Hasselmann equation revisited

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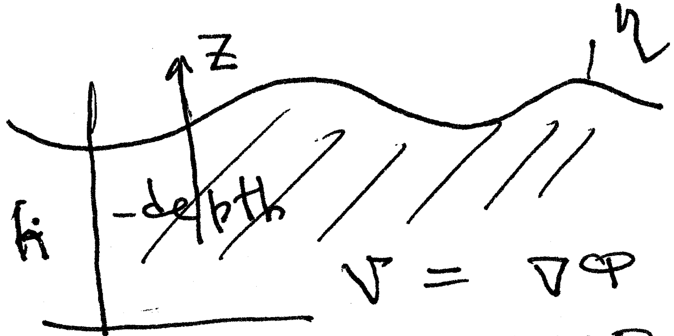
(1)



η = surface elevation

$\psi = \varphi \big|_{z=\eta}$

$v = \nabla \varphi$ $\qquad \Delta \varphi = 0$

$\operatorname{div} v = 0$

$H = T + U$ — total energy

$$\frac{\partial \eta}{\partial t} = \frac{\delta H}{\delta \psi} \qquad \frac{\delta \psi}{\partial t} = -\frac{\delta H}{\delta \eta}$$

After Fourier transformation

$$\frac{\partial \eta_k}{\partial t} = \frac{\delta \hat{H}}{\delta \psi_k^*} \qquad \frac{\partial \psi_k}{\partial t} = -\frac{\delta \hat{H}}{\delta \eta_k}$$

expansion in powers of $k\eta$

$$\hat{H} = H_0 + H_1 + H_2$$

$$H_0 = \frac{1}{2}\int \left\{A_k|\Psi_k|^2 + g\,|\eta_k|^2\right\} dk, \quad A_k = k \tan kH,$$

$$H_1 = \frac{1}{2}\int L^{(1)}(k_1,k_2)\Psi_{k_1}\Psi_{k_2}\eta_{k_3}\,\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\,dk_1\,dk_2\,dk_3,$$

$$H_2 = \frac{1}{2}\int L^{(2)}(k_1,k_2,k_3,k_4)\Psi_{k_1}\Psi_{k_2}\eta_{k_3}\eta_{k_4}$$
$$\times\,\delta(k_1+k_2+k_3+k_4)\,dk_1\,dk_2\,\eta_{k_3}\eta_{k_4}.$$

Here

$$L^{(1)}(k_1,k_2) = -(k_1,k_2) - A_{k_1}A_{k_2}$$
$$L^{(2)}(k_1,k_2,k_3,k_4) = \tfrac{1}{2}\,(k_1^2 A_2 + k_2^2 A_1) + \tfrac{1}{4}A_1A_2(A_{1+3}+A_{2+4} + A_{1+4}+A_{2+3}).$$

Now we can introduce normal variables a_k:

$$\eta_k = \frac{1}{\sqrt{2}}\left(\frac{A_k}{g}\right)^{1/4}(a_k + a_{-k}^*),$$

$$\Psi_k = \frac{i}{\sqrt{2}}\left(\frac{g}{A_k}\right)^{1/4}(a_k - a_{-k}^*).$$

Normal variables obey the following Hamiltonian equations:

$$\frac{\partial a_k}{\partial t} + i\frac{\delta H}{\delta a_k^*} = 0.$$

$$H_0 = \int \omega_k |a_k|^2 \, \mathrm{d}k,$$

$$\begin{aligned} H_1 = \frac{1}{2} \int & V^{(1,2)}_{kk_a k_2} (a_k a^*_{k_1} a^*_{k_2} + a^*_k a_{k_1} a_{k_2}) \\ & \times \delta(k - k_1 - k_2) \, \mathrm{d}k \, \mathrm{d}k_1 \, \mathrm{d}k_2 \\ + \frac{1}{6} \int & V^{(0,3)}_{kk_a k_2} (a_k a_{k_1} a_{k_2} + a^*_k a^*_{k_1} a^*_{k_2}) \\ & \times \delta(k + k_1 + k_2) \, \mathrm{d}k \, \mathrm{d}k_1 \, \mathrm{d}k_2. \quad + \;\ldots \end{aligned}$$

$$\begin{aligned} V^{(1,2)}_{kk_1k_2} = \frac{g^{1/4}}{2\sqrt{2}} \Bigg\{ & \left(\frac{A_k}{A_{k_1} A_{k_2}} \right)^{1/4} L^{(1)}(k_1, k_2) - \left(\frac{A_{k_1}}{A_k A_{k_2}} \right)^{1/4} \\ & \times L^{(1)}(-k, k_1) - \left(\frac{A_{k_2}}{A_k A_{k_1}} \right)^{1/4} L^{(1)}(-k, k_2) \Bigg\}, \end{aligned}$$

$$\begin{aligned} V^{(0,3)}_{kk_1k_2} = \frac{g^{1/4}}{2\sqrt{2}} \Bigg\{ & \left(\frac{A_k}{A_{k_1} A_{k_2}} \right)^{1/4} L^{(1)}(k_1, k_2) + \left(\frac{A_{k_1}}{A_k A_{k_2}} \right)^{1/4} \\ & \times L^{(1)}(k, k_1) + \left(\frac{A_{k_2}}{A_k A_{k_1}} \right)^{1/4} L^{(1)}(k, k_2) \Bigg\}. \end{aligned}$$

