Weak-values Technique for Velocity Measurements

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G. I. Viza et al. Opt Let 38 16 2949 (2013)

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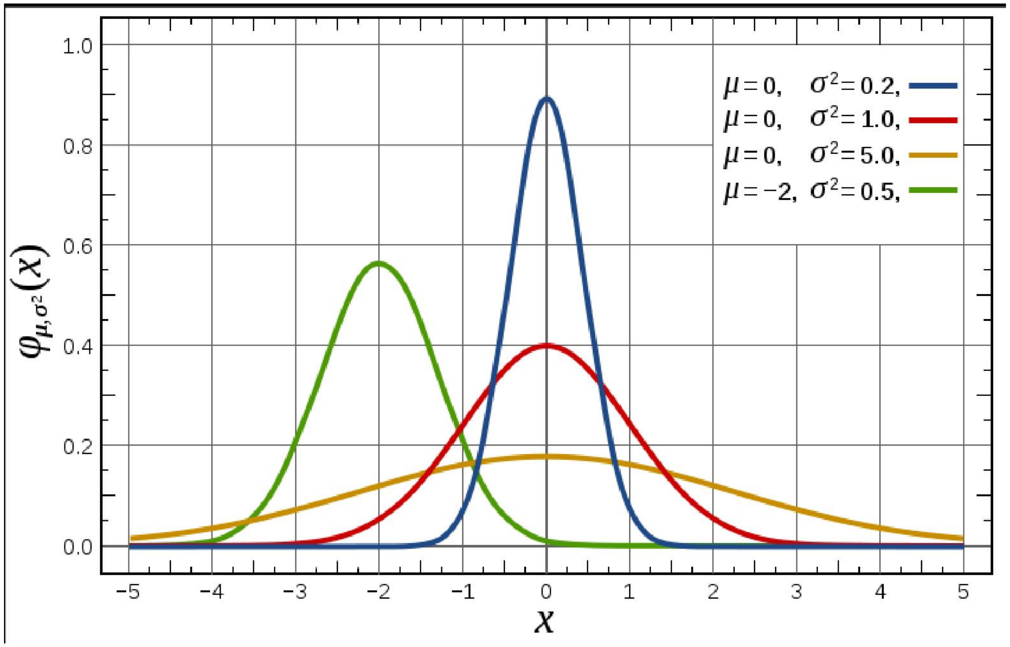
Why are velocity measurements important?

- Y. Yeh and H. Z. Cummins (1964)
  - 40 um/s
- Laser Doppler Anemometry
  - In-vivo medical imaging
  - Quality control vibrometry
- N. Brunner and C. Simmon PRL 105, 010405 (2010)
  - Measuring longitudinal phase with frequency-domain analysis

