Dynamic Cournot Models for Production of Exhaustible Commodities under Stochastic Demand

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Introduction

- **Cournot game framework**: Producers choose quantities of energy to produce and receive profit based on a single market price determined through aggregate supply. In the Cournot game, the strategic variable is quantity.
- **Dynamic game framework**: Producers choose quantities of energy instantaneously to maximize profit over a infinite time horizon.
- Ludkovski and Sircar (2011) studied the stochastic effect of resource exploration in dynamic Cournot Model of exhaustible resources, such as oil.
- We extend the dynamic Cournot model by considering stochastic demand that switches between high and low demand regimes. We study the producers' strategies under stochastic demand.

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The model settings

Exhaustible producer: produce exhaustible resources, such as oil, free of production cost, but undertake costly exploration to replenish his diminishing reserves.

- Reserve Process: $dX_t = -q(X_t)\mathbb{1}_{\{X_t>0\}}dt + \delta dN_t, X_0 = x$,
  - N_t = number of discoveries $\sim$ Poisson process with controlled intensity $a_t\lambda$.
  - $q_t = q(X_t)$ is instantaneous production rate.
- Exploration cost function: $C(a_t) = \kappa a_t + a_t^\beta/\beta$, a_t = exploration effort.
- Instantaneous net return: $q_t p_t - C(a_t)$.

Stochastic demand: price (inverse demand) function:

$$p_t = p(q_t, M_t) = H\mathbb{1}_{\{M_t=H\}} + L\mathbb{1}_{\{M_t=L\}} - q_t,$$

- $M_t \in \{L, H\}$: market demand regime (continuous-time Markov chain), L = low regime, H = high regime.
- Exponential holding rates on the two regimes are λ_0 and λ_1 respectively.

Monopoly Production and Exploration

Value functions of reserves of exhaustible resources in low and high regimes respectively:

$$\bullet v_L(x) = \sup_{q,a} \mathbb{E}\left[\int_0^\infty e^{-rt}(q_t p(q_t, M_t) - C(a_t))\, dt | X_0 = x, M_0 = L\right]$$

$$\bullet v_H(x) = \sup_{q,a} \mathbb{E}\left[\int_0^\infty e^{-rt}(q_t p(q_t, M_t) - C(a_t))\, dt | X_0 = x, M_0 = H\right]$$

- The exhaustible producer chooses optimal **production rates** $q_t := q(X_t)$ and **exploration efforts** $a_t := a(X_t)$ instantaneously at each $t \in [0, +\infty]$ to optimize the overall discounted net profit on infinite time horizon.

HJB Equations

The Hamilton-Jacobi-Bellman equations for the value functions v_0 and $v_1(x)$

$$
\begin{aligned}
0 &= \sup_{q_L}\left[q_L\,(L - q_L) - q_L\,v_L'\right] + \sup_{a_L}\left[a_L\lambda\Delta v_L - C(a_L)\right] \\
&\quad + \lambda_0 v_H - (\lambda_0 + r)v_L\,, \\
0 &= \sup_{q_H}\left[q_H\,(H - q_H) - q_H v_H'\right] + \sup_{a_H}\left[a_H\lambda\Delta v_H - C(a_H)\right] \\
&\quad + \lambda_1 v_L - (\lambda_1 + r)v_H,
\end{aligned}
$$

where $\Delta v_i(x) := v_i(x+\delta) - v_i(x), \quad i = L, H.$

- Production rates control:
  $q_L(x) = \frac{1}{2}\left(L - v_L'(x)\right)^+, \quad q_H(x) = \frac{1}{2}\left(H - v_H'(x)\right)^+.$
- Exploration efforts control:

  $$a_L(x) = [(\lambda\Delta v_L(x) - \kappa)^+]^{\gamma-1}, a_H(x) = [(\lambda\Delta v_H(x) - \kappa)^+]^{\gamma-1},$$

$$\gamma = \frac{\beta}{\beta - 1}.$$

Boundary conditions of the HJB equations

$$v_L(x) = \sup_{a_L \geq 0} \frac{v_H(x)\lambda_0 + v_L(x+\delta)\lambda a_L - C(a_L)}{r + \lambda_0 + \lambda a_L},$$

$$v_H(x) = \sup_{a_H \geq 0} \frac{v_L(x)\lambda_1 + v_H(x+\delta)\lambda a_H - C(a_H)}{r + \lambda_1 + \lambda a_H}.$$

We obtain the boundary condition by taking $x = 0$:

$$v_L(0) = \sup_{a_L \geq 0} \frac{v_H(0)\lambda_0 + v_L(\delta)\lambda a_L - C(a_L)}{r + \lambda_0 + \lambda a_L},$$

$$v_H(0) = \sup_{a_H \geq 0} \frac{v_L(0)\lambda_1 + v_H(\delta)\lambda a_H - C(a_H)}{r + \lambda_1 + \lambda a_H}.$$

Computational Results

- Use the numerical scheme in Ludkovski and Sircar(2011).