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On the next step we perform the canonical transformation excluding cubic terms in the Hamiltonian
To do this we introduce new variables p, q

$$a_k = \frac{1}{\sqrt{2}}(q_k + i p_k) \qquad q_{-k} = q_k^* \quad p_{-k} = p_k^*$$

The functions q_k, p_k obey the equations

$$\frac{\partial q_k}{\partial t} = \frac{\delta H}{\delta p_k^*} \qquad \frac{\delta p_k}{\partial t} = -\frac{\delta H}{\delta q_k^*}$$

We perform the canonical transformation, ~~to~~ excluding cubical terms in the Hamiltonian. To do this we introduce new variables R_k, ξ_k, connected with the "old" variables by relations

$$p_k = \frac{\delta S}{\delta q_{-k}} \qquad S \text{ is generating function}$$

$$\xi_k = \frac{\delta S}{\delta R_{-k}}$$

$$S = \int R_k\, q_k\, \mathrm{d}k + \frac{1}{2}\int A_{kk_1k_2}\, q_k\, q_{k_1}\, R_{k_2} \times \delta(k+k_1+k_2)\, \mathrm{d}k\, \mathrm{d}k_1\, \mathrm{d}k_2 + \frac{1}{3}\int B_{kk_1k_2}\, R_k\, R_{k_1}\, R_{k_2}\, \delta(k+k_1+k_2)\, \mathrm{d}k\, \mathrm{d}k_1\, \mathrm{d}k_2.$$

$$A_{kk_1k_2} = -\frac{1}{4}\left(\frac{L_0+L_1+L_2}{\omega_0+\omega_1+\omega_2} + \frac{L_0+L_1-L_2}{\omega_0+\omega_1-\omega_2}\right) + \frac{1}{4}\left(\frac{L_0-L_1-L_2}{\omega_0-\omega_1-\omega_2} + \frac{L_1-L_0-L_2}{\omega_1-\omega_0-\omega_2}\right),$$

$$B_{kk_1k_2} = -\frac{1}{4}\left(\frac{L_0+L_1+L_2}{\omega_0+\omega_1+\omega_2} + \frac{L_0-L_1-L_2}{\omega_0-\omega_1-\omega_2}\right) - \frac{1}{4}\left(\frac{L_1-L_0-L_2}{\omega_1-\omega_0-\omega_2} + \frac{L_2-L_0-L_1}{\omega_2-\omega_0-\omega_1}\right).$$

Here

$$L_0 = L_{kk_1k_2}, \quad L_1 = L_{k_1kk_2}, \quad L_2 = L_{k_2kk_1},$$

$$\omega_0 = \omega_k, \quad \omega_1 = \omega_{k_1}, \quad \omega_2 = \omega_{k_2}.$$

$$L_{kk_1k_2} = \frac{g^{1/4}\, A_k^{1/4}}{A_{k_1}^{1/4}\, A_{k_2}^{1/2}}\, L^{(1)}_{k_1k_2}.$$

Now

$$H_0 = \frac{1}{2}\int \omega_k\,(|q_k|^2 + |p_k|^2)\, \mathrm{d}k,$$

$$H_1 = \frac{1}{2}\int L_{kk_1k_2}\, q_k\, p_{k_1}\, p_{k_2}\, \delta(k+k_1+k_2)\, \mathrm{d}k\, \mathrm{d}k_1\, \mathrm{d}k_2,$$

$+ \ldots$

New normal variables (for infinite depth)

b_k will be as follows:

$$b_k = \frac{1}{\sqrt{2}}\left(\left(\frac{g}{A_k}\right)^{1/4}\xi_k - \mathrm{i}\left(\frac{A_k}{g}\right)^{1/4}R_k\right). \qquad (3.42)$$

$$\frac{\partial b_k}{\partial t} + i\frac{\delta H}{\delta b_k^*} = 0$$

New normal variables b_k satisfy Zakharov's equation [6]

$$\frac{\partial b_k}{\partial t} + \mathrm{i}\omega_k b_k + \frac{\mathrm{i}}{2}\int T_{kk_1k_2k_3}\, b_{k_1}^*\, b_{k_2}\, b_{k_3}\, \delta_{k+k_1-k_2-k_3}\, dk_1\, dk_2\, dk_3 = 0.$$

$$T_{k_1k_2k_3k_4} = \frac{1}{2}\left(\hat{T}_{k_1k_2k_3k_4} + \hat{T}_{k_2k_1k_3k_4}\right)$$

