A simulations-and-regressions algorithm with application to hydropower storage

Workshop on Electricity, Energy and Commodities Risk Management
Fields Institute

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HEC Montréal

14-16 August 2013

Project Overview

- Initial motivation : the modeling and control of energy production and storage.
- Pure profit optimization : attempt to play storage and variable prices at their best. No risk management *per se.*
- Complex optimization problems : optionnality, stochastic state variables, multiscale seasonalities, long-term decisions.

Project Overview

- Development of a dynamic programming approach based on simulations and regressions.
- The techniques of DP with simulations and regression have become central in financial engineering to solve financial option problems.
- We're valuing an energy instrument that is a set of dependent options ; and to value it, we need to time the sale decisions at best.

Project Overview

Focus here on one application (hydropower) and two state variables : the exogenous spot price of power, and the endogenous water level.

- The endogenous (control-dependent) variable is a key point.
- Two main ideas :
  - Graft the endogenous state variable onto the simulation paths of the exogenous state variable, building paths of "optimal" water levels.
  - Apply a "backwash" technique to both deal with operational limits and avoid clustering in the endogenous variable space.

Related literature

Closely related literature, on gas storage.

- Boogert and De Jong (2008) : probably the first simulations-and-regressions approach to gas storage, but the endogenous variable is discretized.
- Carmona and Ludkowski (2010) : "quasi-simulation" of the endogenous variable.
- Nascimento and Powell (2013) : A.D.P. (approximate dynamic programming, or forward D.P.) approach.

Setting the problem

The **simple** setup, discussed today, includes

- a hydro power production facility which includes storage ;
- the possibility to buy or sell a limited but fixed amount of power at each period ;
- purchases of power increase the water level (see below) ;
- all transactions are at the spot price, which is stochastic ;
- storage is of course bounded above and below.

Setting the problem

A more complete problem setup would include

- a variable local demand at a constant price, which must be satisfied ;
- purchases and sales are on a neighbour market, with stochastic prices ;
- purchases of power help keep water behind the dam (but don't actually increase the level) ;
- water inflows are stochastic.

Setting the problem as a dynamic program

- The goal is to maximize expected net profit over a finite horizon $[0, T]$.
- Natural setup for dynamic programming : knowing the optimal policy from $t+1$ to T, find the optimal policy from t to T by identifying the best policy between t and $t+1$.
- Backward solution is then possible, from time T to time 0, given the final boundary condition.

The Endogenous Variable and the State Equation

- The water level is an endogenous state variable : the production decision at t changes the state of the system at $t+1$.
- Compare : the american option has only an exogenous state variable, the stock price.
- The water levels follow the *state equation*

$$L_{t+1} = h(u_t; L_t);$$

where u is a sales decision and L_t is the water level at time t.

Dynamic programming recursion

Thanks to the optimality principle of dynamic programming, we can compute the *value function* recursively as

$$V_t(S_t, L_t) = \sup_{u_t \in \mathcal{U}(S,L,t)} \left\{ \pi_t\big(u_t; S_t, L_t\big) + \mathbb{E}_t\Big\{ V_{t+1}(S_{t+1}, h(u_t; L_t))\Big\} \right\}$$

where u is the decision variable, π_t is the payoff function on the period from t to $t+1$, the expectation is conditional on time t information.

(The *value function* is the (monetary) value of being in a certain state at a certain time, assuming that the best non-anticipative decisions will be made until the end of time.)

Traditional solution approach for the recursion equation

- The traditional way to solve the continuous time, continuous state variables DP is to discretize all state variables and time.
- This is the technique we use for benchmarking.
- Subject to the curse of dimensionality : beyond a few state variables, the technique is very time-consuming, or even untractable.

A simulations-and-regressions approach

For its simplicity, flexibility and ability to handle greater numbers of state variables, we prefer the dynamic programming approach of Monte Carlo simulations and (simple linear) regressions.

- Monte Carlo simulations are used to generate ahead of time a set of scenarios for the exogenous stochastic variable (e.g. spot price)
- Decisions are discretized.
- For each decision, the profit function is approximated by regressing the profits on the state variable values (for all paths).

