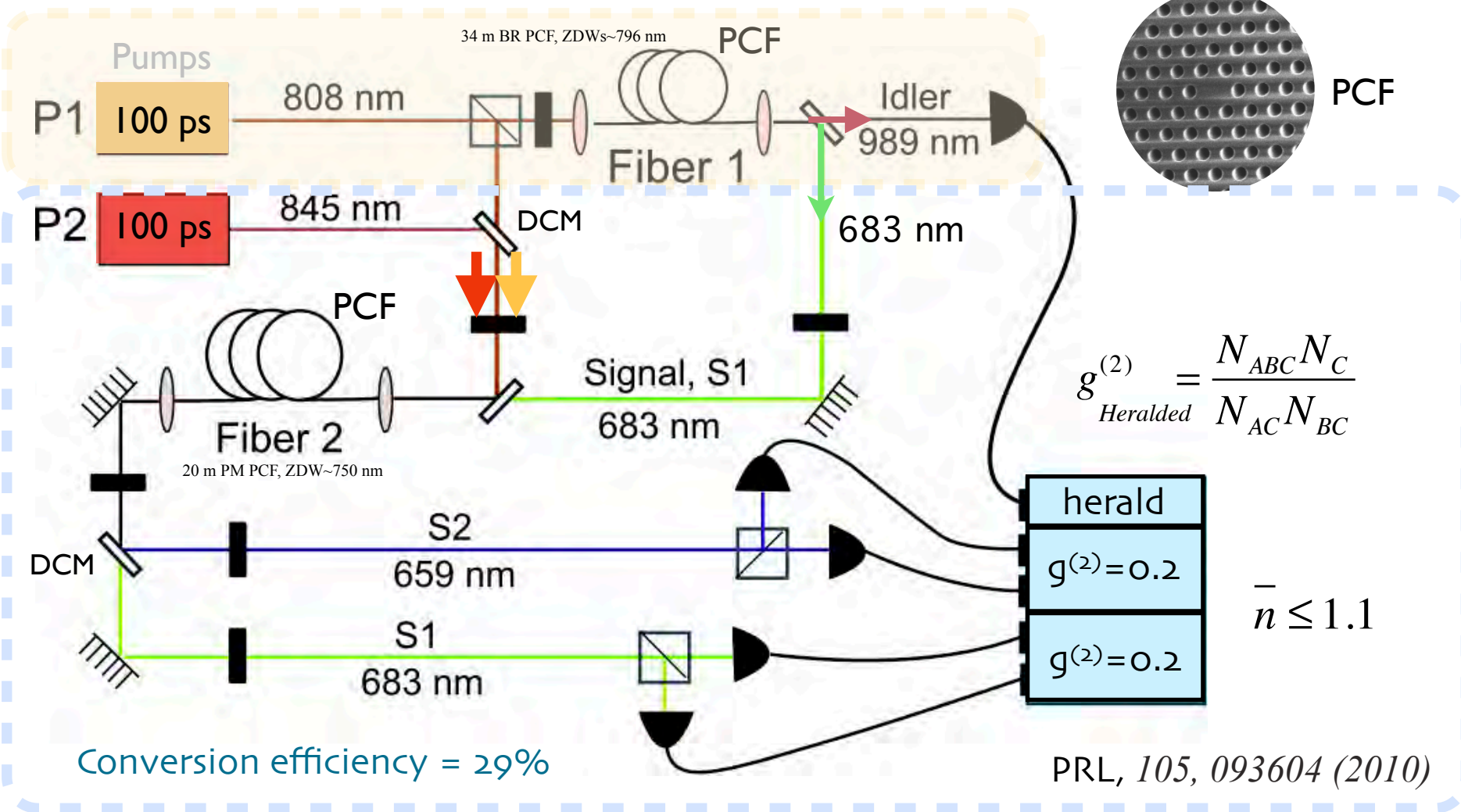
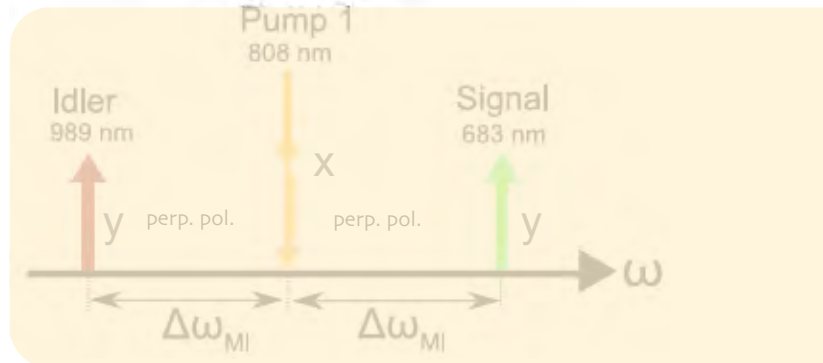
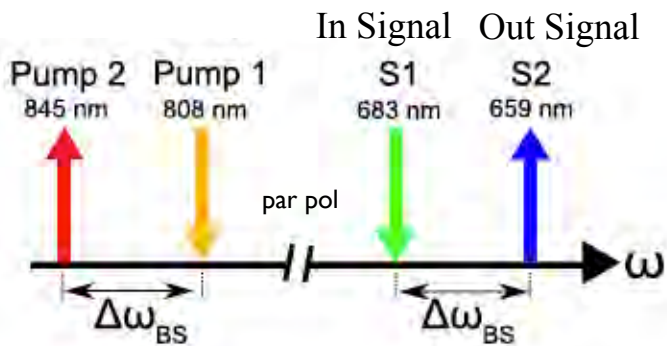


# EXPERIMENTS: Dispersion of Step-Index Fiber and Photonic-Crystal Fiber



Signal 1 has same group velocity as Pump 1.  
Signal 2 has same group velocity as Pump 2.