$$\begin{aligned}
\hat{T}_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} = & -\frac{1}{16\pi^2}\frac{1}{(k_1k_2k_3k_4)^{1/4}}\Big\{ \\
& +\frac{1}{2}\left(k_{1+2}^2 - (\omega_1+\omega_2)^4\right)\left[(\mathbf{k}_1\mathbf{k}_2 - k_1k_2) + (\mathbf{k}_3\mathbf{k}_4 - k_3k_4)\right] \\
& -\frac{1}{2}\left(k_{1-3}^2 - (\omega_1-\omega_3)^4\right)\left[(\mathbf{k}_1\mathbf{k}_3 + k_1k_3) + (\mathbf{k}_2\mathbf{k}_4 + k_2k_4)\right] \\
& -\frac{1}{2}\left(k_{1-4}^2 - (\omega_1-\omega_4)^4\right)\left[(\mathbf{k}_1\mathbf{k}_4 + k_1k_4) + (\mathbf{k}_2\mathbf{k}_3 + k_2k_3)\right] \\
& +\left(\frac{4(\omega_1+\omega_2)^2}{k_{1+2} - (\omega_1+\omega_2)^2} - 1\right)(\mathbf{k}_1\mathbf{k}_2 - k_1k_2)(\mathbf{k}_3\mathbf{k}_4 - k_3k_4) \\
& +\left(\frac{4(\omega_1-\omega_3)^2}{k_{1-3} - (\omega_1-\omega_3)^2} - 1\right)(\mathbf{k}_1\mathbf{k}_3 + k_1k_3)(\mathbf{k}_2\mathbf{k}_4 + k_2k_4) \\
& +\left(\frac{4(\omega_1-\omega_4)^2}{k_{1-4} - (\omega_1-\omega_4)^2} - 1\right)(\mathbf{k}_1\mathbf{k}_4 + k_1k_4)(\mathbf{k}_2\mathbf{k}_3 + k_2k_3)\Big\}
\end{aligned}$$

$$k_{1+2}^2 = |\vec{k}_1 + \vec{k}_2|^2$$

$$k_{1-3}^2 = |\vec{k}_1 - \vec{k}_3|^2$$

$$k_{1-4}^2 = |k_1 - k_4|^2$$

$$H = \int \omega_k b_k b_k^* dk + \frac{1}{4}\int T_{k_1k_2k_3k_4}\, b_{k_1}^* b_{k_2}^* b_{k_3} b_{k_4} \times \delta_{k_1+k_2-k_3-k_4}\, dk_1 dk_2 dk_3 dk_4$$

3. Properties of the new Hamiltonian

A. Symmetry: $T_{1234} = T_{2134} = T_{1243} = T_{3412}$

B. The diagonal part

$$T_{12} = T_{kk_1,kk_1} = -\frac{1}{8\pi^2}\frac{1}{(kk_1)^{1/2}}\left\{ 3k^2k_1^2 + (kk_1)^2 - \frac{4\omega_1\omega_2}{g^2}(kk_1)(k+k_1) + \frac{2(\omega_1+\omega_2)^2\left[(kk_1)^2 - kk_1\right]^2}{\omega_{1+2}^2 - (\omega_1+\omega_2)^2} + \frac{2(\omega_1-\omega_2)^2\left[(kk_1)+kk_1\right]^2}{\omega_{1-2}^2 - (\omega_1-\omega_2)^2} \right\}$$

If $k_1 << k$ $\qquad \cos\theta = \frac{(kk_1)}{kk_1}$

$$T_{12} \simeq \frac{1}{2\pi^2} k_1^2 k \cos\theta$$

If we denote $k_1 = \varepsilon k$ $\quad \varepsilon << 1$

$$T_{12} \simeq \frac{\varepsilon^2}{2\pi^2} k^2 \cos\theta.$$

Two first orders in ε are cancelled.

In one-dimensional case $\quad \cos\theta = 1$

$$T_{12} = \frac{1}{2\pi^2}\begin{cases} k_1^2 k & k_1 < k \\ k_1 k^2 & k < k_1 \end{cases}$$

This is the first misterious cancellation!

(c) Asymptotic behavior of T_{1234}

The "coupling coefficient" T_{k_1,k_2,k_3,k_4} must "be evaluated to the resonant manifold

$$\omega_{k_1} + \omega_{k_2} = \omega_{k_3} + \omega_{k_4}$$

$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{k}_4$$

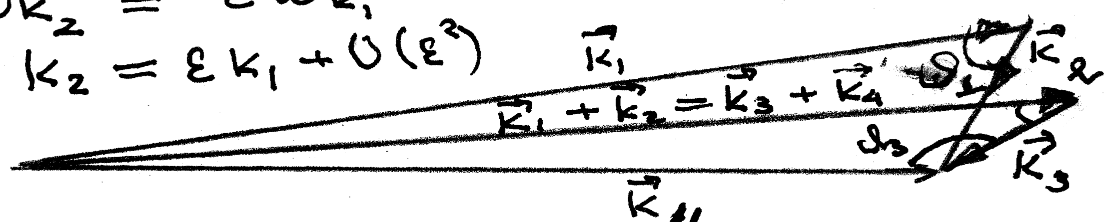
Recall that $\omega_k = \sqrt{gk}$

Let $\omega_{k_3} \ll \omega_{k_1}$ suppose $\omega_{k_3} = \varepsilon^{1/2}\omega_{k_1}$, $k_3 = \varepsilon k_1$

$\varepsilon \ll 1$. Then automatically

$$\omega_{k_2} = \varepsilon^{1/2}\omega_{k_1} + O(\varepsilon^2)$$

$$k_2 = \varepsilon k_1 + O(\varepsilon^2)$$



$$\vec{k}_4 = \vec{k}_1 + O(\varepsilon)$$

$$T_{k_1 k_2 k_3 k_4} \Rightarrow \frac{\varepsilon^2 k_1^3}{4\pi^2}\left[2(\cos\vartheta_1 + \cos\vartheta_3) - \sin(\vartheta_1 - \vartheta_3)(\sin\vartheta_1 - \sin\vartheta_3)\right]$$

$$(\vec{k}_1, \vec{k}_2) = k_1 k_2 \cos\vartheta_1$$

$$(\vec{k}_4, \vec{k}_3) = k_3 k_4 \cos\vartheta_3$$

In the case $\vartheta_1 = \vartheta_3$ we return to the diagonal case. Again two first terms in expansion on ε are cancelled

