Lecture IV: Option pricing in energy markets

Fred Espen Benth

Centre of Mathematics for Applications (CMA)
University of Oslo, Norway

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Introduction

- OTC market huge for energy derivatives
- Highly exotic products:
  - Asian options on power spot
  - Various (cross-commodity) spread options
  - Demand/volume triggered derivatives
  - Swing options
- Payoff depending on spot, indices and/or forwards/futures
- In this lecture: Pricing and hedging of (some) of these exotics

Example of swing options

- Simple operation of a gas-fired power plant: income is

$$\int_t^\tau e^{-r(s-t)} u(s)\,(P(s) - G(s))\, ds$$

- P and G power and gas price resp, in Euro/MWh.
  - Heating rate is included in G...
- $0 \leq u(s) \leq 1$ production rate in MWh
  - Decided by the operator
- Value of power plant

$$V(t) = \sup_{0 \leq u \leq 1} \mathbb{E}\left[\int_t^\tau e^{-r(s-t)} u(s)\,(P(s) - G(s))\, ds \,\middle|\, \mathcal{F}_t\right]$$

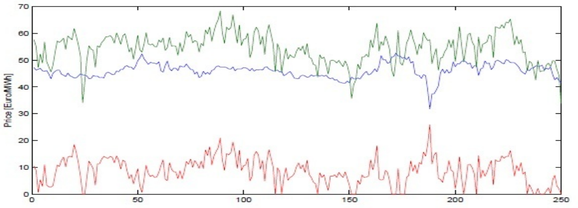
- More fun if there are constraints on production volume....
  - Maximal and/or minimal total production
  - Flexible load contracts, user-time contracts

- Tolling agreement: virtual power plant contract
  - Strip of European call on spread between power spot and fuel
  - Fuel being gas or coal

$$V(t) = \int_t^T e^{-r(s-t)}\mathbb{E}\left[\max\left(P(s) - G(s), 0\right) \mid \mathcal{F}_t\right] ds$$

- Spark spread, the value of exchanging gas with power
  - Dark spread, crack spread, clean spread....

German (EEX) spark spread in 2011



- Green: EEX power (Euro/MWh)
- Blue: Natural gas (Euro/MWH)
- Red: Spark spread, with efficiency factor (heat rate) of 49.13%

Example: Asian options

- European call option on the average power spot price

$$\max\left(\frac{1}{\tau_2-\tau_1}\int_{\tau_1}^{\tau_2} S(u)\,du - K, 0\right)$$

- Traded at NordPool around 2000
  - "Delivery period" a given month
  - Options traded until τ_1, beginning of "delivery"

Example: Energy quanto options

- Extending Asian options to include volume trigger
- Sample payoff

$$\max\left(\frac{1}{\tau_2-\tau_1}\int_{\tau_1}^{\tau_2} S(u)\,du - K_P, 0\right) \times \max\left(\frac{1}{\tau_2-\tau_1}\int_{\tau_1}^{\tau_2} T(u)\,du - K_T, 0\right)$$

- $T(u)$ is the temperature at time u
  - in a location of interest, or average over some area (country)
- Energy quantos on:
  - gas and temperature (demand)
  - or power and wind (supply)
  - Dependency between energy price and temperature crucial

Spread options (tolling agreements)

- Spread payoff with exercise time τ

$$\max\left(P(\tau) - G(\tau), 0\right)$$

- P, G bivariate geometric Brownian motion $\longrightarrow$ Margrabe's Formula
    - Introducing a strike $K \neq 0$, no known analytic pricing formula
- Our concern: valuation for exponential non-Gaussian stationary processes
    - Exponential Lévy semistationary (LSS) models

- Recall definition of LSS process from Lecture III

$$Y(t) = \int_{-\infty}^{t} g(t-s)\sigma(s)\,dL(s)$$

- L a (two-sided) Lévy process (with finite variance)
- σ a stochastic volatility process
- g kernel function defined on $\mathbb{R}_+$
- Integration in semimartingale (Ito) sense
  - $g(t-\cdots) \times \sigma(\cdot)$ square-integrable
- Y is stationary whenever σ is
  - Prime example: $g(x) = \exp(-\alpha x)$, Ornstein-Uhlenbeck process