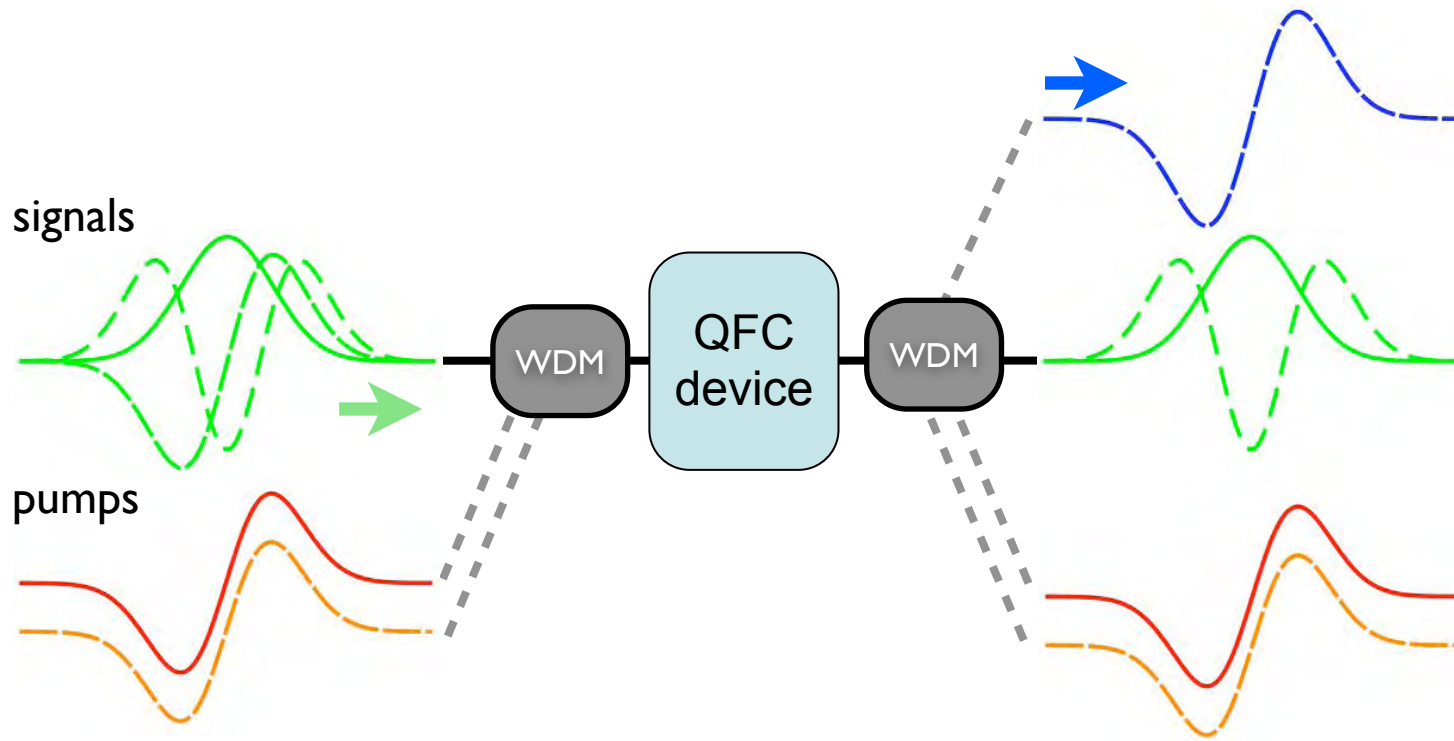
## 2. Conversion



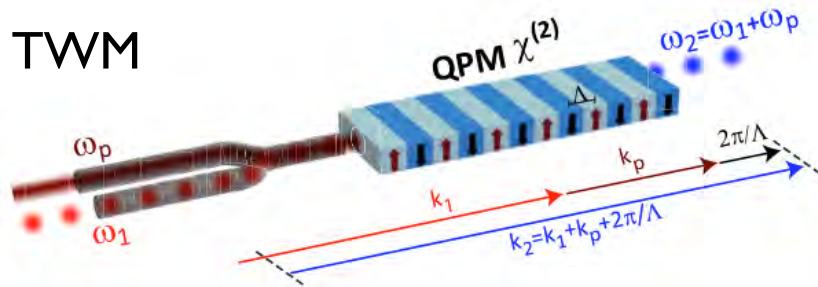
$$g_{Heralded}^{(2)} = \frac{N_{ABC} N_C}{N_{AC} N_{BC}}$$



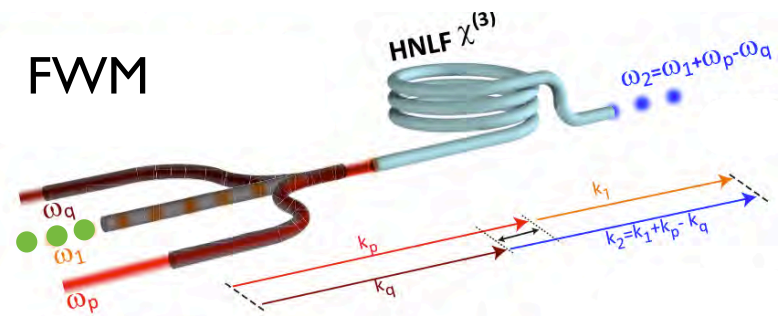
# How effective can QFC be in separating field-orthogonal optical codes?



TWM



FWM



## Modeling QFC by Nonlinear Wave Mixing

$$\begin{aligned}
 \left( \frac{\partial}{\partial z} + \frac{1}{v_g} \frac{\partial}{\partial t} \right) A_g(z,t) &= i\gamma P(z,t) A_b(z,t) \\
 \left( \frac{\partial}{\partial z} + \frac{1}{v_b} \frac{\partial}{\partial t} \right) A_b(z,t) &= i\gamma P^*(z,t) A_g(z,t)
 \end{aligned}$$

pump shapes  
 $P_{TWM}(z,t) = A_p^*(z,t)$   
 $P_{FWM}(z,t) = A_{p1}^*(z,t)A_{p2}(z,t)$   
 pump/coupling  $\gamma$

The equations are linear in  $A_g$  and  $A_b$  signal field operators.

**Solution:**

$$\begin{pmatrix} A_g(t) \\ A_b(t) \end{pmatrix}_{OUT} = \int^t dt' \begin{pmatrix} G_{gg}(t,t') & G_{gb}(t,t') \\ G_{bg}(t,t') & G_{bb}(t,t') \end{pmatrix} \begin{pmatrix} A_g(t') \\ A_b(t') \end{pmatrix}_{IN}$$

**All quantum correlations can be calculated from Green functions.**

Four-wave mixing: McGuinness, MR, CM, Opt. Express 19, 17876 (2011)

Three-wave mixing: Reddy, MR, CM, AM, KR, Opt. Express 21, 13840 (2013)

Christ, Brecht, Mauerer, Silberhorn (NJP 2013)

# Schmidt Mode Decomposition of the Green functions (singular-value decomposition)

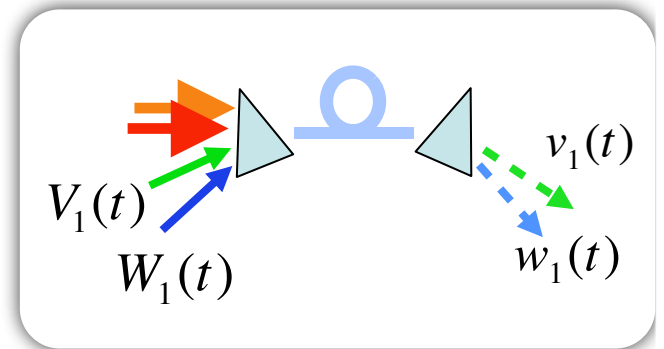
$$\begin{pmatrix} A_g(t) \\ A_b(t) \end{pmatrix}_{OUT} = \sum_n \int^t dt' \begin{pmatrix} \tau_n v_n(t) V_n^*(t') & i\rho_n v_n(t) W_n^*(t') \\ i\rho_n w_n(t) V_n^*(t') & \tau_n w_n(t) W_n^*(t') \end{pmatrix} \begin{pmatrix} A_g(t') \\ A_b(t') \end{pmatrix}_{IN}$$