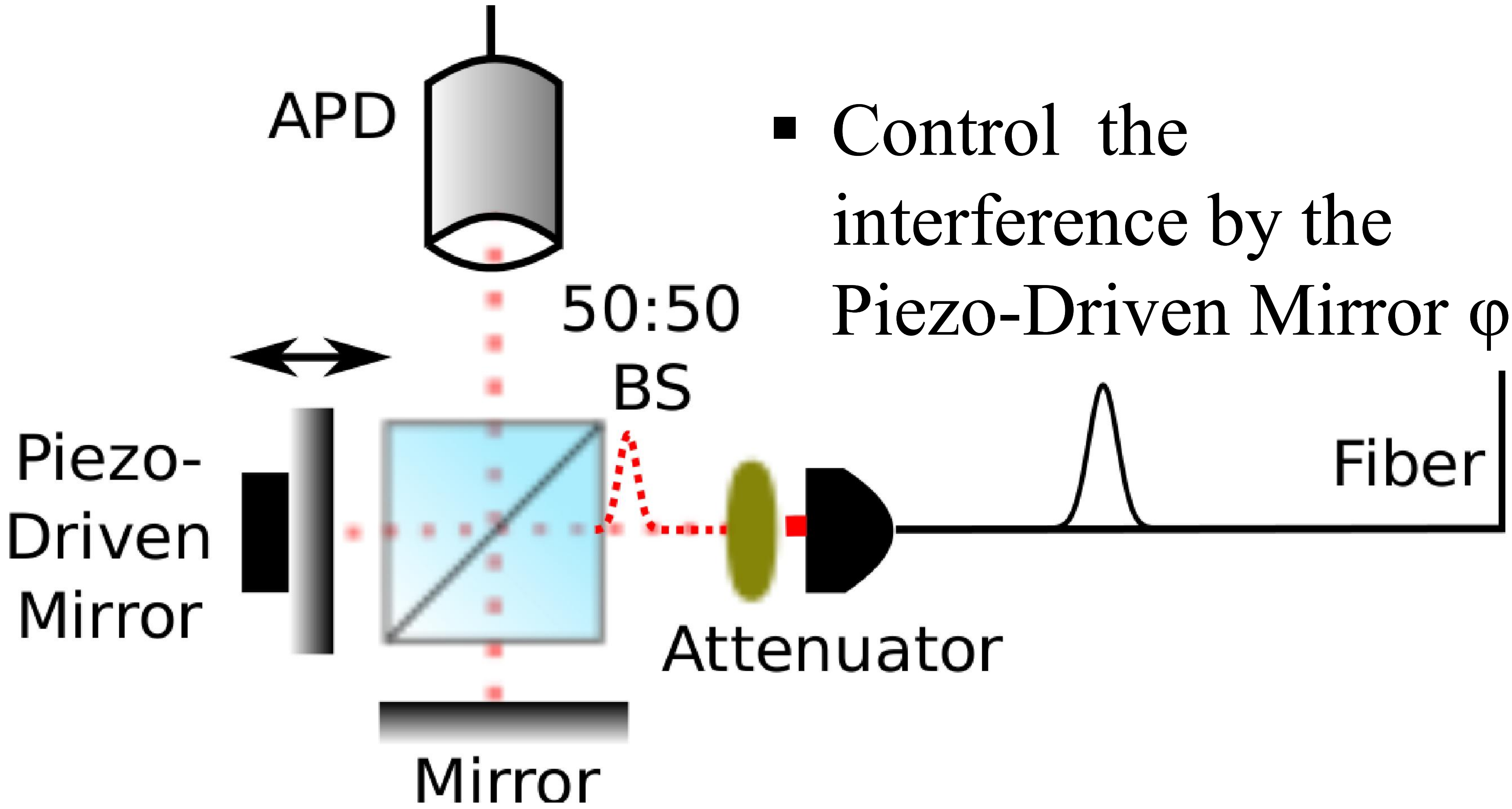
Preparation of State

- Use long temporal Gaussian pulses
- Michelson Interferometer

$$I(t) = I_0(t)\exp\left(\frac{-t^2}{2\tau^2}\right)$$



Small Interaction



- Control the interference by the Piezo-Driven Mirror φ

Observations

- Time Domain: no change
- Frequency Domain: One pulse is red or blue detuned

$$f_d = \pm 2 \tfrac{v}{c} f_0$$

- Technologically difficult to measure frequency and much easier to measure arrival time.