- Bivariate spot price dynamics

$$\ln P(t) = \Lambda_P(t) + Y_P(t)$$
$$\ln G(t) = \Lambda_G(t) + Y_G(t)$$

- $\Lambda_i(t)$ seasonality function, $Y_i(t)$ LSS process with kernel g_i and stochastic volatility σ_i, $i = P, G$
  - The stochastic volatilites are assumed *independent* of U_P, U_G
- $L = (U_P, U_G)$ bivariate (square integrable) Lévy process
  - Denote cumulant (log-characteristic function) by $\psi(x, y)$.
- We suppose that spot model is under Q
  - Pricing measure

Fourier approach to pricing

- To compute the expected value under Q for the spread:
- Factorize out the gas component

$$\Lambda_P(\tau)\mathbb{E}\left[e^{Y_G(\tau)}\left(e^{Y_P(\tau)-Y_G(\tau)} - h\frac{\Lambda_G(\tau)}{\Lambda_P(\tau)}\right)^+ \mid \mathcal{F}_t\right]$$

- Apply the tower property of conditional expectation, conditioning on σ_i,
    - Recall being independent of L
    - $\mathcal{G}_t = \mathcal{F}_t \vee \{\sigma_i(\cdot), i = P, G\}$

$$\Lambda_P(\tau)\mathbb{E}\left[\mathbb{E}\left[e^{Y_G(\tau)}\left(e^{Y_P(\tau)-Y_G(\tau)} - h\frac{\Lambda_G(\tau)}{\Lambda_P(\tau)}\right)^+ \mid \mathcal{G}_t\right] \mid \mathcal{F}_t\right]$$

- For inner expectation, use that Z is the density of an Esscher transform for $t \leq \tau$

$$Z(t) = \mathrm{e}^{\int_{-\infty}^{t} g_G(\tau-s)\sigma_G(s)\,dU_G(s) - \int_{-\infty}^{t} \psi_G(-\mathrm{i}g_G(\tau-s)\sigma_G(s))\,ds}$$

  - $\psi_G(y) = \psi(0, y)$, the cumulant of U_G.
  - The characteristics of L is known under this transform
- This "removes" the multiplicative term $\exp(Y_G(\tau))$ from inner expectation
- Finally, apply Fourier method

- Define , for $c > 1$,

$$f_c(x) = \mathrm{e}^{-cx}\left(\mathrm{e}^x - h\frac{\Lambda_G}{\Lambda_P}\right)^+$$

- $f_c \in L^1(\mathbb{R})$, and its Fourier transform $\widehat{f_c} \in L^1(\mathbb{R})$
- Representation of f_c:

$$f_c(x) = \frac{1}{2\pi}\int_{\mathbb{R}} \widehat{f_c}(y)\mathrm{e}^{\mathrm{i}xy}\,dy$$

- Gives general representation for a random variable X

$$\mathbb{E}[f(X)] = \frac{1}{2\pi}\int \widehat{f_c}(y)\mathbb{E}\left[\mathrm{e}^{\mathrm{i}(y-\mathrm{i}c)X}\right]\,dy$$

Theorem

Suppose exponential integrability of L. Then the spread option has the price at $t \leq \tau$

$$C(t,\tau) = e^{-r(\tau-t)} \frac{\Lambda_P(\tau)}{2\pi} \int_{\mathbb{R}} \widehat{f}_c(y) \Phi_c\Big(Y_P(t,\tau), Y_G(t,\tau)\Big) \Psi_{c,t,\tau}(y)\, dy$$

where, for $i = P, G$,

$$Y_i(t,\tau) = \int_{-\infty}^{t} g_i(\tau - s)\sigma_i(s)\, dU_i(s)$$

$$\Phi_c(u,v) = \exp\left((y - \mathrm{i}c)u + (1 - (\mathrm{i}y + c))v\right)$$

and

$$\Psi_{c,t,\tau}(y) = \mathbb{E}\left[e^{\int_t^\tau \psi((y-\mathrm{i}c)g_P(\tau-s)\sigma_P,((c-1)\mathrm{i}-y)g_G(\tau-s)\sigma_G(s))\, ds} \mid \mathcal{F}_t\right]$$