**with**  $\rho_n^2 + \tau_n^2 = 1$     $\rho_n^2 = \text{conversion}$ ,  $\tau_n^2 = \text{nonconversion}$

Temporal Schmidt modes reduce problem to low-dimensional state space:

$$\text{if } \begin{pmatrix} A_g(t') \\ A_b(t') \end{pmatrix}_{IN} = \begin{pmatrix} a_g V_1(t') \\ a_b W_1(t') \end{pmatrix}$$

$$\text{then } \begin{pmatrix} A_g(t) \\ A_b(t) \end{pmatrix}_{OUT} = \begin{pmatrix} (\tau_1 a_g + i\rho_1 a_b) v_1(t) \\ (i\rho_1 a_g + \tau_1 a_b) w_1(t) \end{pmatrix}$$

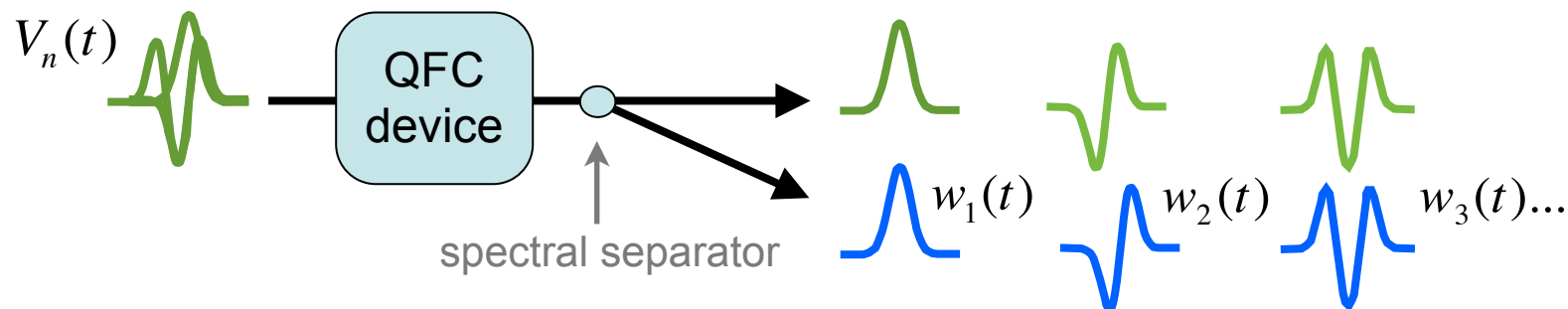


Operators undergo a pair-wise beam-splitter transformation



## Figure of Merit for Temporal-mode Selectivity

$$G_{bg}(t, t') = i \sum_n \rho_n w_n(t) V_n^*(t')$$



$$\eta_n = |\rho_n|^2 = \text{conversion efficiency}$$

$$\text{separability} \equiv \frac{\eta_{\text{Target}}}{\sum_n \eta_n} \leq 1$$

$$S \equiv \text{Selectivity} \equiv \text{separability} \times \eta_{\text{Target}}$$

$$S = \frac{|\eta_{\text{Target}}|^2}{\sum_n \eta_n} \leq 1$$

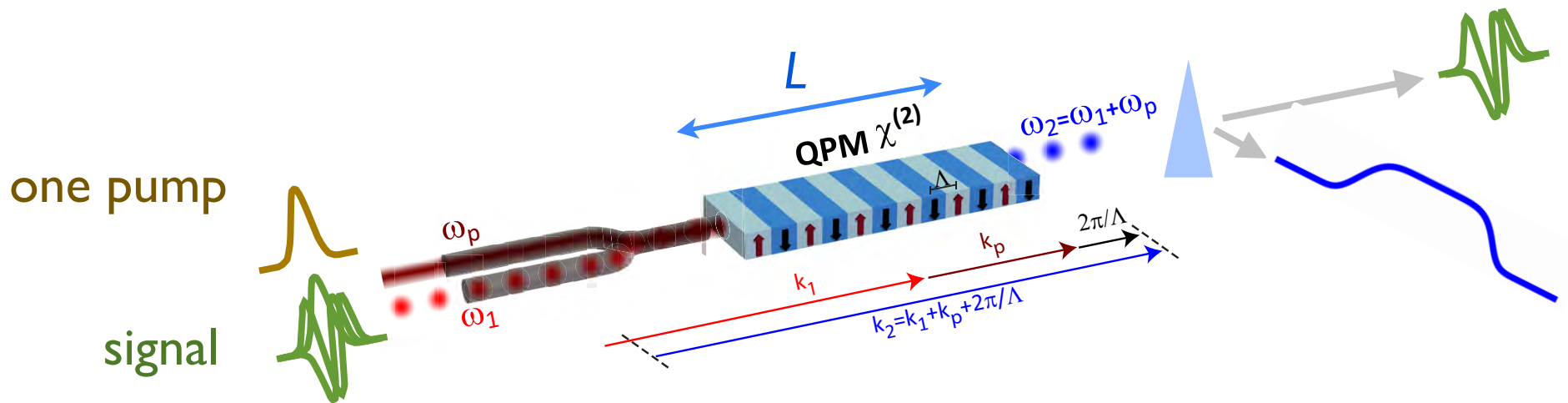
ideally:  $S = 1$

Reddy, MR, CM, AM, KR,  
Opt. Express 21, 13840 (2013)

Apply to three-wave mixing:

# Three-wave Mixing

Optimum case: Pump velocity matches 'green' signal velocity. Blue is slower.

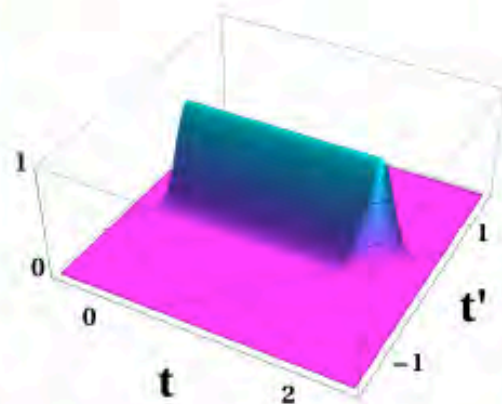
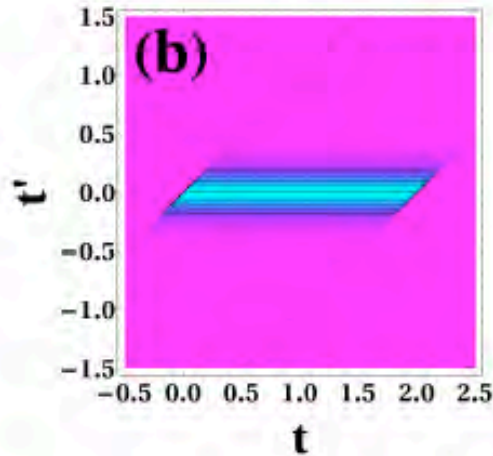