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One more misterious cancellation

Let all wave vectors $\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4$ are collinear and $k_1 > 0$, $k_2 > 0$ $k_3 > 0$ while $k_4 < 0$. In this case the resonant manifold can be univormided as follows

$$k_1 = A^2(\xi^2 + \xi + 1)^2 \quad k_2 = A^2 \xi^2 (\xi + 1)^2 \quad k_3 = A^2(\xi + 1)^2$$

$$k_4 = -A^2 \xi^2$$

$$\omega_1 = \frac{A}{g^{1/2}}(\xi^2 + \xi + 1) \quad \omega_2 = \frac{A\xi}{g^{1/2}}(\xi + 1) \quad \omega_3 = \frac{A}{g^{1/2}}(\xi + 1)$$

$$\omega_4 = \frac{A}{g}\xi$$

Then $T = T(A, \xi) \equiv 0!$

(Dyachenko, Zakharov 1994)

This remarkable fact makes possible to simplify the Hamiltonian by performing an additional canonical transformation (only in one-dimensional case) by exculding the „old" terms in the quartic Hamiltonian

$$b_k \rightarrow c_k$$

In new variables the coupling coefficient is drastically simplified. Now

$$T_{k k_1 k_2 k_3} = \frac{1}{16\pi^2} \Theta(k)\,\Theta(k_1)\,\Theta(k_2)\,\Theta(k_3) \times$$

$$\Big[k k_1 (k + k_1) + k_2 k_3 (k_2 + k_3) + k k_2 |k - k_2| + k k_3 |k - k_3| +$$

$$+ k_1 k_2 |k_1 - k_2| + k_1 k_3 (k_1 - k_3) \Big]$$

The new „compact reduced Hamiltonian" descriled interaction of wave propagating in one direction. It is very convinient for numerical simulation

The corresponding dynamic equation has nice solitonic solutions, but this equation is <u>non-integrable</u>, because the six-wave amplitude

$S_{k k_1 k_2 k_3 k_4 k_5}$ evaluated to the resonant manifold

$$\omega_k + \omega_{k_1} + \omega_{k_2} = \omega_{k_3} + \omega_{k_4} + \omega_5$$
$$k + k_1 + k_2 = k_3 + k_4 + k_5$$

is non-zero. (Dyachenko, Kachulin, Zakharov 2013).

<u>Conjecture:</u> (not prove yet)

In all orders of the perturbation theory waves propagating in one direction do not generate waves propagating in the opposite direction.

It means that the envelop solitons <u>live forever</u>. They do not loose energy by a backward radiation

4 The Hasselmann kinetic equation

To perform the statistical description of an ensemble of water waves one has to introduce the pair correlation function

$$\langle b_k b^*_{k'} \rangle = N_k \, \delta_{k-k'}$$

N_k is the "wave action spectrum", obeying the Hasselmann kinetic equation

$$\frac{\partial N_k}{\partial t} = S_{nl} + S_{in} + S_{diss}$$

Here S_{in} the wind input term, S_{diss} is responsible for dissipation due to wave breaking. These term are known only approximately from empirical date. But S_{nl} can be derived from the first principles

$$S_{nl} = \pi \int |T_{k k_1 k_2 k_3}|^2 \, \delta(\vec{k} + \vec{k}_1 - \vec{k}_2 - \vec{k}_3) \, \delta(\omega_k + \omega_{k_1} - \omega_{k_2} - \omega_{k_3}) \times$$
$$\times \left(N_{k_1} N_{k_2} N_{k_3} + N_k N_{k_2} N_{k_3} - N_k N_{k_1} N_{k_2} - N_k N_{k_1} N_{k_3} \right) \times$$
$$\times \, dk_1 \, dk_2 \, dk_3$$

The fundamental question: what are the solutions of the equation

$$S_{nl} = 0$$

?

Answer: a general solution symmetric with the reflection $N(k_x, k_y) = N(k_x, -k_y)$ depends on three arbitrary constants P, Q, R

$$N(k) = \frac{P^{1/3}}{k^4}\, G\left(\frac{Q\omega_k}{P}, \frac{R\omega_k}{k}, \cos\theta\right)$$

This is a Kolmogorov-type solution, P, R are fluxes of energy and momentum to high wave numbers, Q is the flux of wave action directed to small wave numbers

One can introduce polar coordinates on the K-plane $K_x = \frac{\omega^2}{g}\cos\varphi$ $K_y = \frac{\omega^2}{g}\sin\varphi$. Then in a very crude approximation one can replace the S_{nl} by an elliptic operator

$$S_{nl} \simeq \frac{H_0}{g^{4}}\left(\frac{\partial^2}{\partial\omega^2} + \frac{2}{\omega^2}\frac{\partial^2}{\partial\varphi^2}\right)\omega^{15} N^3$$