Observations 2

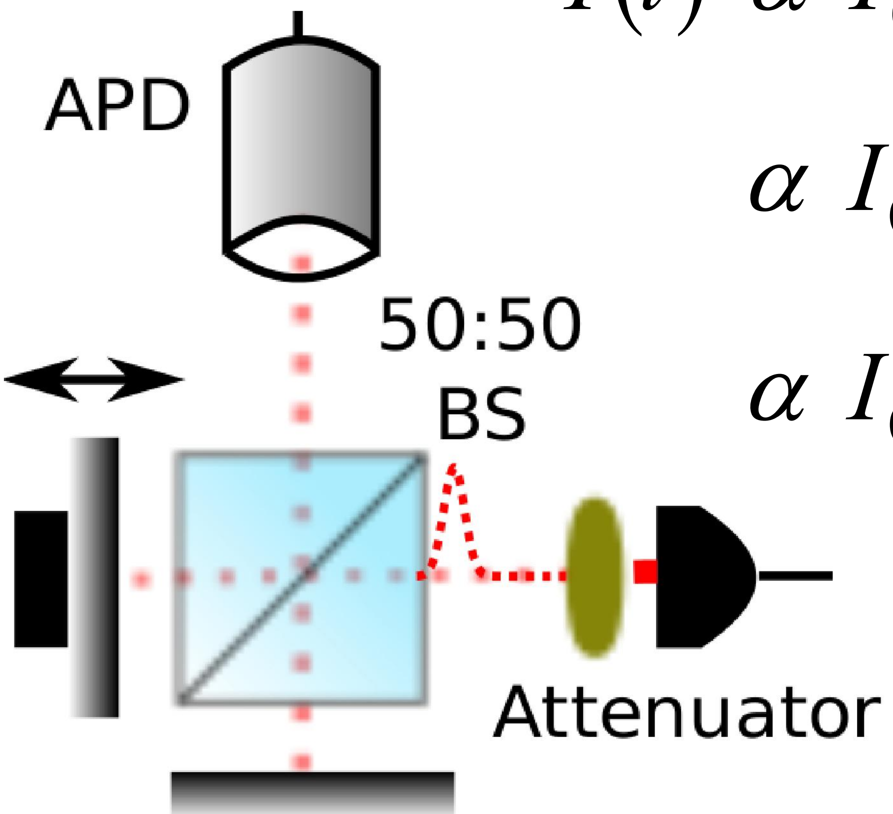
- Phase:
  - Point by point interference
  - Mirror is imparting a phase within the coherence length of the laser
- Pulse:
  - Pulse length >> Coherence length of the laser >> φ

Theory

$$I(t) \; \alpha \; I_0(t)\left|1-\exp\left(i2\varphi+i2kx(t)\right)\right|^2$$

$$\alpha \; I_0 \sin^2(\varphi)\exp\left(\frac{-t^2}{2\tau^2}\right)\left|\frac{\sin(\varphi+kvt)}{\sin(\varphi)}\right|^2$$

$$\alpha \; I_0 \sin^2(\varphi)\exp\left[\frac{-1}{2\tau^2}\left(t-\frac{2kv\tau^2}{\varphi}\right)^2\right]$$



Weak-values

- Coupled velocity/frequency to time
- Pre-selection: temporal Gaussian pulse
- Small interaction:
  - Tiny velocity disturbing the frequency domain
- Post-selection: angle close to destructive interference
- Result: A shift in arrival time

Limitations

- **Cramer-Rao Bound;** the lowest bound of the variance over an unbiased estimator. When achieved, the estimator is efficient. = 1/F

$$P(t;\delta t)=\frac{1}{\sqrt{2\pi\tau^2}}\exp\left(\frac{-(t-\delta t)^2}{2\tau^2}\right)$$

- **Fisher Information**

$$F=\int dt P(t;\delta t)\left[\frac{d}{d\delta t}\ln(P(t;\delta t))\right]^2$$

- Fisher Information:

$$N \sin^2(\varphi) \int dt P(t;\delta t)\left[\tfrac{d}{dt} \ln P(t;\delta t)\right]^2$$

$$\approx \frac{N\varphi^2}{\tau^2}$$

- Uncertainty of δt is bounded by: $\Delta(\delta t) \geq \frac{\tau}{\varphi\sqrt{N}}$
- Error in estimating velocity is bounded by:

$$\Delta v_{CRB} = \frac{\Delta(\delta t)\varphi}{2k\tau^2} = \frac{1}{2k\tau\sqrt{N}}$$