Reddy, MR, CM, LM, KR,  
Opt. Express 21, 13840 (2013)

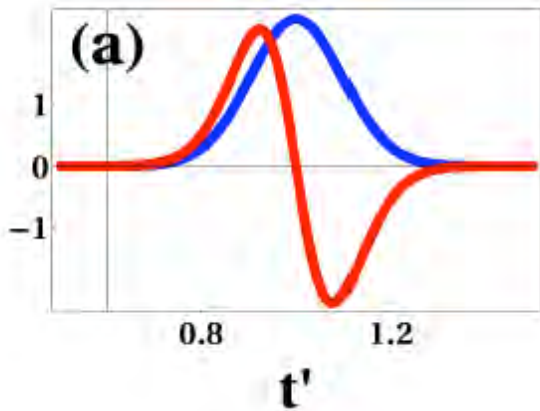
# Three-wave Mixing - Low conversion efficiency ultrashort pump pulse

Optimum case: Pump velocity matches green signal velocity

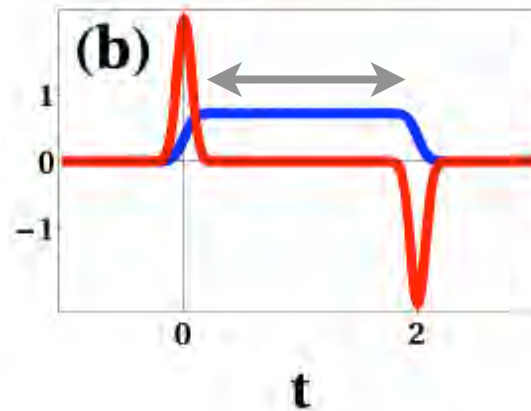
Green function



Schmidt modes: input



Schmidt modes: output



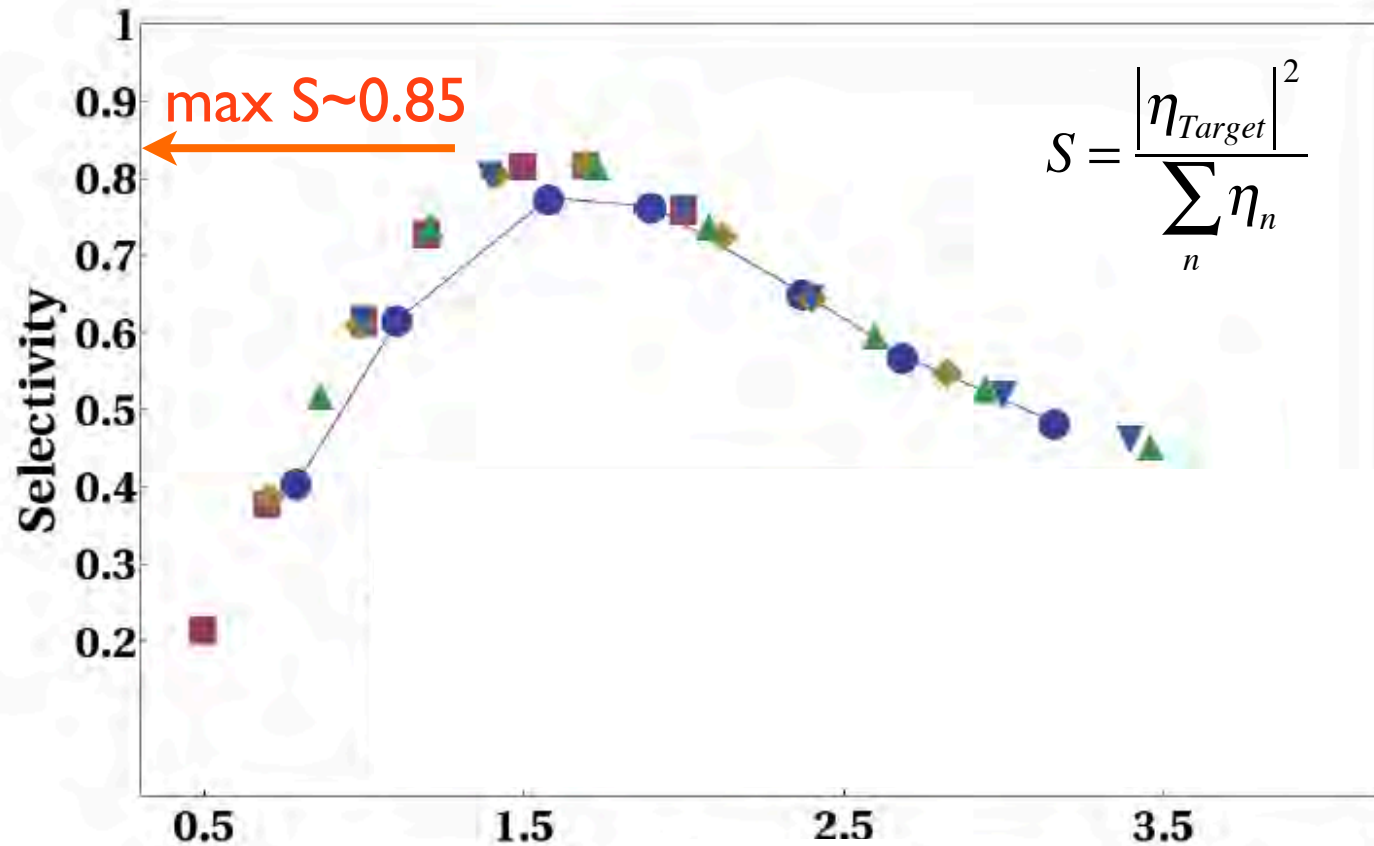
$$\text{duration} = \left( \frac{1}{v_g} - \frac{1}{v_b} \right) L$$

Separability is 0.94, but Selectivity is low when conversion efficiency is low.



