No-Wait Flowshop Scheduling is as Hard as Asymmetric Traveling Salesman Problem

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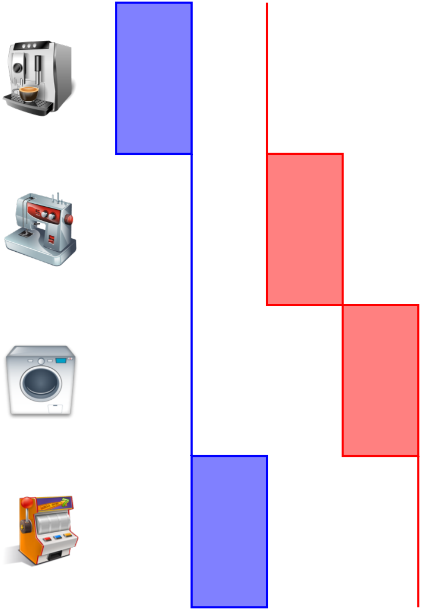
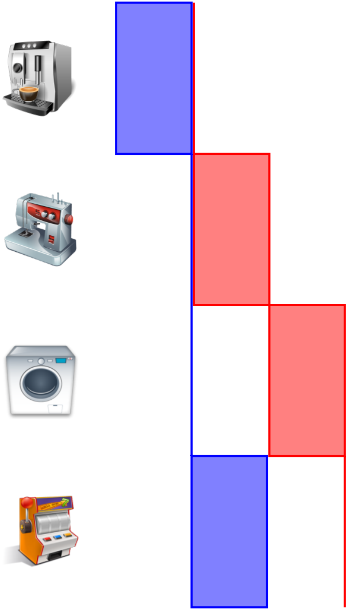
Think: production line for steel manufacturing, or production units with no intermediate storage capacity.

Example



Example – OK



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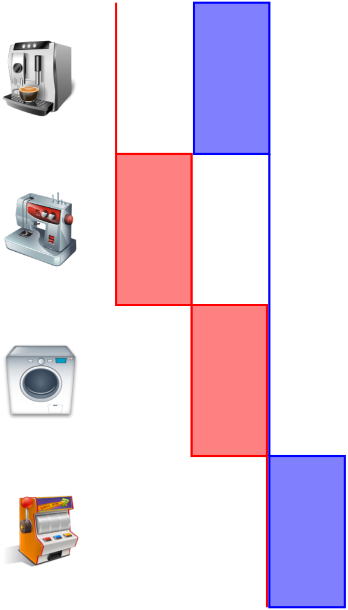

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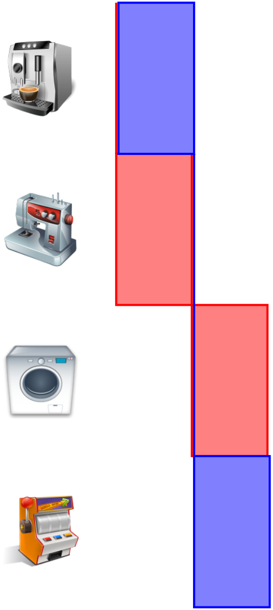
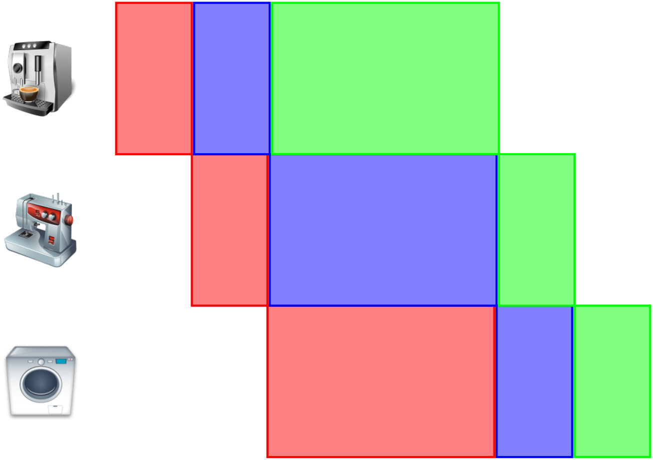
Example – Not OK!



Not OK!