In this model case, which inherit the most important properties of S_{nl}

$$G(\omega,\varphi) = \frac{1}{(2\pi H_0)^{1/3}}\,\frac{g^{4/3}}{\omega^5}\left(P + \omega Q + \frac{R}{\omega}\cos\varphi\right)^{1/3}$$

For exact equation $G(\omega,\varphi)$ is not known even numerically. Only isotropic powerlike solutions are known. They are two (and only two, this is a rigorous theorem (so called KZ spectra)

$$N_1(k) = c_p \frac{P^{1/3}}{k^4} \quad \text{KZ spectrum for the direct cascade of energy}$$

$$N_2(k) = c_a \frac{Q^{1/3}}{k^{23/6}} \quad \text{KZ spectrum for the inverse cascade of wave action}$$

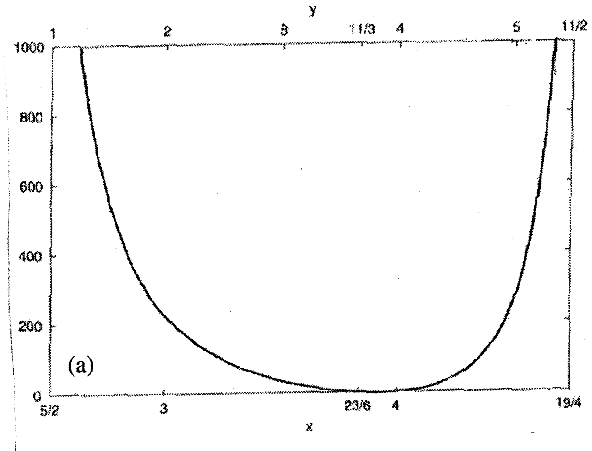
Both KZ spectra systematically observed in laboratory and numerical experiments as well as direct measurements of energy spectra in ocean

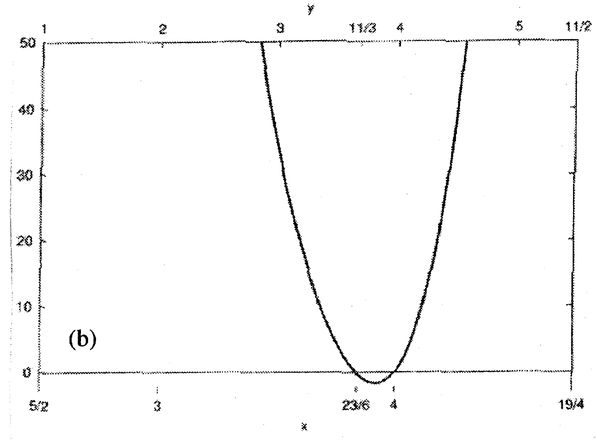
Let us look for isotropic powerlike solution assuming that

$$N_k = \frac{\lambda}{k^x} \qquad \text{Then}$$

$$S_{nl} = \frac{\lambda^3}{k^{3x - 19/2}} F(x)$$

Function $16\pi^2 F(x)$ found numericaley is plotted below





This i zoom on x – axis

The „window of opportunity" for x is $\frac{5}{2} < x < 19/4$

The Kolmogorov constants are expressed as follows

$$C_p = \left(\frac{3}{2\pi F'(4)}\right)^{1/3} \qquad C_q = \left(\frac{3}{2\pi \left|F'\left(\frac{23}{6}\right)\right|}\right)^{1/3}$$

Their numerical values are

$$C_p = 0{,}219 \qquad C_q = 0{,}237$$

Here η multiplied due to $16\pi^2 g^2$ to closer to experimental data. Knowledge of asymptotic behavior of $T_{k k_1 k_2 k_2}$ makes possible to simplify interaction of short and long waves

Suppose the spectrum of long waves is isotropic and known. It is $N_0(k)$. Then the behavior of short waves is described by the following diffusion equation

$$\frac{\partial N}{\partial t} = \frac{D}{k} \frac{\partial}{\partial k} k^3 \frac{\partial N}{\partial k}$$