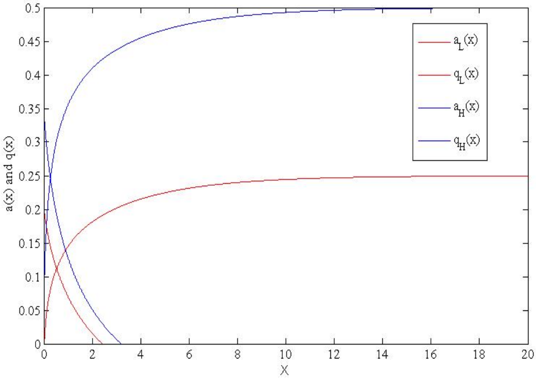

Figure : Optimal production rates $q_L(x)$, $q_H(x)$ (increasing) and optimal exploration rates $a_L(x)$, $a_H(x)$ (decreasing) under stochastic demand $p(x, M_0) = 1\mathbb{1}_{\{M_0=H\}} + \frac{1}{2}\mathbb{1}_{\{M_0=L\}} - q(x)$. The parameters are $\delta = 1$, $\lambda_0 = 1/3$, $\lambda_1 = 1/5$, $C(a) = 0.1a + a^2/2$.

Interpretation of q(x) and a(x)

- Optimal production rates
  - In both the two regimes, production rates $q(x)$ increase in reserves level x.
  - The marginal increment of $q(x)$ is monotonically decreasing, and $q_L(x) \rightarrow \frac{L}{2}$, $q_H(x) \rightarrow \frac{H}{2}$ as x goes to large enough.
- Optimal exploration efforts
  - $a_L(x)$ and $a_H(x)$ decrease in reserve level x, more remaining reserves needs less exploration efforts.
  - Low regime efforts $a_L(x)$ is less than high regime efforts $a_H(x)$, since in low regime production is less than high regime, thus less exploration efforts are needed.

Limit behavior of the model

- As $\lambda_0, \lambda_1 \to \infty$, mean holding time in each regime gets small, the frequency of regime-switching increases, the market gets more volatile:
  - **Production rates**: Production moves down in low regime and moves up in high regime. Producer holds reserves in low regime for expanding production in high regime to make more profit. Less mean holding time in low regime ensures that the discounted extra profit in high regime can cover the loss in low regime due to reduced production.
  - **Exploration efforts**: Exploration efforts move up in low regime and down in high regime. Since producer holds reserves in low regime for production in high regime, exploration efforts in high regime is not needed that much. In low regime more efforts are made to explore resources for production in high regime.

Limit behavior of the model

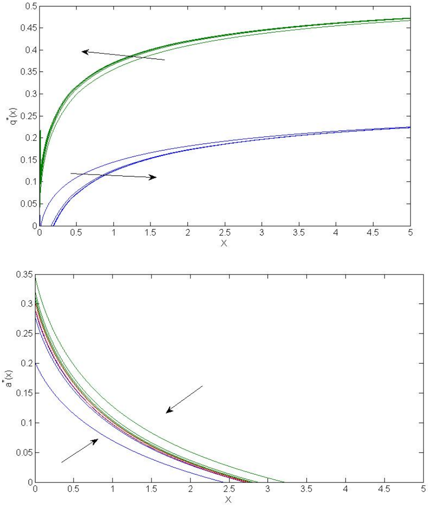

Figure : Top panel: production rate $q_L(x)$ shifts to the right, $q_H(x)$ shifts to the left. Bottom panel:exploration rates $a_L(x)$ shifts to the right, $a_H(x)$ shifts to the left. $\lambda_0 = m/3$, $\lambda_1 = m/5$, $m = 1, 5, 10, 15, 20$.

Duopoly with a green producer

We consider the duopoly with a green producer:

- **Exhaustible resources producer** (player 1): produces exhaustible resource with zero cost, but needs exploration cost. Instantaneous net return

$$q_t^1 p_t - C(a_t)$$

- **Green producer** (player 2): produces inexhaustible resources,e.g. solar power, but need positive fixed marginal production costs $c > 0$. Instantaneous net return

$$q_t^2 p_t - c q_t^2$$

Price (inverse demand) function :
$p_t = p(q_t, M_t) = H\mathbb{1}_{\{M_t = H\}} + L\mathbb{1}_{\{M_t = L\}} - q_t^1 - q_t^2$, $M_t \in \{L, H\}$.