# Three-wave Mixing - High conversion efficiency ultrashort pump pulse

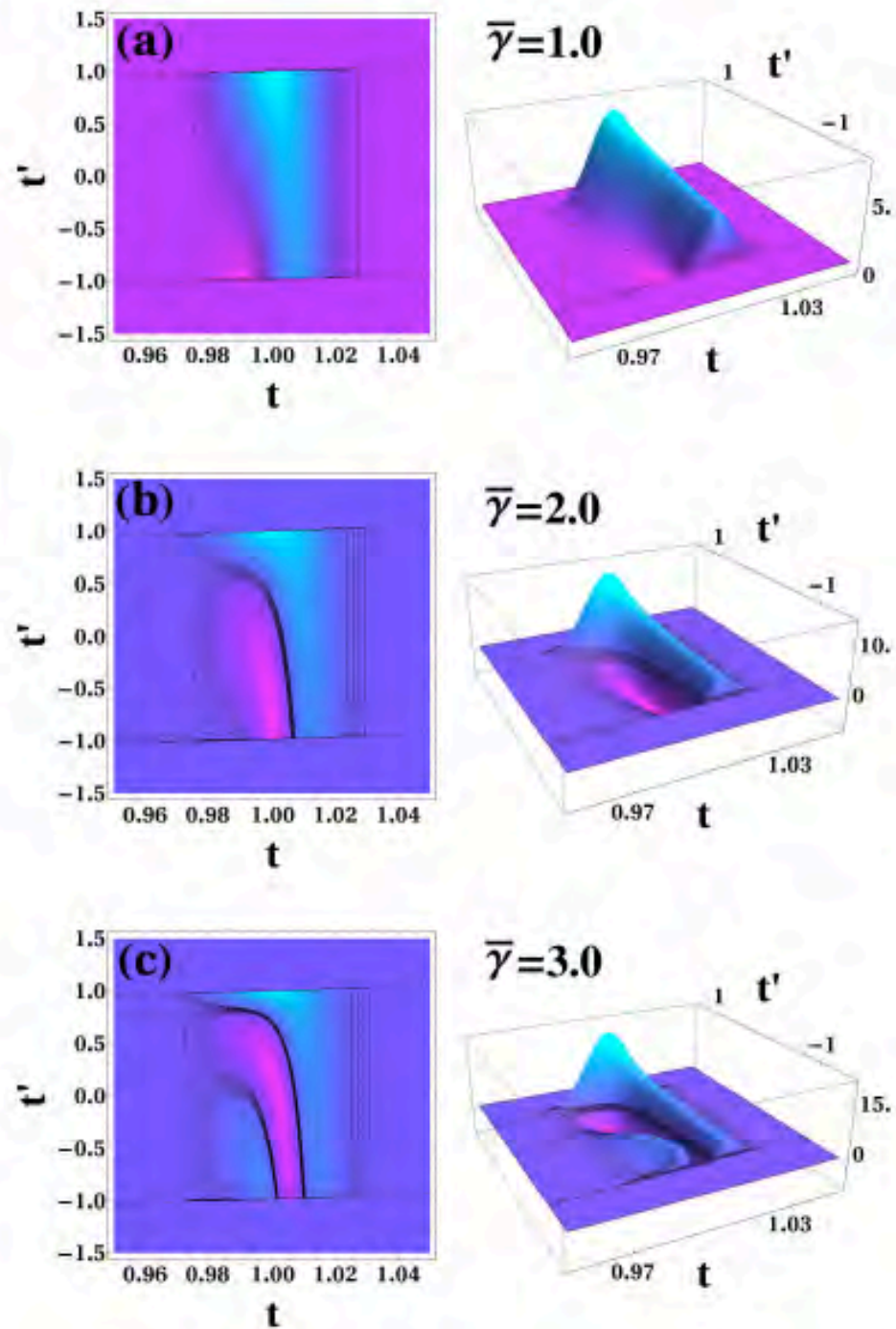
Optimum case: Pump  
velocity matches green  
signal velocity



pump strength,  $\gamma \sqrt{\frac{L}{\left(\frac{1}{v_g} - \frac{1}{v_b}\right)}}$

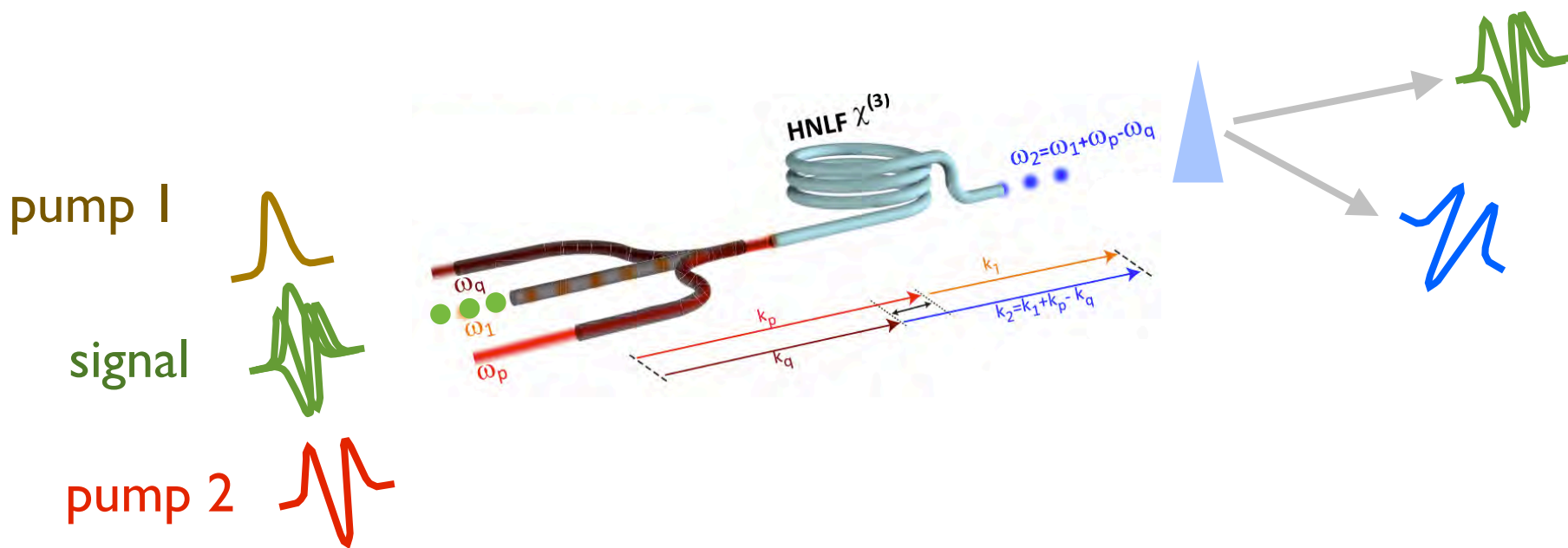
Origin of limited Selectivity:  
oscillations in the Green  
function make it  
non-separable.

Consequence of 'Rabi  
oscillations' between blue  
and green.



# Four-wave Mixing

One pump selects the input mode shape;  
Other pump determines the output mode shape.



