Who is Crazier: Bayes or Fisher? A Missing (Data) Perspective on Fiducial Inference

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Let's Meet the Crazies

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- Why should finding $\mathbf{U}|\mathbf{x}$ be any easier than finding $\theta|\mathbf{x}$?

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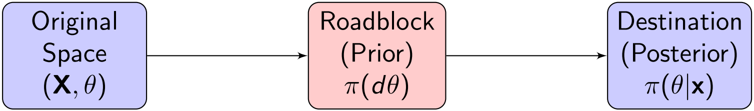

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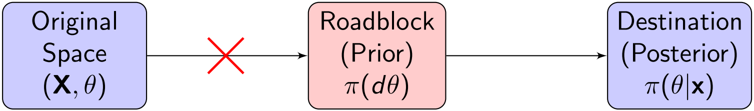

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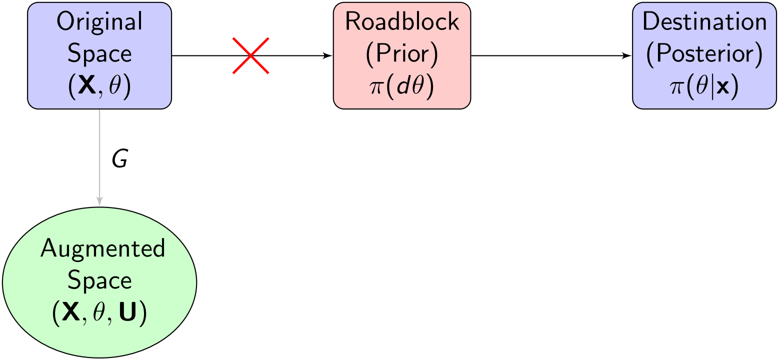

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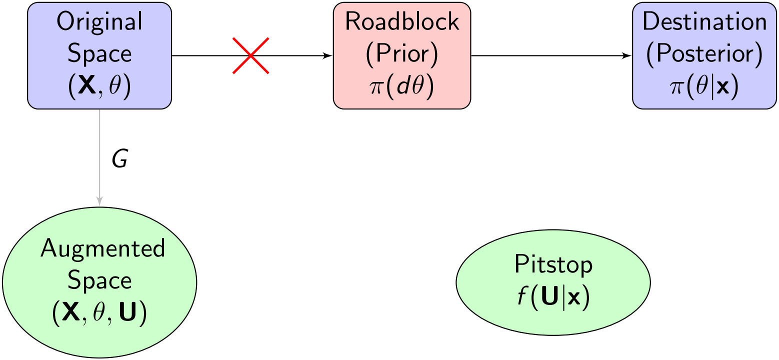

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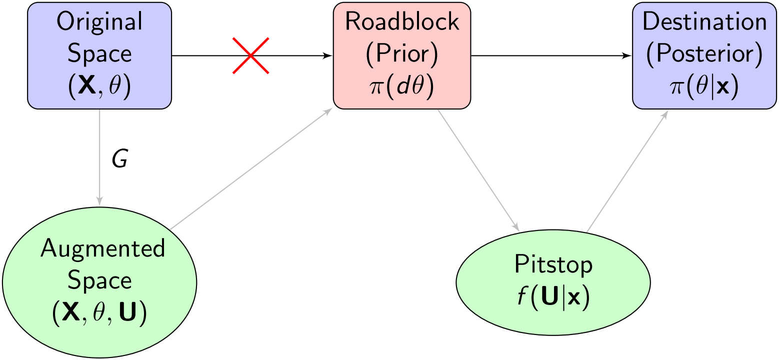