Curiouser and curiouser



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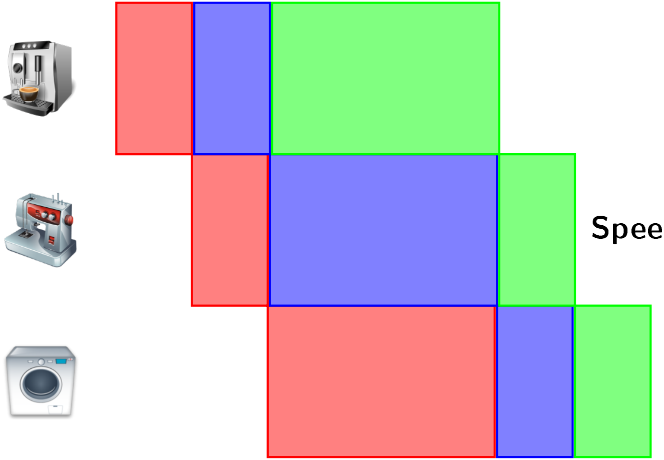
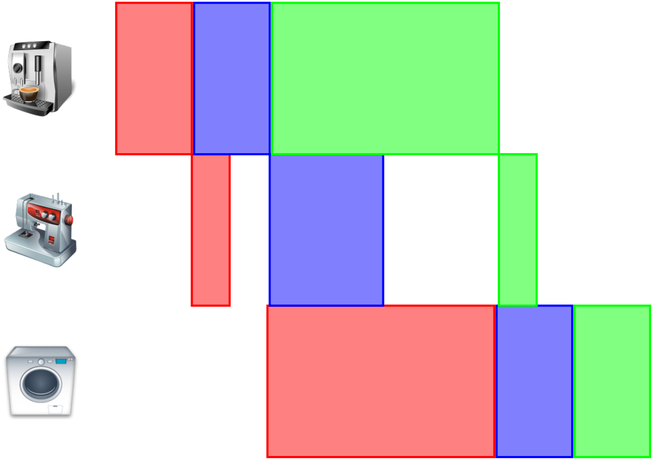


Speed up x2

Curiouser and curiouser



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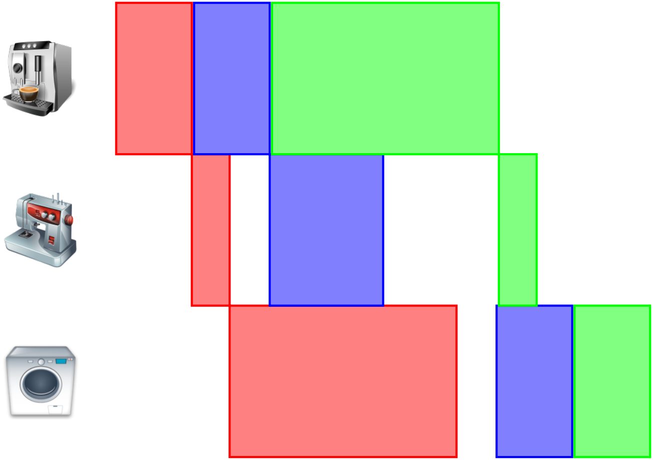

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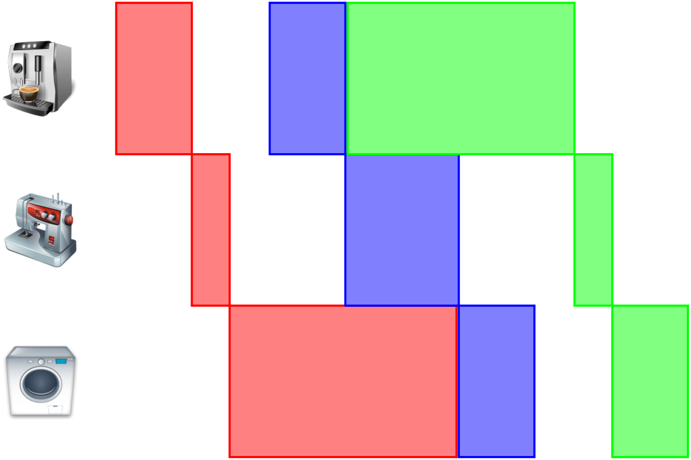