CM, LM, MR, KR, PRA 053829 (2012)



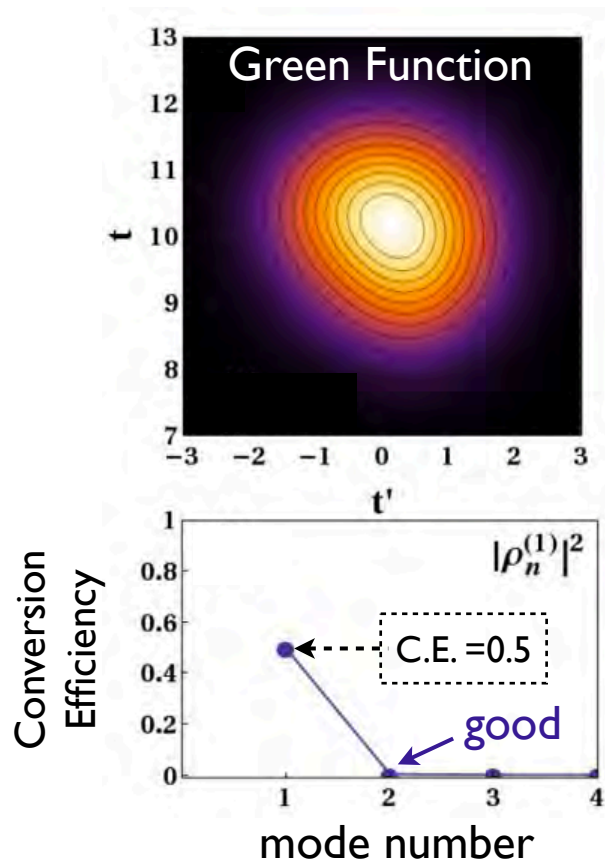
# Four-Wave Mixing

Much the same as TWM, with the shape of the medium replaced by the shape of the second pump.

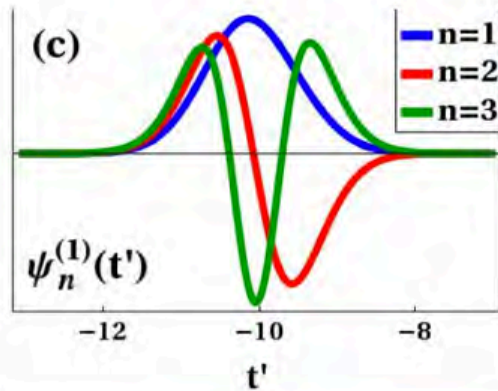


Optimum case: pump 1 velocity matches green signal velocity and pump 2 matches blue signal velocity. Complete collision occurs.

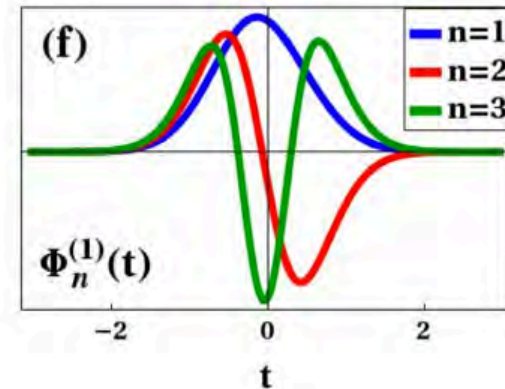
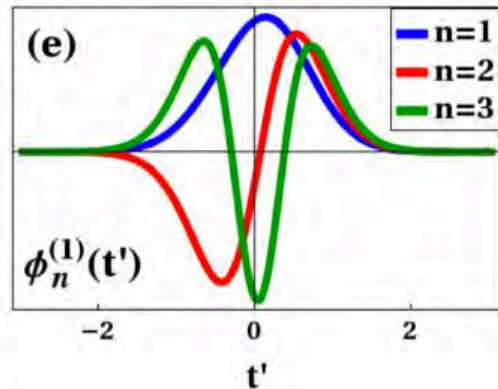
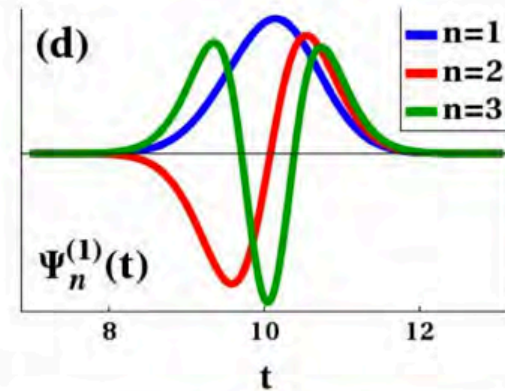
$$\bar{\gamma} = 0.83$$



Schmidt modes: input



Schmidt modes: output



Selectivity ~ 0.5

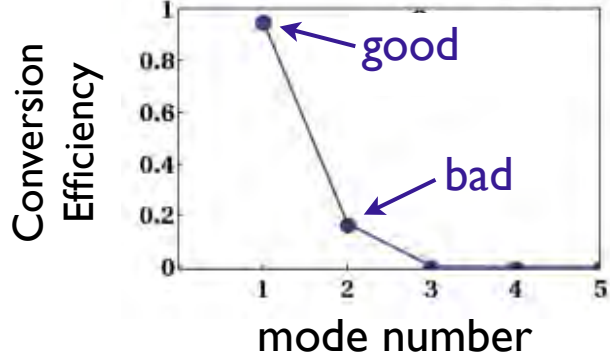
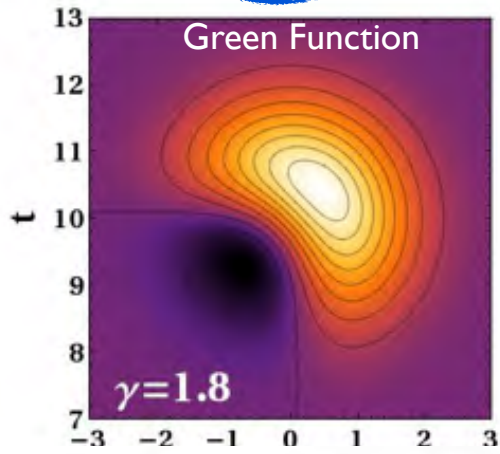
increase  $\bar{\gamma}$



