Propagation of Long-crested Water Waves

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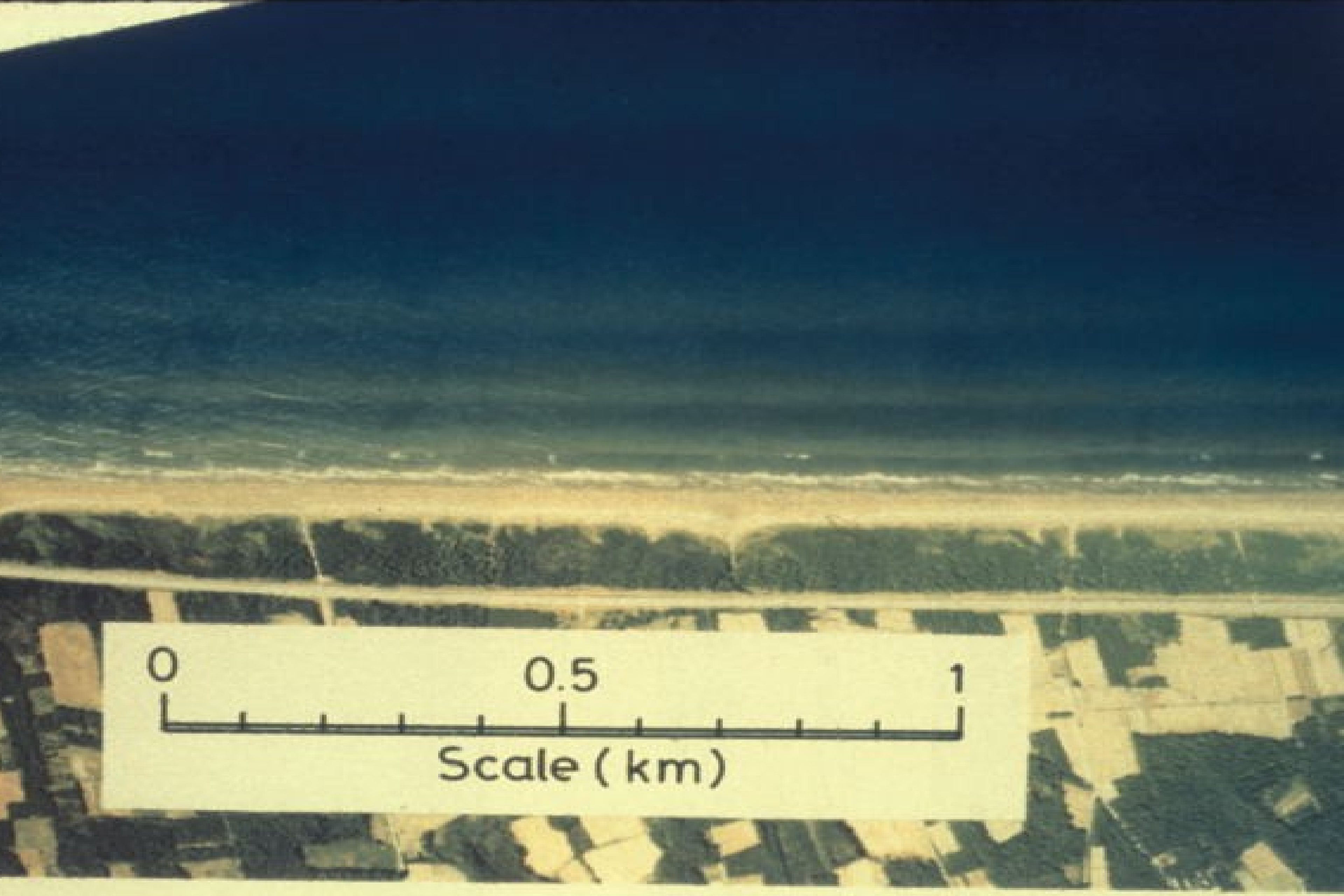
Examples of Long-crested Waves