What about the endogenous variable (water level) ?

Solution through simulations-and-regressions

Let the value function be known at $t+1$

$$V_{t+1}\left(S_{t+1}^{(k)}, L_{t+1}^{(k)}\right)$$

for each spot price path $k \in 1, \ldots, K$.
Define the *backward* state equation

$$\overleftarrow{h}(u_t; L_{t+1}) = L_t$$

and the water level at time t which *depends on* u_t

$$L_t^{(k)}(u_t) = \overleftarrow{h}\left(u_t; L_{t+1}^{(k)}\right)$$

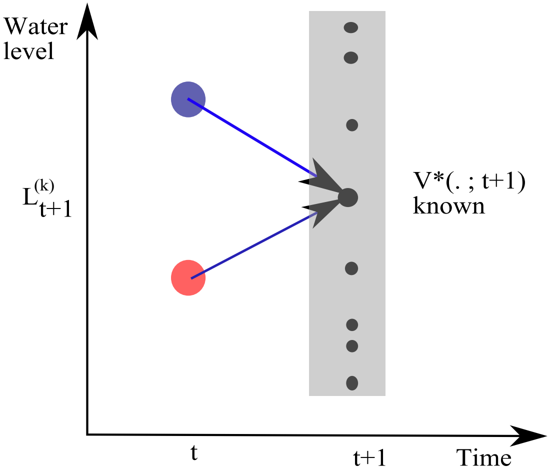
We can regress

$$\pi_t\left(u_t; S_t^{(k)}, L_t^{(k)}(u_t)\right) + V_{t+1}\left(S_{t+1}^{(k)}, L_{t+1}^{(k)}\right) \quad \text{on } \left(S_t^{(k)}, L_t^{(k)}(u_t)\right)$$

for each possible decision u_t. Note that the "antecedent levels" $L_t^{(k)}(u_t)$ are functions of the decision.

We obtain a regression surface *for each possible discrete decision.*

What is an adequate path-k, time-t water level $L_t^{(k)}$?



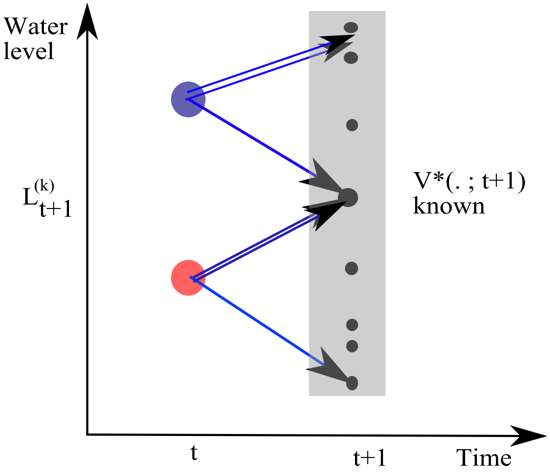
Creating Paths of Water Levels

- Endogenous water level variable cannot be simulated ahead of time, like the spot prices.
- However, the value expectations must rely on optimal paths.
- We build water level paths backwards, using the regression surfaces.
- These **water level paths** are not actual simulations, but each is *matched* with a **spot price path.**

Creating Paths of Water Levels : forward-optimal paths

- So, which time-t water level L_t is the "right" level, given a time-(t+1) water level L_{t+1} ?
- Certainly not the level with the highest value, that would be "backward optimal".
- We need a time-t level that "forward optimally" leads to the (known) time-(t+1) level.
- The regression surfaces are computed already, so just use them repeatedly (small numerical overhead)

What is an adequate path-k, time-t water level $L_t^{(k)}$?



Clusters, Bounds and Backwash

Two problems crop up with this “fausse-simulation” technique.