Smallest resolvable velocity

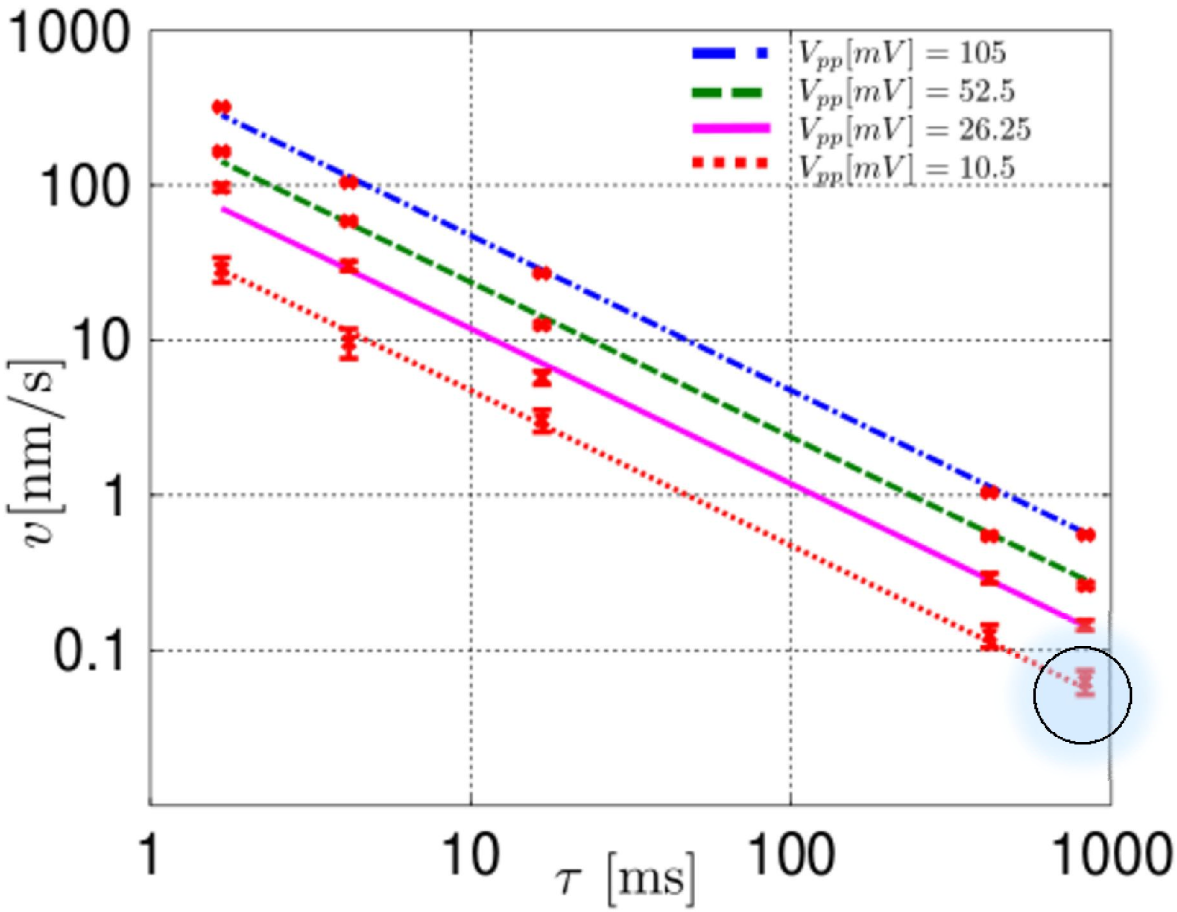
$$P(t;\delta t) = \frac{1}{\sqrt{2\pi\tau^2}} \exp\left(\frac{-(t-\delta t)^2}{2\tau^2}\right)$$

- Our Signal-to-Noise ratio is straight forward for a Gaussian: time shift divided by the uncertainty of the time shift

- SNR = $\frac{\delta t}{\tau}\varphi\sqrt{N} = \frac{v}{\Delta v}$

Starling, et al. PRA 82, 063822 (2010)