Duopoly with a green producer

- $v_L(x), v_H(x)$: value functions of the exhaustible producer in low and high regimes;

$$v_L(x) = \sup_{q^1,a} E\left[\int_0^\infty e^{-rt}(q_t^1 p(q_t^1,(q_t^2)^*, M_t) - C(a_t))\,dt | X_0 = x, M_0 = L\right]$$

$$v_H(x) = \sup_{q^1,a} E\left[\int_0^\infty e^{-rt}(q_t^1 p(q_t^1,(q_t^2)^*, M_t) - C(a_t))\,dt | X_0 = x, M_0 = H\right]$$

- $g_L(x), g_H(x)$: value functions of the green producer in low and high regimes.

$$g_L(x) = \sup_{q^2} E\left[\int_0^\infty e^{-rt}(q_t^2(p((q_t^1)^*, q_t^2, M_t) - c))\,dt | X_0 = x, M_0 = L\right]$$

$$g_H(x) = \sup_{q^2} E\left[\int_0^\infty e^{-rt}(q_t^2(p((q_t^1)^*, q_t^2, M_t) - c))\,dt | X_0 = x, M_0 = H\right]$$

HJB equations of the value functions

- $$\sup_{q_0^1}\left[q_0^1\left(L-q_0^1-(q_0^2)^*\right)-v_0'(x)q_0^1\right]+\sup_{a_0}\left[a_0\lambda\Delta v_0(x)-C(a_0)\right]$$
  $$+\lambda_0 v_1(x)-(\lambda_0+r)v_0(x)=0,$$
- $$\sup_{q_1^1}\left[q_1^1\left(H-q_1^1-(q_1^2)^*\right)-v_1'(x)q_1^1\right]+\sup_{a_1}\left[a_1\lambda\Delta v_1(x)-C(a_1)\right]$$
  $$+\lambda_1 v_0(x)-(\lambda_1+r)v_1(x)=0,$$
- $$\sup_{q_0^2}\left[q_0^2\left(L-(q_0^1)^*-q_0^2-c\right)\right]-g_0'(x)(q_0^1)^*+a_0\lambda\Delta g_0(x)$$
  $$+\lambda_0 g_1(x)-(r+\lambda_0)g_0(x)=0,$$
- $$\sup_{q_1^2}\left[q_1^2\left(H-(q_1^1)^*-q_1^2-c\right)\right]-g_1'(x)(q_1^1)^*+a_1\lambda\Delta g_1(x)$$
  $$+\lambda_1 g_0(x)-(r+\lambda_1)g_1(x)=0.$$

Nash equilibrium of the two players' strategies

• Optimal production rates:

- $(q_L^{oil}(x))^* = \frac{1}{2}\max\Big(L - (q_L^{green}(x))^* - v_L'(x), 0\Big)$,
- $(q_L^{green}(x))^* = \frac{1}{2}\max\Big(L - (q_L^{oil}(x))^* - c, 0\Big)$;
- $(q_H^{oil}(x))^* = \frac{1}{2}\max\Big(H - (q_H^{green}(x))^* - v_H'(x), 0\Big)$,
- $(q_H^{green}(x))^* = \frac{1}{2}\max\Big(H - (q_H^{oil}(x))^* - c, 0\Big)$.

• Optimal exploration efforts:

- $a_L(x) = [(\lambda \Delta v_L(x) - \kappa)^+]^{\gamma-1}$,
- $a_H(x) = [(\lambda \Delta v_H(x) - \kappa)^+]^{\gamma-1}, \gamma = \frac{\beta}{\beta-1}, C(a) = \kappa a + \frac{a^\beta}{\beta}$

Computational results

The exhaustible producer

- The production rate is increasing in x, and increasing in c, since green production is disencouraged as c increases.

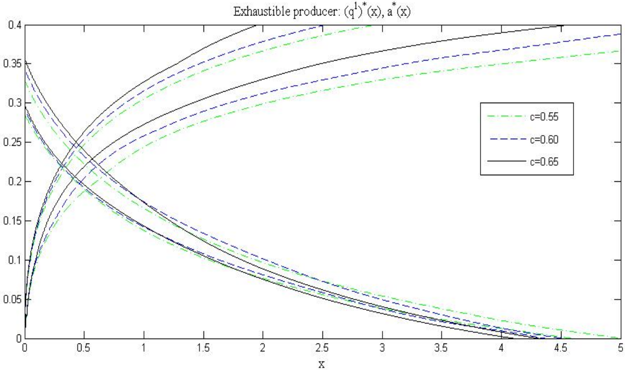

Figure : Exhaustible producer's optimal production rate $q_L^{oil}(x)$ and $q_H^{oil}(x)$ for $c = 0.55, 0.60, 0.65$, and $L = 0.75, H = 1$.