- Note: spread price *not* a function of the current power and gas spot, but on $Y_i(t,\tau), i = P, G$
- Recalling theory from Lecture III: $Y_i(t,\tau)$ isl given by the logarithmic forward price....

$$\ln f_i(t,\tau) = X_i(t,\tau,\sigma_i(t)) + Y_i(t,\tau)$$

- No stochastic volatility, $\sigma_i = 1$: X_i is a deterministic function

Some remarks on hedging

- Power spot not tradeable, gas requires storage facilities
- Alternatively, hedge spread option using forwards!
- But incomplete model, so only partial hedging possible
  - Quadratic hedging, for example
  - May also depend on stochastic volatility, making model "more incomplete"
- In real markets: forwards on power and gas deliver over a given time period
  - Further complication, as we cannot easily express spread in such forwards
  - Further approximation of partial hedging strategy

Asian options

Asian options

- Options on the average spot price over a period
  - Traded at NordPool up to around 2000 for "monthly periods"
- Recall payoff function

$$\max\left(\frac{1}{\tau_2-\tau_1}\int_{\tau_1}^{\tau_2} S(u)\,du - K, 0\right)$$

- Geometric LSS spot model:

$$\ln S(t) = \Lambda(t) + Y(t)$$

- Y an LSS process with kernel g and stochastic volatility σ

- Pricing requires simulation
  - Propose an efficient Monte Carlo simulation of the path of an LSS process
- Suppose that $g_\lambda(u) := \exp(\lambda u) g(u) \in L^1(\mathbb{R})$ and its Fourier transform is in $L^1(\mathbb{R})$

$$Y(t) = \frac{1}{2\pi} \int_{\mathbb{R}} \widehat{g}_\lambda(y) \widehat{Y}_{\lambda,y}(t)\, dy$$

- $\widehat{Y}_{\lambda,y}(t)$ complex-valued Ornstein-Uhlenbeck process

$$\widehat{Y}_{\lambda,y}(t) = \int_{-\infty}^{t} \mathrm{e}^{(\mathrm{i}y-\lambda)(t-s)} \sigma(s)\, dL(s)$$

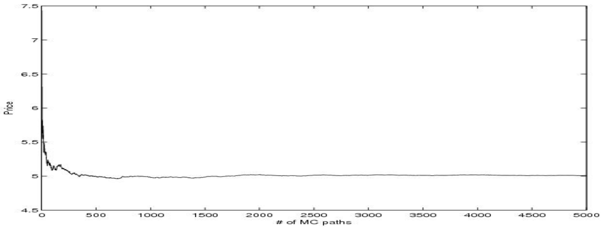
- Paths of Ornstein-Uhlenbeck processes can be simulated iteratively

$$\widehat{Y}_{\lambda,y}(t+\delta) = \mathrm{e}^{(\mathrm{i}y-\lambda)\delta}\widehat{Y}_{\lambda,y}(t)+\mathrm{e}^{(\mathrm{i}y-\lambda)\delta}\int_t^{t+\delta}\mathrm{e}^{(\mathrm{i}y-\lambda)(t-s)}\sigma(s)\,dL(s)$$

- Numerical integration (fast Fourier) to obtain paths of Y
  - Extend g to $\mathbb{R}$ if $g(0) > 0$
  - Let $g(u) = 0$ for $u < 0$ if $g(0) = 0$.
  - Smooth out g at $u = 0$ if singular in origo
- Error estimates in L^2-norm of the paths in terms of time-stepping size δ

- Asian call option on Y over $[0, 1]$, with strike $K = 5$
- Y BSS-process, with $\sigma = 1$, $Y(0) = 10$, and kernel (modified Bjerksund model)

$$g(u) = \frac{1}{u+1}\exp(-u)$$



Issues of hedging

- Let $F(t, \tau_1, \tau_2)$ be forward price for contract delivering power spot S over τ_1 to τ_2: At $t = \tau_2$,

$$F(\tau_2, \tau_1, \tau_2) = \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} S(u)\, du$$