Horizon – Freak Wave

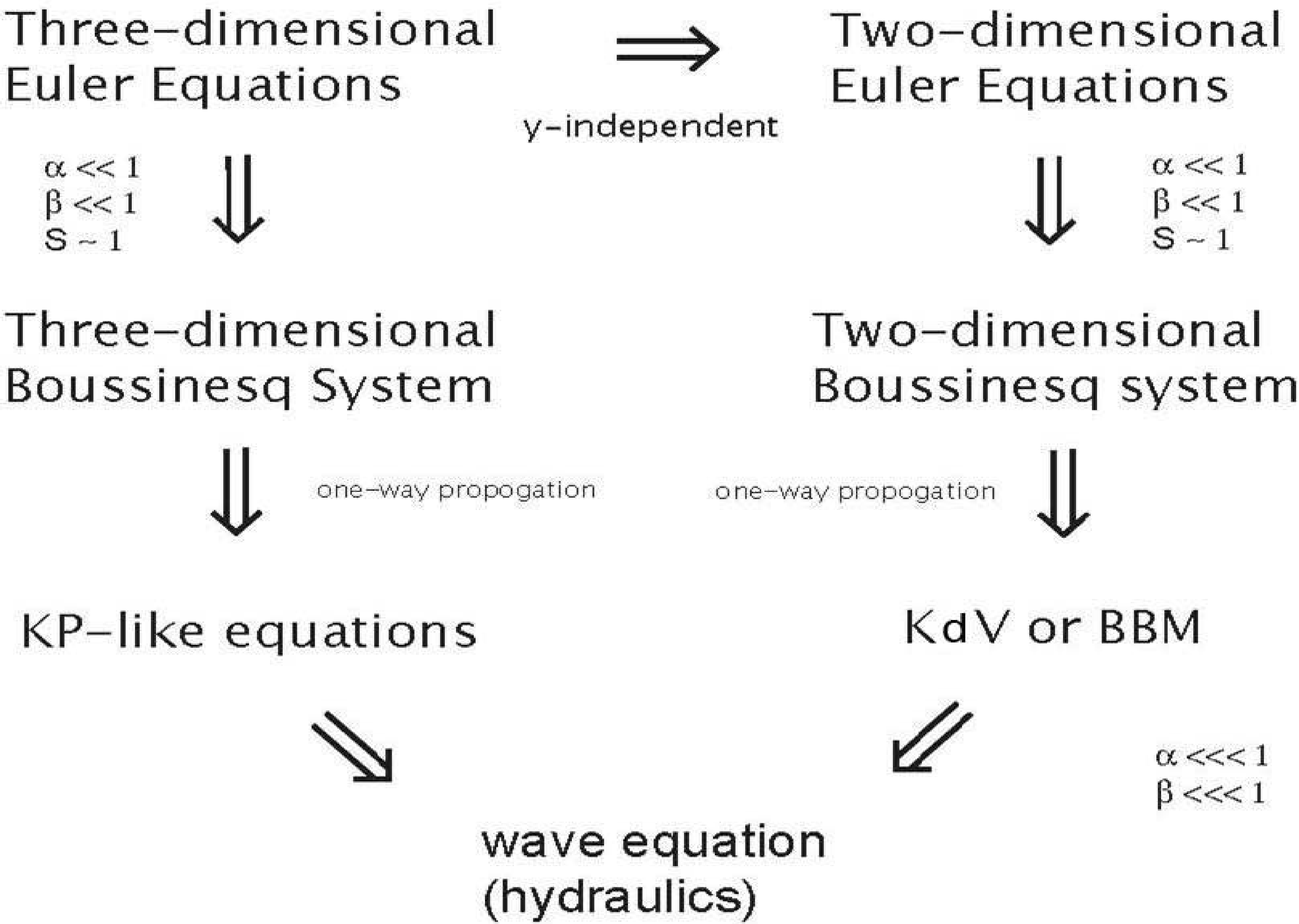
Evidence of the Persistance of Long-crested Waves



(a) Sept. 6, 1975 (11h. 29m.). Reference point No.42 to No.54.

otograph 1 Multiple longshore bars at Hakui Beach, Japan taken by the Geographical Survey Institute, Ministry of Construction. Number of reference points

Analysis of an Example Problem



Three-dimensional Euler Equations
y-independent
Two-dimensional Euler Equations
α << 1
β << 1
S ~ 1
α << 1
β << 1
S ~ 1
Three-dimensional Boussinesq System
Two-dimensional Boussinesq system
one-way propogation
one-way propogation
KP-like equations
KdV or BBM
α <<< 1
β <<< 1
wave equation
(hydraulics)