where

$$D = \frac{5}{8} \pi^{-3} g^{3/2} \int_0^\infty q^{17/2} N^2(q)\, dq$$

Similar equations are used in financial mathematics

Conclusion: formation of „fat" powerlike spectra in ocean is very fast

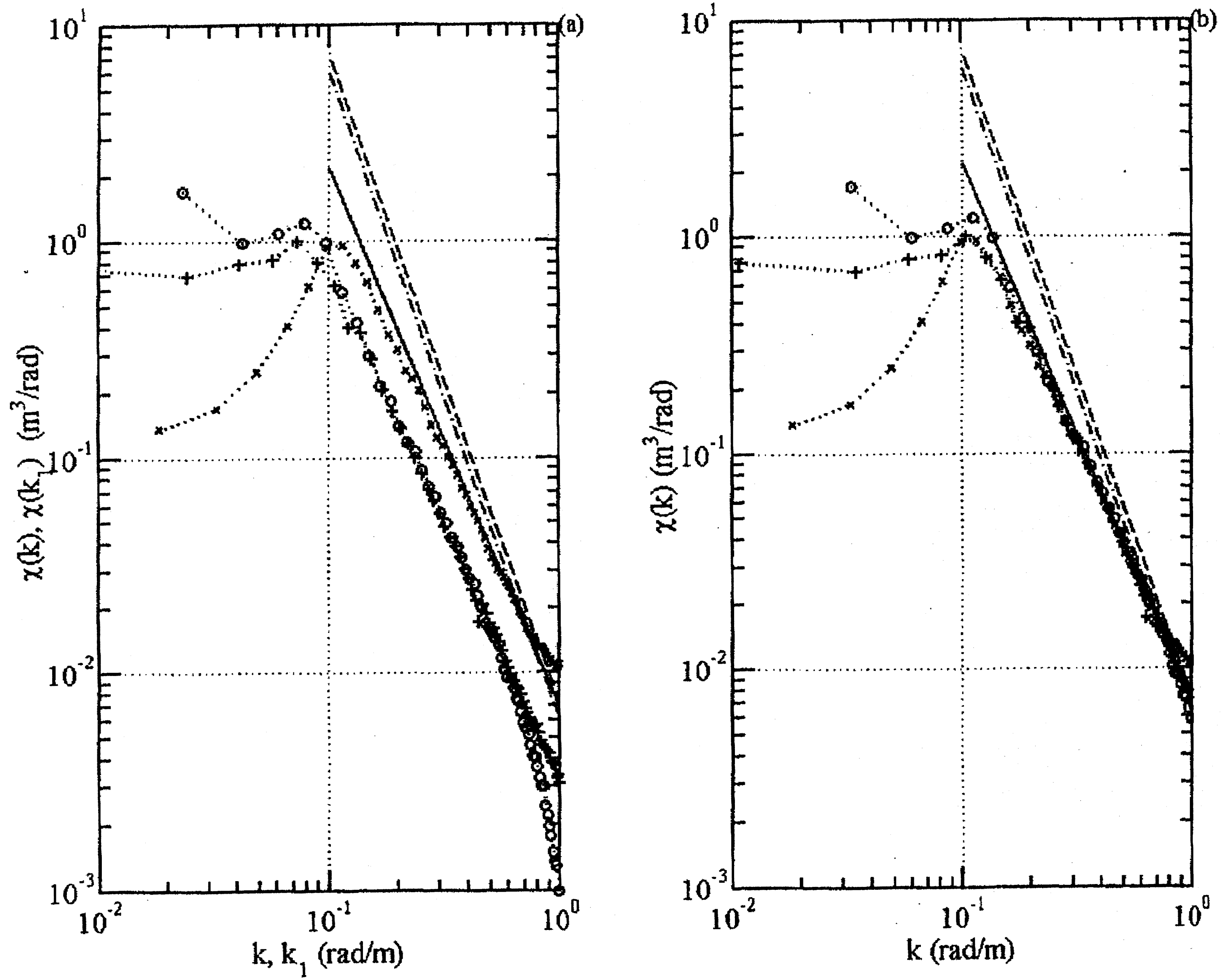


FIG. 7. (a) A comparison of three 1D wavenumber spectra. Crosses: omnidirectional spectrum, pluses: marginal ɔectrum in the flight direction, circles: traverse spectrum in the flight direction, solid curves: $\chi(k) = 0.06u_* g^{-0.5}k^{-2.5}$?hillips 1985), dashed–dotted curves: $\chi(k) = 0.006k^{-3}$ (Phillips 1977), and dashed curves: $\chi(k) =$.$002(u_*/c)(c_m/c)k_m k^{-4}$ (Hwang et al. 1996; Hwang 1997). (b) Same as (a) but the transect wavenumber is adjusted by ıe projection equation (4).

5 Energy balance in wind-driven sea

In the stationary spectral band

$$S_{nl} + S_{in} + S_{diss} = 0$$

is balance equation. Which term is this balance is the most important? In majority of empiric model the "source terms" $S_{in} + S_{diss}$ can be presented as follows

$$S_{in} + S_{diss} = \gamma(k)\, N(k)$$

$\gamma(k) > 0$ in the excitation area

$\gamma(k) < 0$ in the dissipation area (at large wave number).

Notice that the S_{nl} can be presented in the form

$$S_{nl.} = F_k - \Gamma_k N_k$$

$$F_k = \pi \int |T_{k k_1 k_2 k_3}|^2 \delta(k + k_1 - k_2 - k_3)\, \delta(\omega_k + \omega_{k_1} - \omega_{k_2} - \omega_{k_3}) \times$$
$$\times N_{k_1} N_{k_2} N_{k_3}\, dk_1 dk_2 dk_3 > 0$$

$$\Gamma_k = \pi \int |T_{k k_1 k_2 k_3}|^2 \delta(k + k_1 - k_2 - k_3)\, \delta(\omega_k + \omega_{k_1} - \omega_{k_2} - \omega_{k_3}) \times$$
$$\times (N_{k_1} N_{k_2} + N_{k_1} N_{k_3} - N_{k_2} N_{k_3})\, dk_1 dk_2 dk_3$$