- Asian option: call option on forward with exercise time τ_2
- In power and gas, forwards are traded with delivery period
  - Hence, can price, but also *hedge* using these
  - Analyse based on forward price model rather than spot!
- Problem: many contracts are *not* traded in the settlement period
  - Can hedge up to time τ_1
  - ..but not all the way up to exercise τ_2

Example: quadratic hedging

- Hedge option with payoff X at exercise τ_2, using $\psi(s)$ forwards
- Assume Levy (jump) dynamics for the forward price
    - Martingale dynamics
- Can only trade forward up to time $\tau_1 < \tau_2$

$$V(t) = V(0) + \int_0^{t\wedge\tau_1} \psi(s) dF(s) + \mathbf{1}_{\{t>\tau_1\}}\psi(\tau_1)(F(t,\tau_1,\tau_2) - F(\tau_1,\tau_1,\tau_2))$$

- Predictable strategies ψ being integrable with respect to F.

- Minimize quadratic hedging error

$$\mathbb{E}[(X - V(\tau_2))^2]$$

- Solution:
  - Classical quadratic hedge up to time τ_1,
  - thereafter, use the constant hedge

$$\psi_{\min} = \frac{\mathbb{E}[X(F(\tau_2) - F(\tau_1)) \,|\, \mathcal{F}_{\tau_1}]}{\mathbb{E}[(F(\tau_2) - F(\tau_1))^2 \,|\, \mathcal{F}_{\tau_1}]}$$

Example: geometric Brownian motion

- X call option with strike K at time τ_2
- $t \mapsto F(t, \tau_1, \tau_2)$ geometric Brownian motion with constant volatility σ
  - We suppose the forward is tradeable only up to time $\tau_1 < \tau_2$
- Quadratic hedge
  - N is the cumulative standard normal distribution function

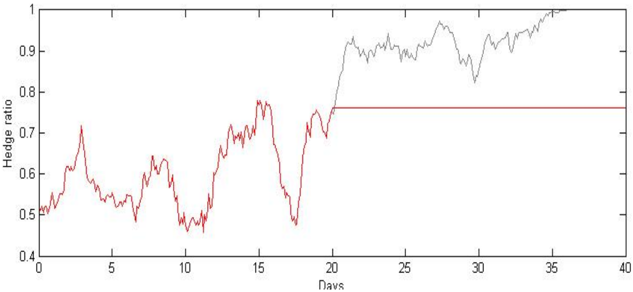
$$\psi(t) = \begin{cases} N(d(t)), & t \leq T_1 \\ \psi_{\min}, & t > T_1 \end{cases}$$

where

$$\psi_{\min} = \frac{F(\tau_1)e^{\sigma^2(\tau_2-\tau_1)}N(\sigma\sqrt{\tau_2-\tau_1}+d(\tau_1)) - (K+F(\tau_1))N(d(\tau_1)) + KN(d(\tau_1)-\sigma\sqrt{\tau_2-\tau_1})}{F(\tau_1)(e^{\sigma^2(\tau_2-\tau_1)}-1)}$$

$$d(t) = \frac{\ln(F(t,\tau_1,\tau_2)/K) + 0.5\sigma^2(\tau_2-t)}{\sigma\sqrt{\tau_2-t}}$$

- Empirical example:
  - Annual vol of 30%, $\tau_1 = 20, \tau_2 = 40$ days
  - ATM call with strike 100
  - Quadratic hedge jumps 1.8% up at τ_1 compared to delta hedge



Quanto options

- Recall payoff of an energy quanto option

$$\max\left(\frac{1}{\tau_2-\tau_1}\int_{\tau_1}^{\tau_2} S(u)\,du - K_P, 0\right) \times \max\left(\mathrm{T}_{\text{index}}(\tau_1,\tau_2) - K_T, 0\right)$$