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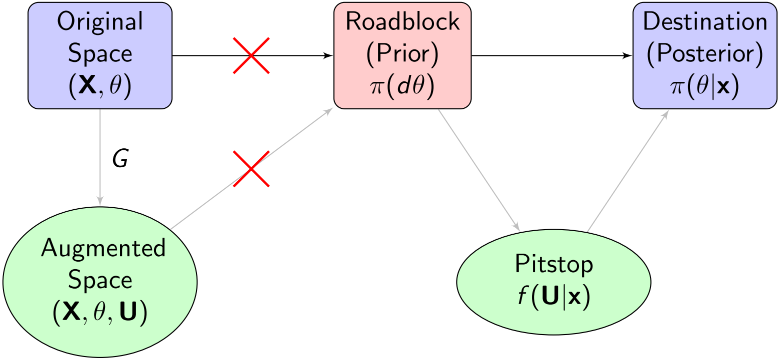

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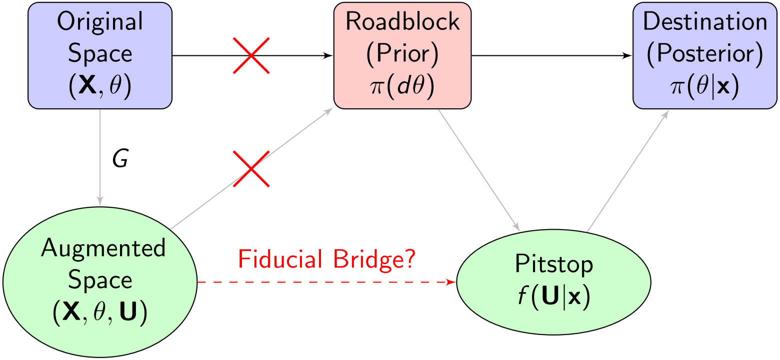

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The key point is that knowing θ *is equivalent to knowing* $\mathbf{U}$*; in other words, inference on* θ *is equivalent to predicting the value of the unobserved* $\mathbf{U}$. (Martin, Zhang, and Liu 2010)

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Can we predict **U** without any knowledge of the nuisance?

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- Let (X, Y) have fiducial equation

$$X = \varepsilon_1 \text{ and } Y = \rho\varepsilon_1 + \sqrt{1-\rho}\varepsilon_2 \text{ where } \varepsilon_i \text{ i.i.d. } N(0,1)$$

- X, Y are marginally ancillary but not jointly ancillary.
- $P(X = x|\varepsilon, \rho) = 1\{x = \varepsilon_1\}$ is free of ρ.
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- Situation reverses by switching the roles of X and Y in the fiducial equation.

Didn't You Say You Were Ancillary?

- Does there exist a statistic, $A(\mathbf{x})$, s.t. $f(A(\mathbf{x})|\mathbf{U})$ is free of the nuisance π?
    - How about statistics ancillary for θ?

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- Situation reverses by switching the roles of X and Y in the fiducial equation.
- Whether or not $A(\mathbf{x})$ is free of π given $\mathbf{U}$ depends on G.

Representational Ancillarity

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Definition of R-Ancillarity

A statistic $A(\mathbf{x})$ is representationally ancillary w.r.t. G if there exists a representation A_G s.t. $A(G(\theta, \mathbf{U})) = A_G(\mathbf{U})\ \forall \theta$.

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- In Basu's paradox, X, Y are both ancillaries but they are not R-ancillaries with respect to the same G.

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- Fiducial inference depends on G because whether a statistic is ancillary for π depends on whether it is R-ancillary for G.
- Definition of ancillarity as *distributional independence* of $A(\mathbf{x})$ from θ is insufficient.
- We require the notion of *representational* (or functional) *independence*.

Is Fiducial Hazardous?

	Cox Hazard Model	Fiducial

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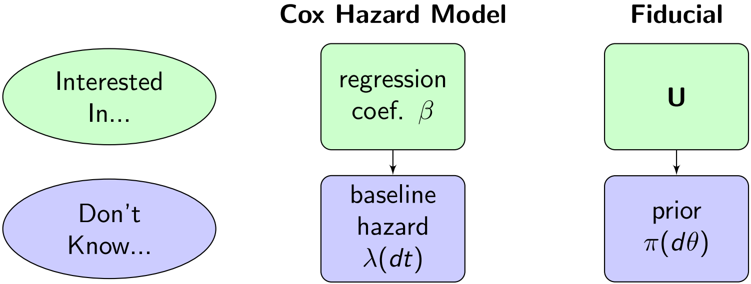
Interested In...	regression coef. β	**U**

Is Fiducial Hazardous?