One must compare

γ_k and Γ_k

Comparision of Γ_k and γ_k

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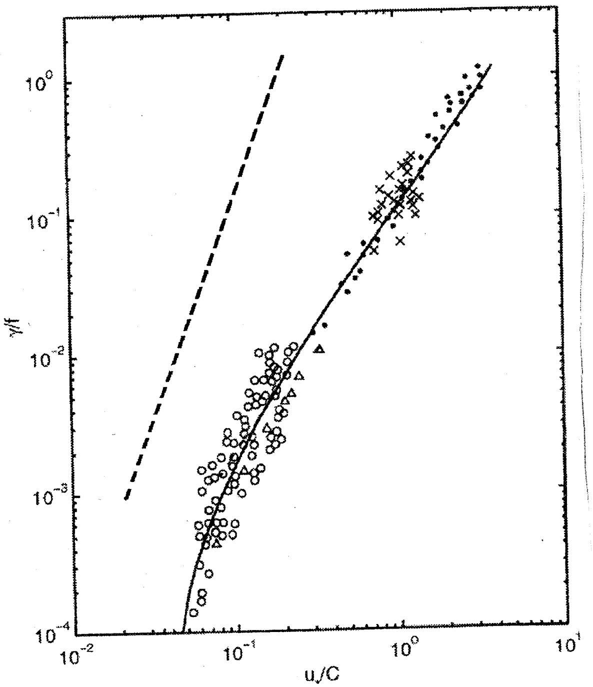


Figure 3. Comparison of the experimental data on the wind-induced growth rate $2\pi\gamma_{in}(\omega)/\omega$ taken from [26] and the damping due to four-wave interactions $2\pi\Gamma(\omega)/\omega$, calculated for the narrow in angle spectrum at $\mu \simeq 0.05$ using equation (6.11) (dashed line).

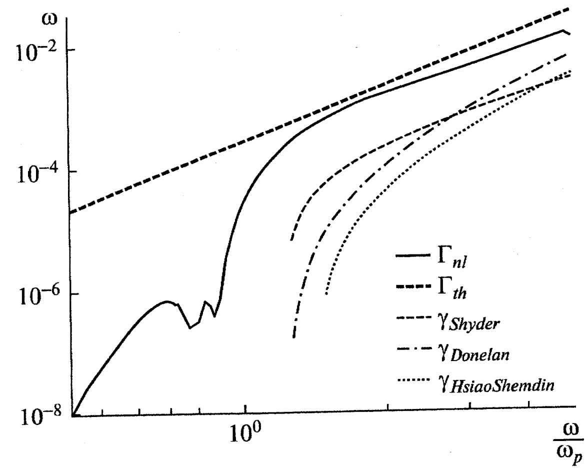
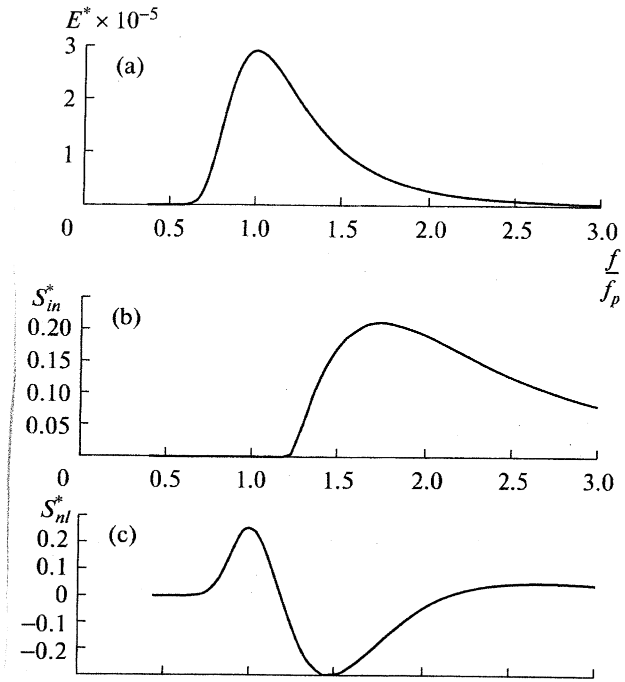


Fig. 3. Nonlinear damping coefficient Γ_k given by theoretical estimate (21) and by the numerical simulation (dashed and solid bold curves, correspondingly). Conventional dependencies of wind growth increments [3] are shown by thin curves with authors' names in legend.



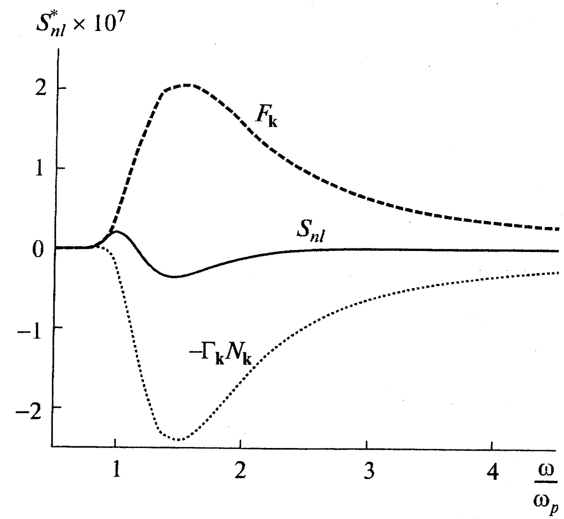


Fig. 2. Decomposition of the collision integral S_{nl} in Fig. 1c (solid line) for the case by Komen et al. [7] into nonlinear forcing (dashed) and damping (dotted) terms (see (13)–(15)).