- $\mathrm{T}_{\text{index}}(\tau_1,\tau_2)$ temperature index measured over $[\tau_1,\tau_2]$
  - CAT index, say, or HDD/CDD
- Consider idea of viewing the contract as an option on two forwards
  - Product of two calls,
  - One on forward energy, and one on temperature (CAT forward)
- Main advantages
  - Avoid specification of the risk premium in the spot modelling
  - Can price "analytically" rather than via simulation

Case study: bivariate GBM

- Consider bivariate GBM model

$$dF_P(t,\tau_1,\tau_2) = \sigma_P(t,\tau_1,\tau_2)F_P(t,\tau_1,\tau_2)\,dW_P(t)$$
$$dF_T(t,\tau_1,\tau_2) = \sigma_T(t,\tau_1,\tau_2)F_T(t,\tau_1,\tau_2)\,dW_T(t)$$

- σ_P, σ_T deterministic volatilities, W_P, W_T correlated Brownian motions
- May express the price of the quanto as a "Black-76-like" formula

- Price of quanto at time $t \le \tau_1$ is

$$C(t) = e^{-r(\tau_2 - t)} \{ F_P(t) F_T(t) e^{\rho \sigma_P \sigma_T} N(d_P^{***}, d_T^{***}) - F_P(t) K_T N(d_P^{**}, d_T^{**}) - F_T(t) K_P N(d_P^{*}, d_T^{*}) + K_P K_T N(d_P, d_T) \}$$

where

$$d_i = \frac{\ln(F_i(t)/K) - 0.5\sigma_i^2}{\sigma_i}, \quad d_i^{**} = d_i + \sigma_i, i = P, T$$
$$d_i^{*} = d_i + \rho\sigma_j, \quad d_i^{***} = d_i + \rho\sigma_j + \sigma_i, i, j = P, T, i \neq j$$

- $N(x, y)$ bivariate cumulative distribution function with correlation ρ, equal to the one between W_P, and W_T
- σ_P and σ_T integrated volatility

$$\sigma_i^2 = \int_t^{\tau_2} \sigma_i^2(s, \tau_1, \tau_2)\, ds, \quad i = P, T$$

Empirical study of US gas and temperature

- Temperature index in quanto is based on Heating-degree days

$$\mathrm{T}_{\mathrm{index}}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \max(c - T(u), 0)\, du$$

- $F_T(t, \tau_1, \tau_2)$ HDD forward
- HDD forward prices for New York
  - Use prices for 7 first delivery months
- NYMEX gas forwards, monthly delivery
  - Use prices for coming 12 delivery months
- 3 years of daily data, from 2007 on

- Approach modelling of $F(t, \tau_1, \tau_2)$ by $F(t, \tau)$, forward with fixed maturity date
  - Choose the maturity date τ to be middle of delivery period
  - Price dynamics only for $t \leq \tau$!
- Two factor structure (long and short term variations)

$$dF_i(t,\tau) = F_i(t,\tau)\left\{\gamma_i \, dW_i + \beta_i e^{-\alpha_i(\tau - t)} \, dB_i\right\} \; i = G, T$$

- Estimate using Kalman filtering
  - W and B strongly negatively correlated for both gas and temperature
  - W's negatively correlated , B's positively

- Compute quanto-option prices from our formula
  - The period τ_1 to τ_2 is December 2011
  - Current time t is December 31, 2010
  - Use market observed prices at this date for $F_G(t), F_T(t)$
- Prices benchmarked against independent gas and temperature
  - Quanto option price is equal to the product of two call options prices, with interest rate $r/2$

strikes K_G, K_T	1100,3	1200,5	1300,7
dependence	596	231	108
independence	470	164	74

- Note: long-dated option, long-term components most influencial
  - These are negatively correlated, approx. -0.3

Conclusions

- European-style options can be priced using transform-based methods
  - Example: spread options
- Path-dependent options require simulation of LSS processes
  - Suggested a method based on Fourier transform
  - Paths simulated via a number of OU-processes
- Considered "new" quanto option
  - Priced using corresponding forwards
  - Case study from US gas and temperature market
- Discussed hedging based on minimizing quadratic hedge error
  - Particular consideration of no-trading constraint in delivery period

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Coordinates:

- fredb@math.uio.no
- folk.uio.no/fredb/
- www.cma.uio.no