Computational results

The green producer

• Green production rate is decreasing in reserves level x.
• Green production decreases as cost c increases in both high and low regimes.

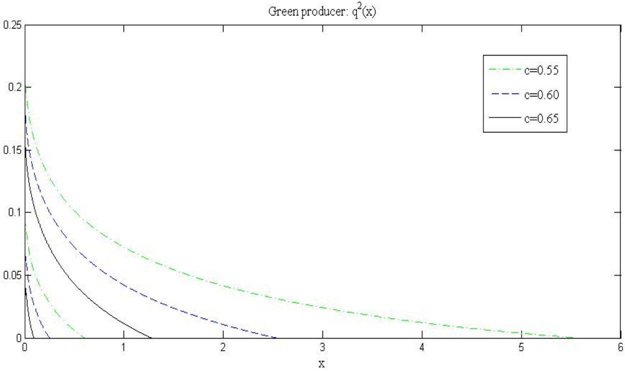

Figure : Green producer's optimal production rate $q_L^{green}(x)$ and $q_H^{green}(x)$ for $c = 0.55, 0.60, 0.65$, and $L = 0.75, H = 1$.

Saturation and Blockading

- **Saturation of exhaustible resources exploration**: exploration efforts are made if reserves level x is small, and stopped if reserves level x is large. We are interested in the critical reserve level $x > 0$ where exploration is closed.
  - $x_{sat} := \sup\{x > 0 : a(x) > 0\}$.
- **Blockading of green production**: exhaustible production increases in x, green production decreases in x. We are interested in critical reserves level $x > 0$ where the green production is blockaded.
  - $x_b := \sup\{x > 0 : q^{green}(x) > 0\}$.

Saturation and Blockading

as a function of green production cost c.

Consider x_{sat} and x_b as functions of green production cost c:

- x_{sat}:
  - when c is small ($c < 0.55$ in the figure), green producer is the leader, exhaustible producer is disencouraged and lowers exploration.
  - As c increases moderately, the exhaustible producer begins to lead the market, and therefore increase exploration efforts.
  - When c is large ($c > 0.6$ in the figure), green production is highly discouraged or even blockaded, the exhaustible producer becomes effective leader of the market, thus exploration efforts no longer increases with c.
- x_b: increase in c discourages green production and thus lower the blockading level x_b.

Saturation and Blockading

as a function of green production cost c.

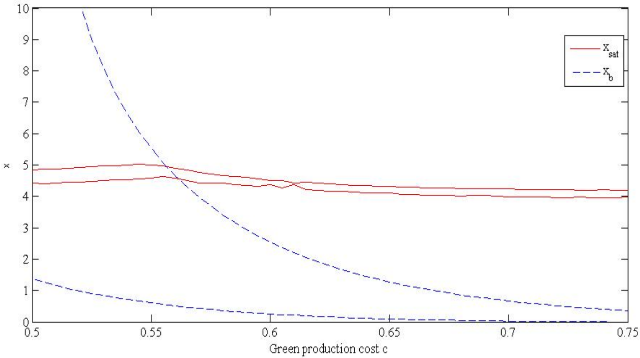

Figure : The higher curves are for high regime, the lower curves are for low regime. We take $L = 0.75, H = 1$.

Saturation and Blockading

as a function of start-up exploration cost κ.

Consider x_{sat} and x_b as a function of start-up exploration cost κ.

- Recall that the cost function of exploration efforts is $C(a) = \kappa a + a^\beta/\beta$
- *Exhaustible producer*: As κ increases, exploration gets more costly, exploration efforts a decreases. Thus x_{sat} decreases in κ in both regimes.
- *Green producer*: As κ increases, exploration efforts of exhaustible resources decreases, and thus less reserves can be used for production. The exhaustible production decreases, and the green production increases. Therefore x_b increases in κ.

Saturation and Blockading

as a function of start-up exploration cost κ.