- Problem 1 : little control over the building of the water level paths, so water levels can go out-of-bounds (leakage) and can cluster. (And they do !)
- Problem 2 : need to take account of the water level operational bounds wisely. We *do* need information about crossing the bounds.
- Solution :
  - Add a penalty term for violations of the dam upper and lower levels.
  - Let water level paths go out-of-bounds, and use that info in the regressions.
  - When a path goes too far out-of-bounds, *backwash* it randomly to the feasible area, thereby smoothing clusters.

Clusters, Bounds and Backwash

Solution :

- Add a penalty term for violations of the dam upper and lower levels.
- Let water level paths go out-of-bounds, and use that info in the regressions. This takes care of problem 2.
- When a path goes too far out-of-bounds, *backwash* it randomly to the feasible area, thereby smoothing clusters. This takes care of problem 1.

Summary of the algorithm

Initialization :

1. Choose a set of basis functions for the state variables, S_t and L_t ;
2. Randomly generate K paths for the exogenous variable S_t, $(t = 0, 1, \ldots, T)$;
3. Randomly generate K time-T levels of the endogenous variable L_T, within the range $[L_{\min}, L_{\max}]$;
4. Compute time-T values according to a boundary condition.

Backward recursion : for all times from $t = T - 1$ to $t = 0$:

1. Compute the regression surfaces $\widetilde{V}^u(S, L)$, $u \in \{\texttt{+1,0,-1}\}$ using a payoff with penalty for going out of bounds.
2. For each of the $3K$ candidate levels $L_t^{(k)}$, compute a forwardoptimal decision.
3. Associate a level $L_t^{(k)}$ for each path k, according to the decisions in the above step. If $L_t^{(k)}$ is too far out of bounds, randomly reassign it to a random, acceptable water level (the backwash technique)
4. Compute the K values $V_t\left(S_t^{(k)}, L_t^{(k)}\right)$ as a sum of payoffs until time T along path (k). In the case of paths whose level has been reassigned in step 3, use instead the value on the regression surface.

Out-of-sample tests : retain solely the regression parameters.

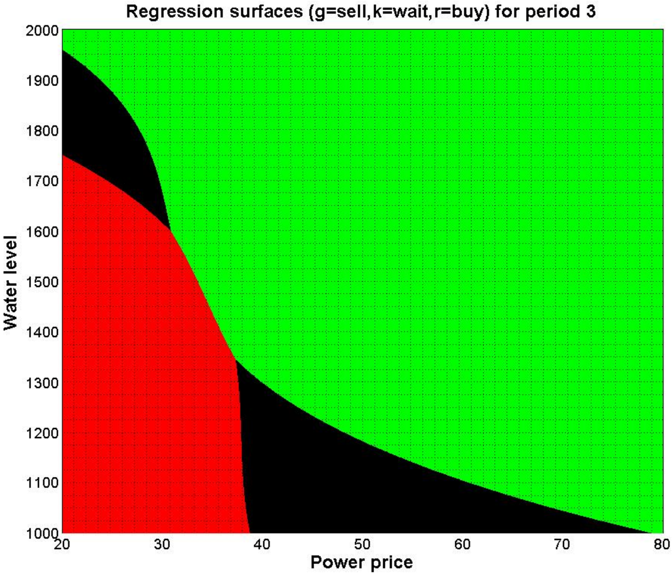
Convergence ?

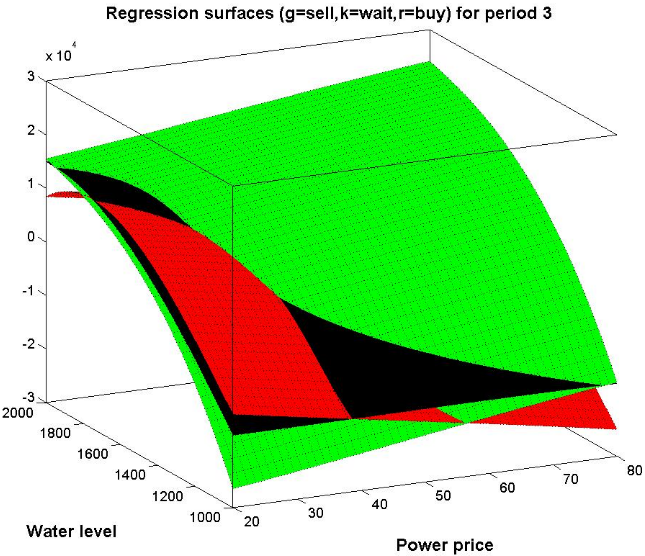
- Sources of approximations come from the simulation, the regression on basis, the backwash.
- Tsitsiklis-Van Roy vs Longstaff-Schwartz approaches.
- Given the backwash procedure, this algorithm is in fact hybrid of TVR and LS.