The a-b-c-d systems

$$V_t + \nabla\eta + \epsilon\left(\frac{1}{2}\nabla|V|^2 + a\nabla\triangle\eta - b\triangle V_t\right) = 0$$

$$\eta_t + \nabla\cdot V + \epsilon\left(\nabla\cdot(\eta V) + c\triangle\nabla V - d\triangle\eta_t\right) = 0$$

where $$a = \frac{1-\theta^2}{2}\mu \qquad b = \frac{1-\theta^2}{2}(1-\mu)$$

$$c = \frac{\theta^2-\frac{1}{3}}{2}\lambda \qquad d = \frac{\theta^2-\frac{1}{3}}{2}(1-\lambda)$$

and $\theta \in [0,1], \lambda, \mu \in \mathcal{R}$ present themselves as possible models for the surface waves.

Two-dimensional Versions
(negligable variation in the y-direction)

$$\begin{aligned} \eta_t + u_x + \epsilon((u\eta)_x + a u_{xxx} - b\eta_{xxt}) &= 0 \\ u_t + \eta_x + \epsilon(u u_x + c\eta_{xxx} - d u_{xxt}) &= 0 \end{aligned}$$

where
$$\begin{aligned} a &= \frac{1}{2}\left(\theta^2 - \frac{1}{3}\right)\lambda \\ b &= \frac{1}{2}\left(\theta^2 - \frac{1}{3}\right)(1-\lambda) \\ c &= \frac{1}{2}(1-\theta^2)\mu \\ d &= \frac{1}{2}(1-\theta^2)(1-\mu) \end{aligned}$$

$$0 \le \theta \le 1, \lambda, \mu \in \mathcal{R}, \quad a + b + c + d = \frac{1}{3}$$

coupled KdV system :

$$\eta_t + u_x + (u\eta)_x + \frac{1}{6}u_{xxx} = 0$$

$$u_t + \eta_x + uu_x + \frac{1}{6}\eta_{xxx} = 0$$

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regularized Boussinesq system :

$$\eta_t + u_x + (u\eta)_x - \frac{1}{6}\eta_{xxt} = 0$$

$$u_t + \eta_x + uu_x - \frac{1}{6}u_{xxt} = 0$$

Boussinesq's 'original' system:

$$\begin{aligned} \eta_t + u_x + (u\eta)_x &= 0 \\ u_t + \eta_x + uu_x - u_{xxt} &= 0 \end{aligned}$$

Bona-Smith system:

$$\begin{aligned} \eta_t + u_x + (u\eta)_x - b\eta_{xxt} &= 0 \\ u_t + \eta_x + uu_x + c\eta_{xxx} - bu_{xxt} &= 0 \end{aligned}$$

The a-b-c-d systems

$$V_t + \nabla\eta + \epsilon\left(\frac{1}{2}\nabla|V|^2 + a\nabla\triangle\eta - b\triangle V_t\right) = 0$$

$$\eta_t + \nabla\cdot V + \epsilon\left(\nabla\cdot(\eta V) + c\triangle\nabla V - d\triangle\eta_t\right) = 0$$

where $\quad a = \dfrac{1-\theta^2}{2}\mu \qquad b = \dfrac{1-\theta^2}{2}(1-\mu)$

$$c = \frac{\theta^2 - \frac{1}{3}}{2}\lambda \qquad d = \frac{\theta^2 - \frac{1}{3}}{2}(1-\lambda)$$

and $\quad \theta \in [0,1], \lambda, \mu \in \mathcal{R}$ present themselves as possible models for the surface waves.

If we set $\theta = \sqrt{\frac{2}{3}}$ and $\mu = \lambda = 0$, the Boussinsq equations become the coupled system

$$V_t + \nabla \eta + \epsilon \left(\frac{1}{2} \nabla |V|^2 - \frac{1}{6} \triangle V_t \right) = 0,$$

$$\eta_t + \nabla \cdot V + \epsilon \left(\nabla \cdot (\eta V) - \frac{1}{6} \triangle \eta_t \right) = 0.$$

A simle rescaling of the independent and dependent variables gives the tidy form

$$V_t + \nabla\eta + \nabla|V|^2 - \triangle V_t = 0,$$

$$\eta_t + \nabla\cdot V + \nabla\cdot(\eta V) - \triangle\eta_t = 0.$$

Happy Birthday, Walter!!
And best wishes for many, many more!!