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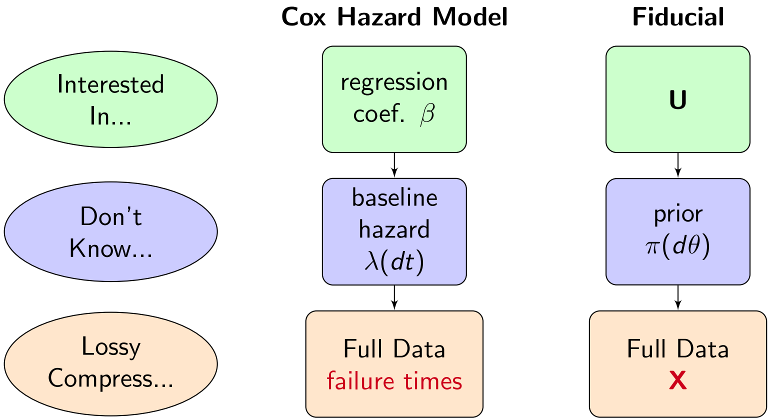

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Interested In...	regression coef. β	$\mathbf{U}$
Don't Know...	baseline hazard $\lambda(dt)$	prior $\pi(d\theta)$
Lossy Compress...	Full Data failure times	Full Data $\mathbf{X}$
Inference From...	λ ancillary β non-ancillary ranks	π ancillary $\mathbf{U}$ non-ancillary $A(\mathbf{x})$

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- Why does conditioning on ancillary statistics recover second order information?
- **Fiducial Answer**: It increases our efficiency of predicting $\mathbf{U}$.

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- **What are we forgetting by doing this?**

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x-Compatible Regions

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  2. **How** we came to observe $\mathbf{U_0} \in \mathcal{M}(\mathbf{x})$.

We Observe and Observe that We Observe

x-Compatible Regions

- If we generate data $\mathbf{x} = G(\theta_0, \mathbf{U}_0)$, we learn that $\mathbf{U}_0$ resides in

$$\mathcal{M}(G(\theta_0, \mathbf{U}_0)) = \{\mathbf{U} : \exists \theta \in \Theta \text{ s.t. } G(\theta_0, \mathbf{U}_0) = G(\theta, \mathbf{U})\}.$$

- **Hannig (2009) assumes that**

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- To correct use the information in (1), **we need to condition on the how**, i.e., condition on the observation process.

Bernoulli with Restricted Parameter Space

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- $\mathbb{I}^{obs} = X\mathbb{I}_1 + (1-X)\mathbb{I}_0$ (Note: $\mathbb{I}^{obs} = 1$)
- For $x \in \mathbb{X}$, define

$$O_x = \begin{cases} 1 & \text{if } \mathbb{I}^{obs} = \mathbb{I}_x \\ 0 & \text{otherwise} \end{cases}$$

A Sleight of Hand

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- $\mathcal{M}(X)$ contains the information $\mathbb{I}_x$ through $\mathbb{I}^{obs}$ **and** $\{O_x\}$.

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- We **ignore how we learned** $\{\mathbb{I}_x = t\}$.

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Confidence Validity (Rubin 1976)

- The posterior $f(U|U \in \mathcal{M}(x))$ leads to valid confidence regions for U if the observation process is ignorable

$$P\left(\mathbb{I}_x = 1|O_0 = o_0, O_1 = o_1, U, \pi\right) = P\left(\mathbb{I}_x = 1|U\right)$$

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- The information $U \in \mathcal{M}(x)$ seems *free*. But **to use it correctly** requires paying the price of a prior on θ.