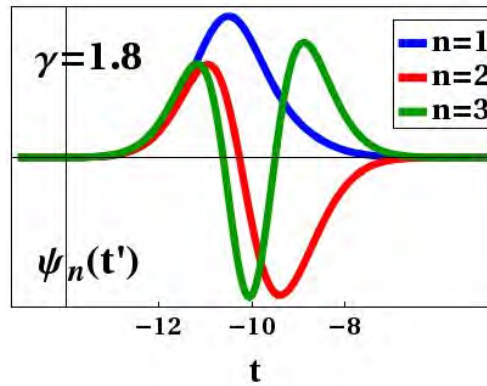
# Four-Wave Mixing



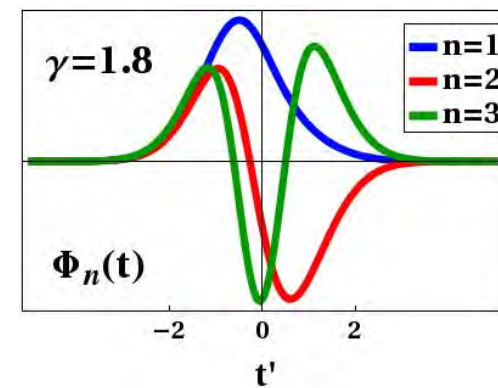
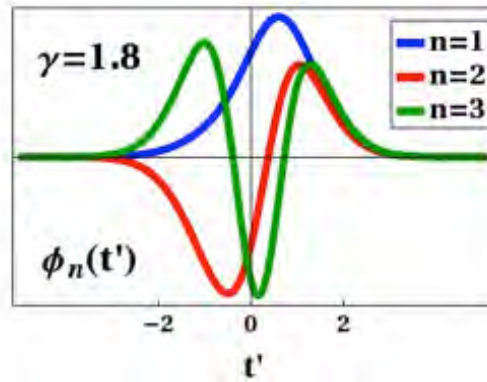
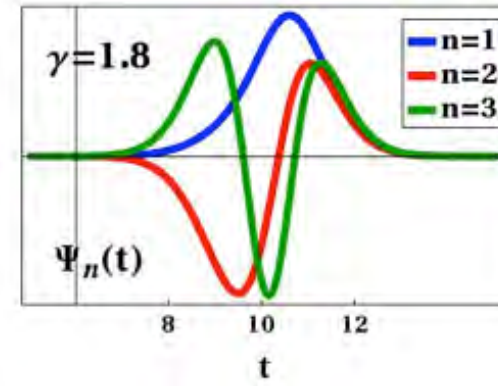
$\bar{\gamma} = 1.8$



Schmidt modes: input



Schmidt modes: output



Selectivity ~ 0.7

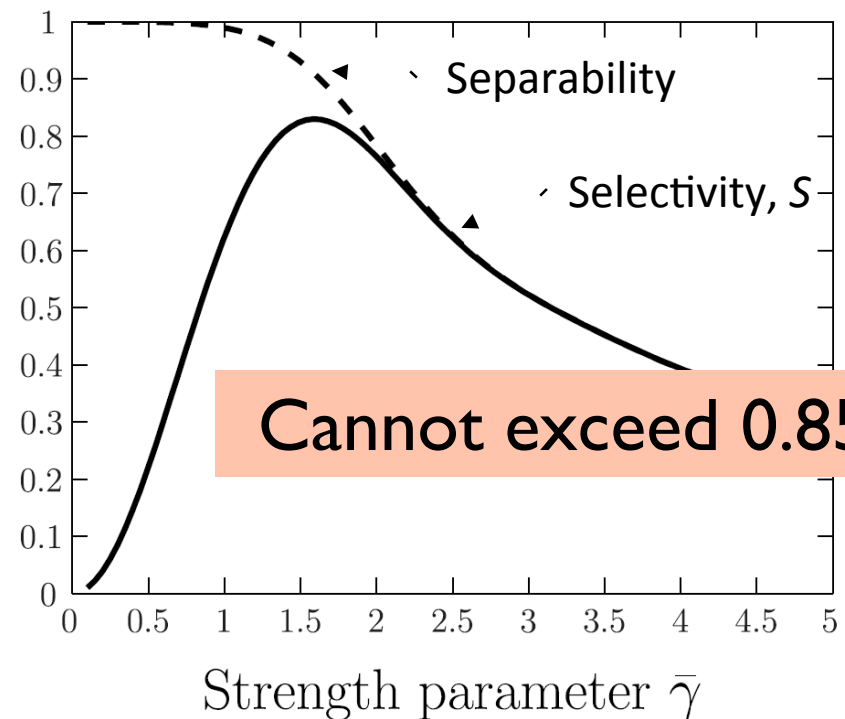
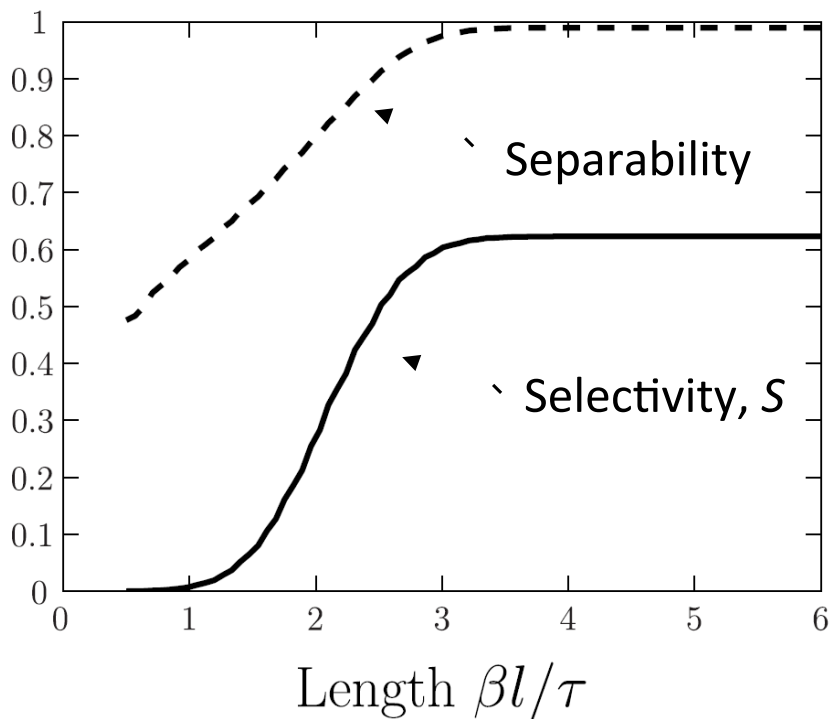
## Four-Wave Mixing

$$\eta_n = |\rho_n|^2 = \text{conversion efficiency}$$

$$S \equiv \text{Selectivity} \equiv \text{separability} \times \eta_{\text{Target}}$$