Illustration with a simple example

We consider a simple but interesting case of four half-days.

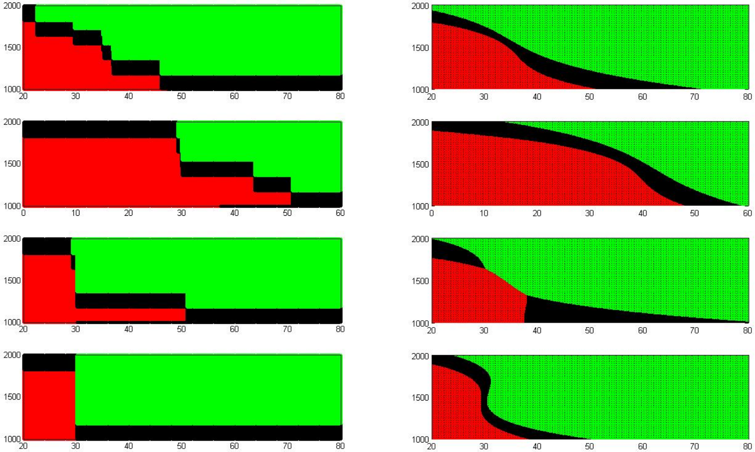
- Average price is 50$, except during the 2nd and 5th periods (average of 30$). Prices are serially independent.
- Three regimes : buy, sell, do nothing.
- Number of simulations in the learning phase : 50 000.
- Comparison is done with a fully discretized DP as benchmark.


Regression surfaces (g=sell,k=wait,r=buy) for period 3
Water level
2000
1900
1800
1700
1600
1500
1400
1300
1200
1100
1000
20
30
40
50
60
70
80
Power price


Regression surfaces (g=sell,k=wait,r=buy) for period 3
x 10^4
3
2
1
0
-1
-2
-3
2000
1800
1600
1400
1200
1000
Water level
20
30
40
50
60
70
80
Power price

Benchmark policies vs Simulation and Regression policies

Spot prices on x-axis ; water levels on y-axis.



Numerical results on a larger model

- Algorithm is run on a problem with the same state variables (price and water level) but 16 weeks long at two periods per day (224 time steps)
- Spot price follows a geometric brownian motion (we want to benchmark !)
- Daily, weekly, monthly seasonalities on the spot prices.
- We do out-of-sample testing against a finely discretized dynamic program. The benchmark value is 247 500 $.
- Obtain results within two percent of the optimal value :

Npath	Mean	Stdv
25000	242 765 $	277 $
50000	242 897 $	161 $
75000	242 900 $	128 $

Numerical results : pretty good or pretty bad ?

Pretty good or pretty bad ? Well, both...

- The quality of the results (sim-and-reg vs benchmark) is influenced by the bases and by the backwash procedure.
- Polynomial bases do their best, but are clearly imperfect. This is however could be rather good news for the backwash technique.
- Note that sim-and-reg and benchmark results are similarly impacted by the discretization of the decision.

Conclusions

- The classical simulations-and-regressions technique is extended to a more general problem with an endogenous (control-dependent) state variable.
- Neither the exogenous nor the endogenous variables are discretized.
- Simulation based, so very flexible with respect to the modeling of the exogenous, stochastic variables.

Future and on-going work

- Introduce more exogenous and endogenous variables.
- Introduce decisions that kick in only after a number of periods.
- Risk management.
- Non-energy applications.

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