Γ

Conclusion: S_{nl} is dominating term!
This fact makes possible to develop an analytical theory of the wind-driven sea

6 Quadruplete form of the Hasselmann equation

Calculation of the S_{nl} demands three-dimensional integration. Let us choose variables as follows

Let us denote $\vec{K}_B = \frac{\vec{k}_1 + \vec{k}_2}{2}$

If $\vec{K}_B$ is the unit vector ~~along~~ directed along the real axis. Suppose

$$\omega_1 + \omega_2 = \omega_3 + \omega_4 = 2S$$

Then frequencies can be parametrized as follow

$$\omega_1 = S(1-\lambda) \qquad \omega_2 = S(1+\lambda)$$

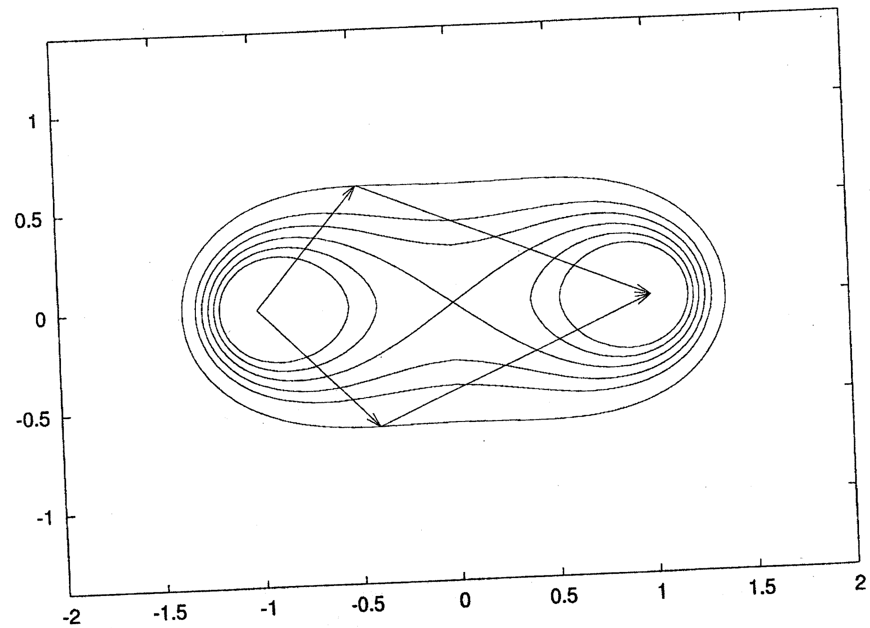
$$\omega_3 = S(1-\mu) \qquad \omega_4 = S(1+\mu)$$

One can choos S, λ, μ as integration variables. S is the "modulus of the standart quadruplete

$$\frac{1}{\sqrt{2}} < S < \infty$$

Modulus s defines a curve of genus 2 on the
" plane of standard quadruplets. λ, μ define two
points on this curve. If $s=1$, this ~~tis~~ the
Phillips curve. One can choos a finite set of
moduli and finite set of points λ, μ, replacing
integration by summation. At the moment the
modellers use only one point $s=1$ $\lambda = 1/4$, $\mu=0$

We are working on elaboration of the code A
including approximaly hundred points.
question of optimal choice of their configuration
is really crucial. Maybe a dozen of points
in enough to obtain a very good approximation



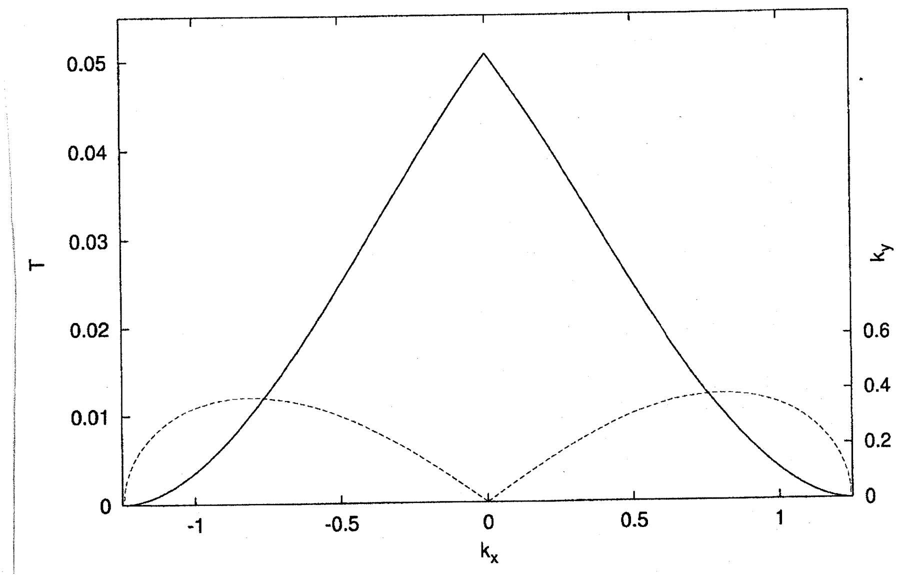


Figure 2: The example of the T coeffcicient behavior. The figure shows the values of T on the Phillips curve with $\mathbf{k_3} = \mathbf{k_4} = \mathbf{1}$

Plane of standart q

Behavior of T on the Phillips curve

Congratulations to Walter Craig
and Happy New Year!