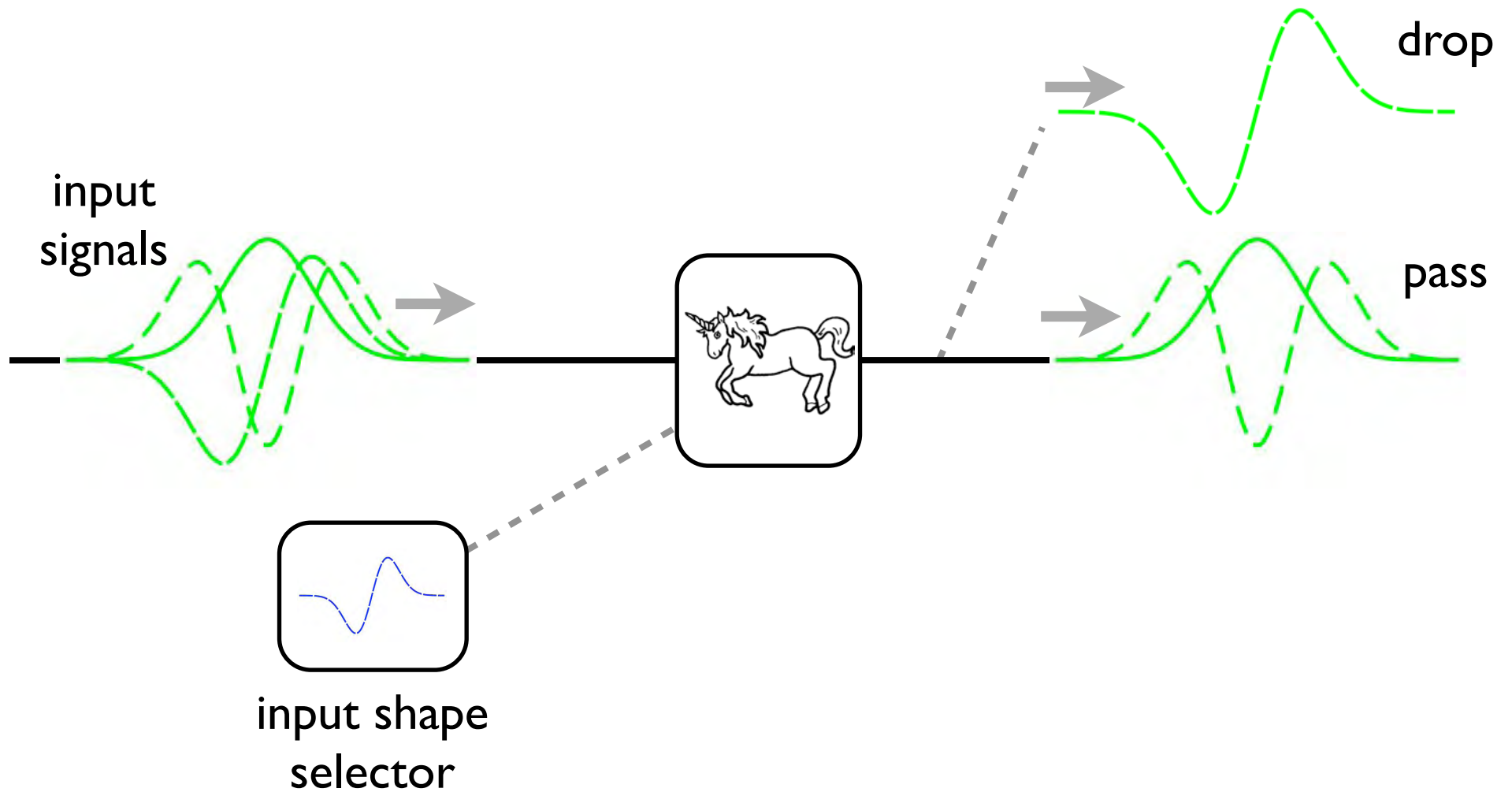
$$\text{separability} \equiv \frac{\eta_{\text{Target}}}{\sum_n \eta_n} \leq 1$$

$$S = \frac{|\eta_{\text{Target}}|^2}{\sum_n \eta_n} \leq 1$$

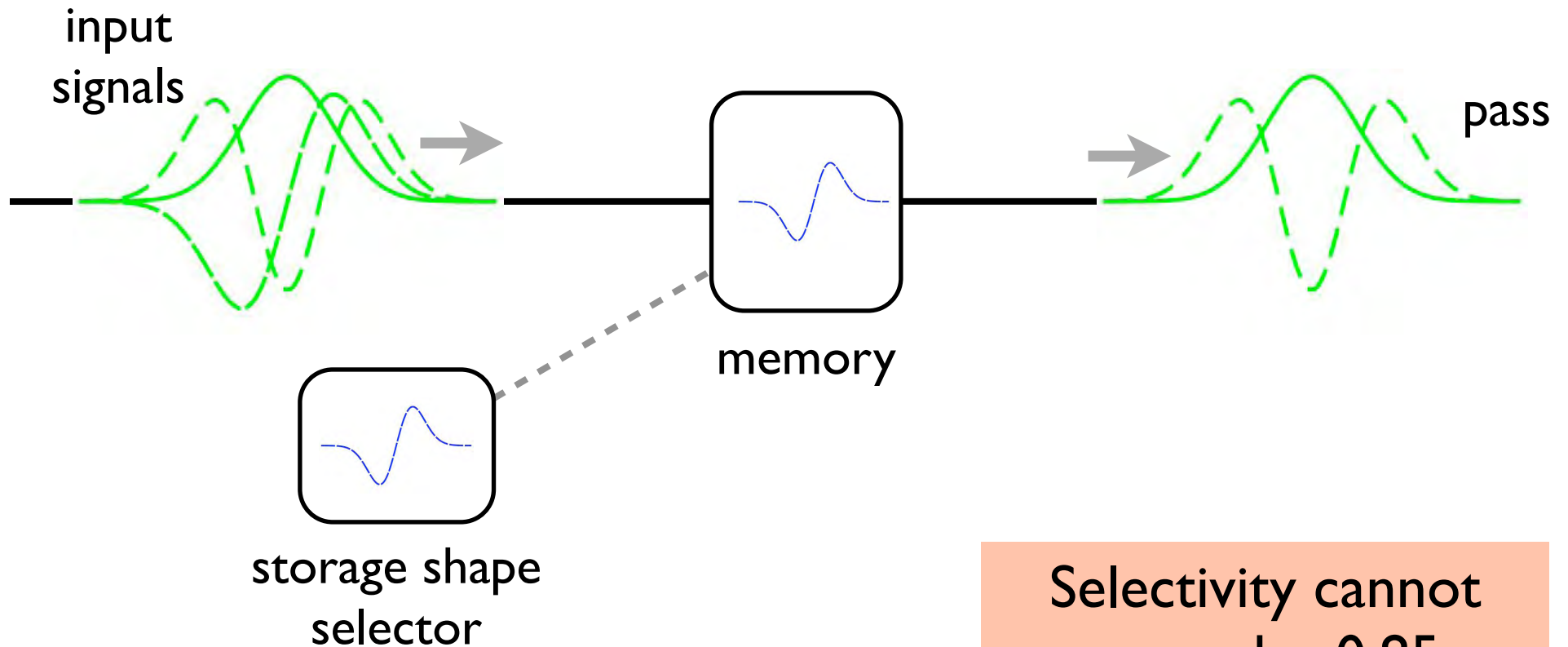


Cannot exceed 0.85

# Still Mythical: a drop device with 100% Selectivity



# Atomic Ensemble Quantum Memory with Temporal-Mode Selectivity

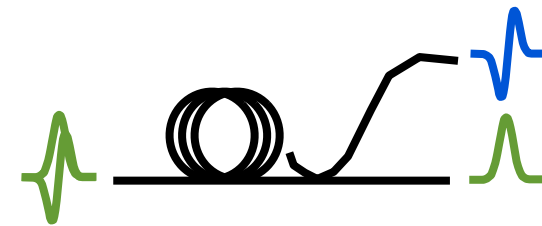


Selectivity cannot  
exceed  $\sim 0.85$   
J Nunn et al (2007)

Quantum conversion between near-frequency channels, for Quantum Internet



Temporal-mode-selective routing, shaping of single-photon qubits. Pulse-shape-division multiplexing with temporally orthogonal codes.



Dileep Reddy, University of Oregon

C. J. McKinstrie, Bell Laboratories

Lasse Mejling and Karsten Rottwitt  
Technical University of Denmark

NSF



unicorn