Results



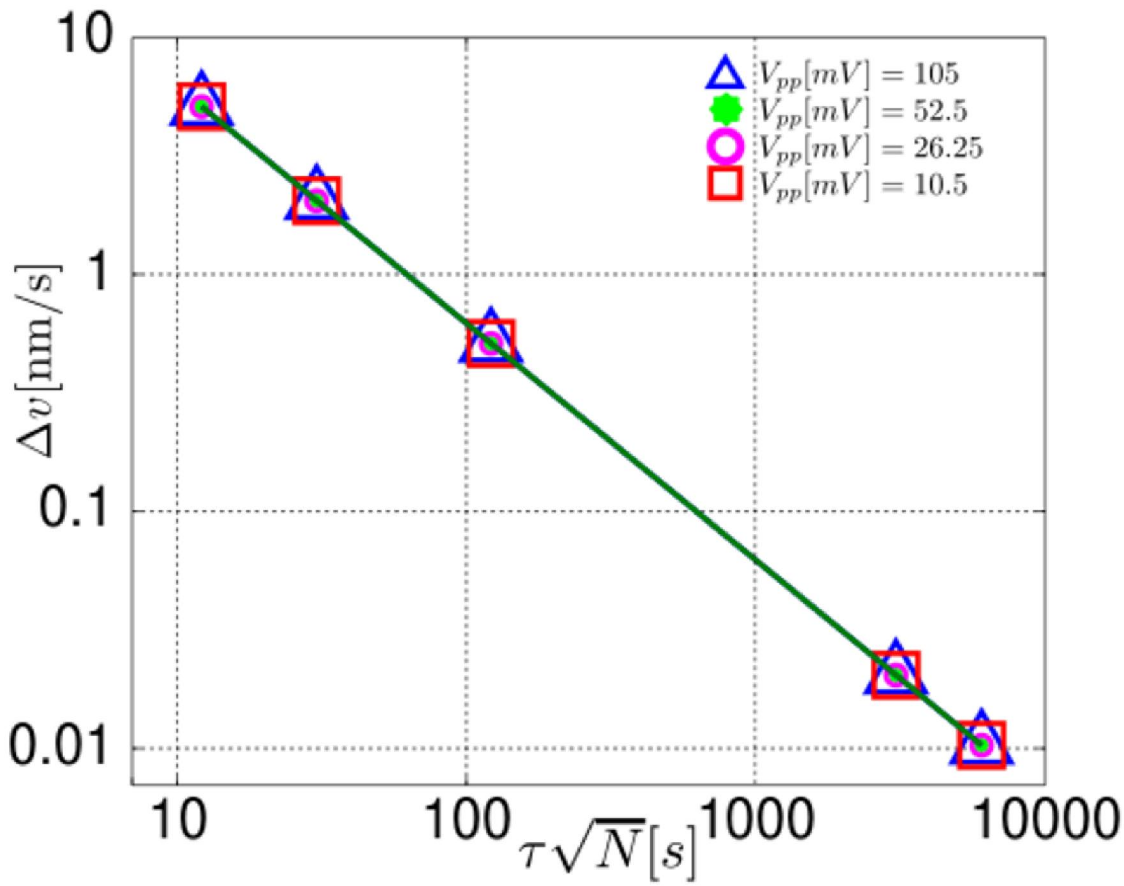
$$\delta t = \frac{2kv\tau^2}{\varphi}$$

$$\tau = \{1.7 - 833\}\ ms$$

$$\varphi \approx 0.3\, rad$$

$$v = 60 \pm 11\ {}^{pm}\!/\!_{s}$$

Uncertainty Results



- Estimator is efficient
- There is not better estimator to produce smaller uncertainties.

Sub picometer/second

$$\delta t = \frac{2kv\tau^2}{\varphi}$$

- What tricks can we do to achieve sub picometer/second?
  - post-selecting angel smaller
  - pulses longer

Results 2

- Our estimator changed from Gaussian to only the tip. Only the top 12% peak to peak intensity variation.

V_pp [mV]	Φ [rad]	v [pm/s]
2.0	0.275	1.4±0.5
1.0	0.276	0.6±0.4
0.5	0.279	0.4±0.4

Conclusion and Remarks

- The new estimator is not efficient but it helped us resolve slower velocities.
- Instability of the interferometer limited our integration time. The path length was constant for less than 15mins at a time.
- A weak-values technique allowed us to reach a CRB for velocity measurements.
- Achieved 400fm/s with a $\tau = 50s$.
  - In each run we took (10min) the mirror moved the distance of about 2 hydrogen atoms!

Thank you

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  - Barak Dayan
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