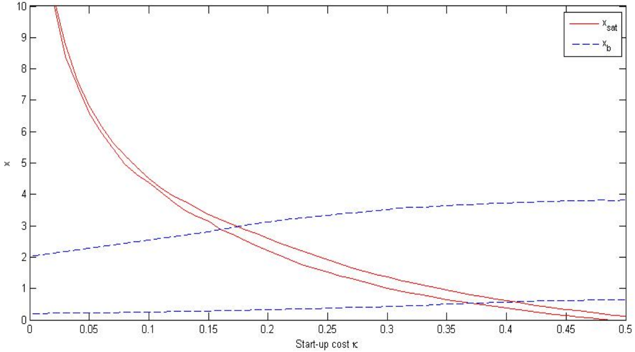

Figure : x_{sat} and x_b as a function of start-up exploration cost κ. The higher curves are for high-demand regime, the lower curves are for low-demand regime. We take $L = 0.75, H = 1$, and $c = 0.6$.

Exhaustible production shut-down

- Since the high regime is more profitable than the low regime, the exhaustible producer may shut down production and hold reserves in low regime and restart production after switch to the high regime to gain more profit.
- When the difference between the two regimes, i.e. the difference between H and L, are large, the profitability in high regime is much larger than the low regime, and therefore the producer may shut down production and hold until switches to the high regime to gain more profit.
- Notation: $x_{start} := \inf\left\{x > 0 : q_L^{oil}(x) > 0\right\}$.

Exhaustible production shut-down

- x_{start} as a function of H

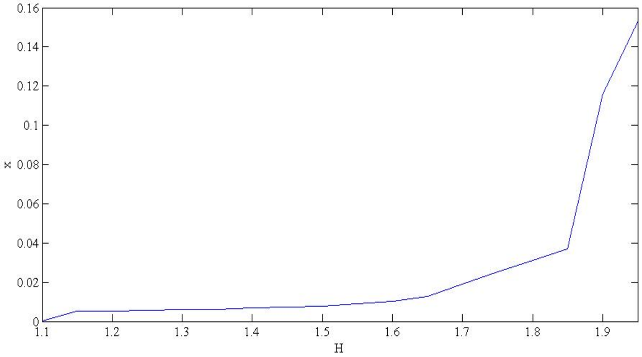

Figure : x_{start} as a function of H. We take $L = 0.75, c = 0.6$.

Exhaustible production shut-down

x_{start} as a function of low regime holding rate λ_0

As λ_0 increases, the mean holding time in the low regime dcreases. The green producer chooses to reduce the production and restart after switch to the high regime to gain more profit. Therefore x_{start} is a increasing function in low demand holding rate λ_0.

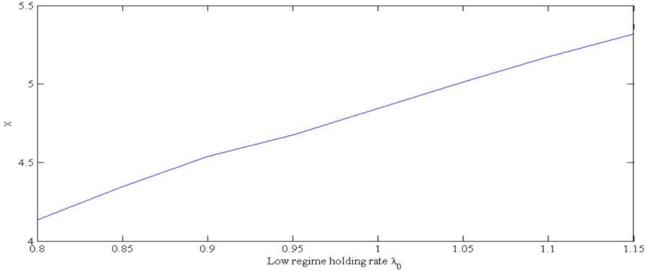

Figure : x_{start} as a function of low regime holding rate λ_0. $L = 0.5$, $H = 1$, $c = 0.3$.

Exhaustible production shut-down

- x_{start} as a function of green production cost c
  - When c is small ($c << L$), green production is not blockaded in both low and high regimes, green producer leads the market in both the two regimes. So there is no reason to shut down exhaustible production in low regime, unless there are some other reasons (e.g. $H >> L$).
  - When c is large ($c \rightarrow H$), green production is highly blockaded in high regime and totally blockaded in low regime. Green production cost c plays little role in affecting x_{start}.
  - When c is moderate (around L), the effect of c on x_{start} is ambiguous, since as c increases in this region, both high and low regimes are in favor of exhaustible producer.

Note: please see the next slide for illustration.

Exhaustible production shut-down

- x_{start} as a function of green production cost c.
  - The effect of c on x_{start} is ambiguous.

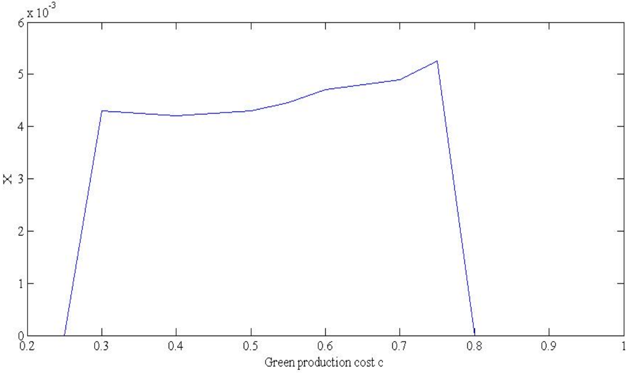

Figure : x_{start} as a function of green production cost c. We take $L = 0.5, H = 1$.