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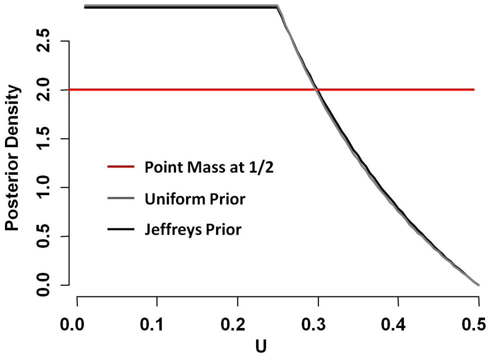

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- **Question 1:** So when is this information free? When is $\mathbf{U}|\mathcal{M}(\mathbf{x})$ objective—free of π?

- **Question 2:** What is maximal amount of free information about $\mathbf{U}$? What is the "best" objective posterior for $\mathbf{U}$?

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- **What is the smallest we can make $\mathcal{A}_G$? Does a smallest region even exist?**

From the Cave to Utopia?

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- Can we achieve Utopia?

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Utopia Can Always Be Achieved

$$\textbf{Utopia} = \bigcup_{\theta_0 \in \Theta} \mathcal{M}\left(G\left(\theta_0, \mathbf{U}_0\right)\right) = \bigcap_{A:\ R\text{-ancillary for } G} \mathcal{A}_G\left(\mathbf{U}_0\right) = \textbf{Cave}$$

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- **Utopia represents an upper bound on the informativeness of an R-ancillary. It is always achieved.**

Making Relevant Subsets Relevant Again

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if and only if $\forall \mathbf{U} \in \mathbb{U}$, $\forall \theta \in \Theta$, $\mathcal{M}\left(G\left(\theta, \mathbf{U}\right)\right) = \mathcal{U}_G\left(\mathbf{U}\right)$ where $\mathcal{U}_G\left(\mathbf{U}\right)$ is the Utopia set containing $\mathbf{U}$.

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- $\mathcal{M}(G(\theta, \mathbf{U}))$ is the information we get back about $\mathbf{U}$ if data are generated using θ.
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- C_θ attains posterior probability $1-\alpha$ if (1) and (2) hold.

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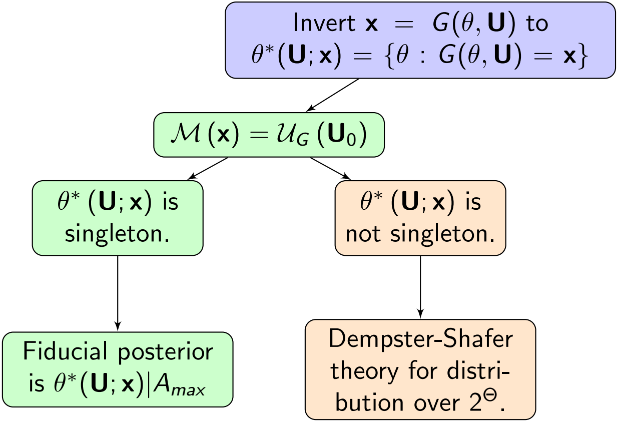

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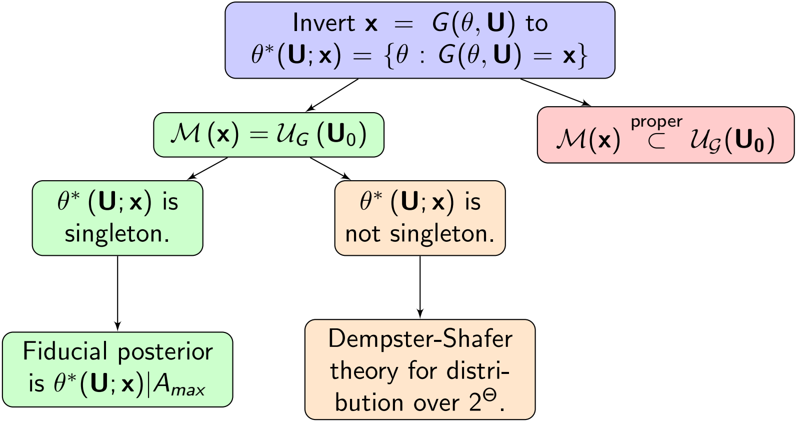

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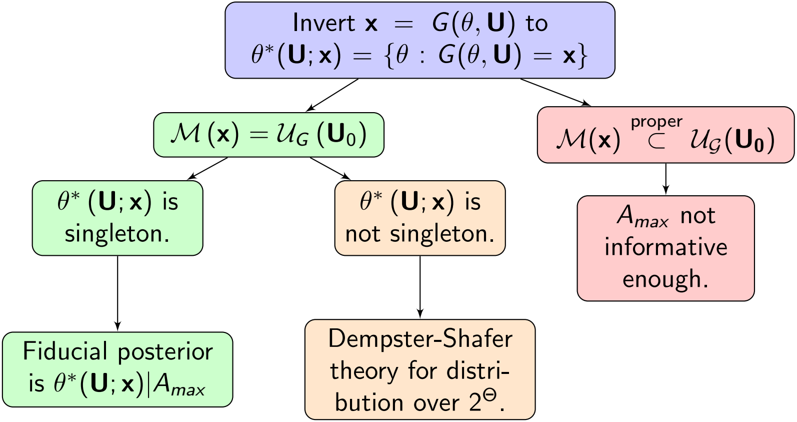

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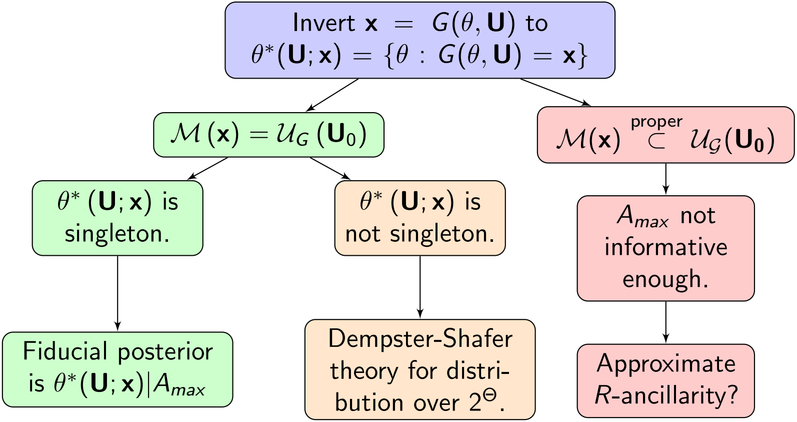

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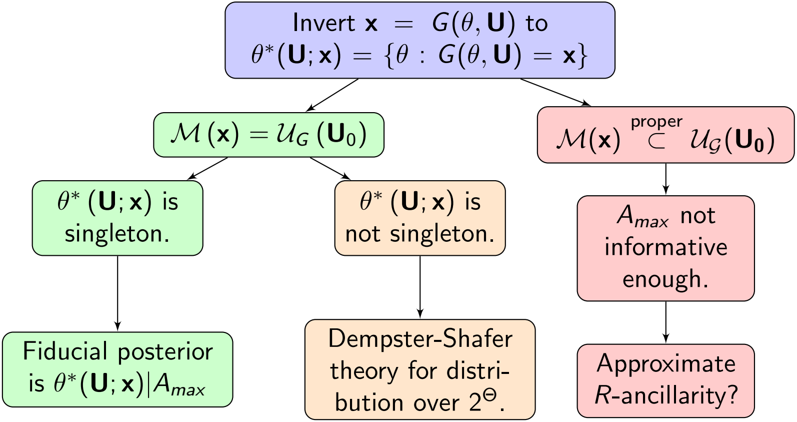

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  - $\theta^*(\mathbf{U}; \mathbf{x})$ is singleton. → Fiducial posterior is $\theta^*(\mathbf{U}; \mathbf{x}) | A_{max}$
  - $\theta^*(\mathbf{U}; \mathbf{x})$ is not singleton. → Dempster-Shafer theory for distribution over 2^{Θ}.
- $\mathcal{M}(\mathbf{x}) \overset{\text{proper}}{\subset} \mathcal{U}_{\mathcal{G}}(\mathbf{U_0})$ → A_{max} not informative enough. → Approximate R-ancillarity?

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- **Non-uniqueness**: How does one choose G?

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