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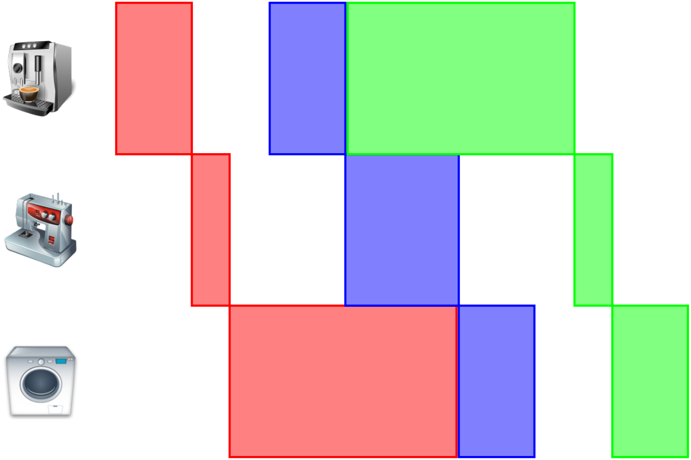

Example due to Spieksma and Woeginger (2005).

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Is No-Wait Flowshop an easy case of ATSP?

Our results

Theorem (Main Result 1)

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Theorem (Main Result 2)

There is an $O(\log m)$-approximation algorithm for No-Wait Flowshop.

Overview

1. Reduction to ATSP
2. Encoding semi-metrics in No-Wait Flowshop
3. $O(\log m)$-approximation

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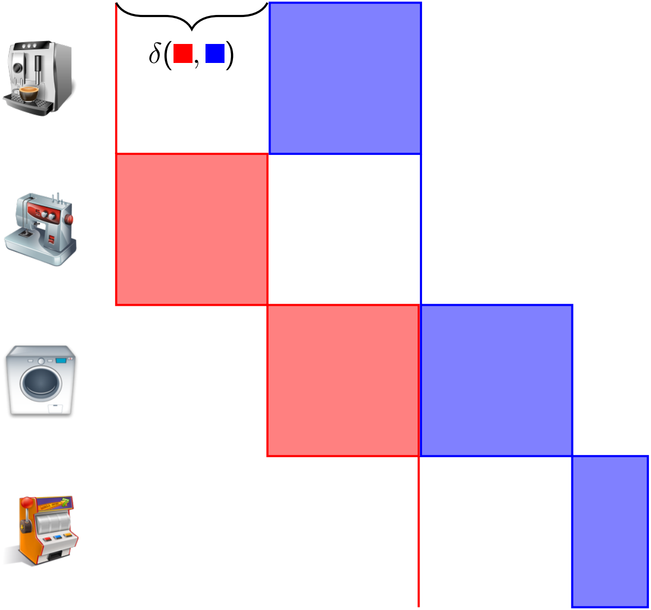

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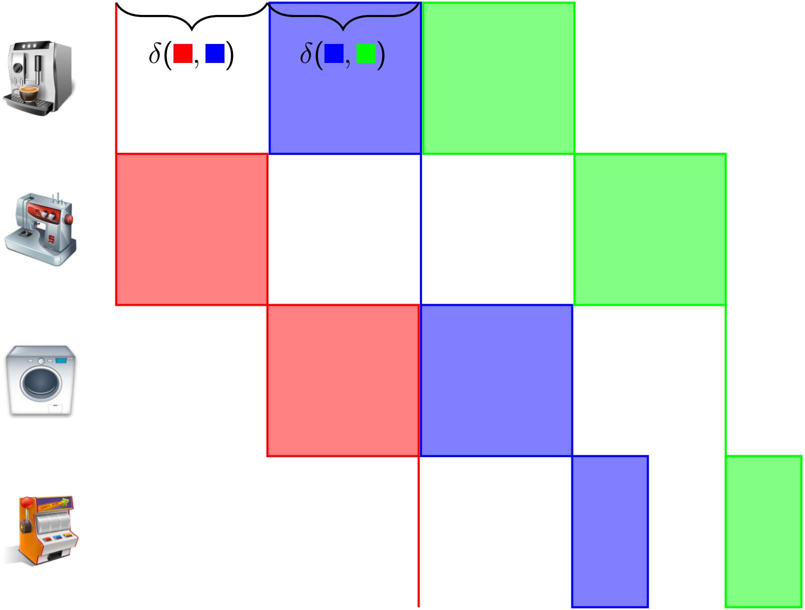
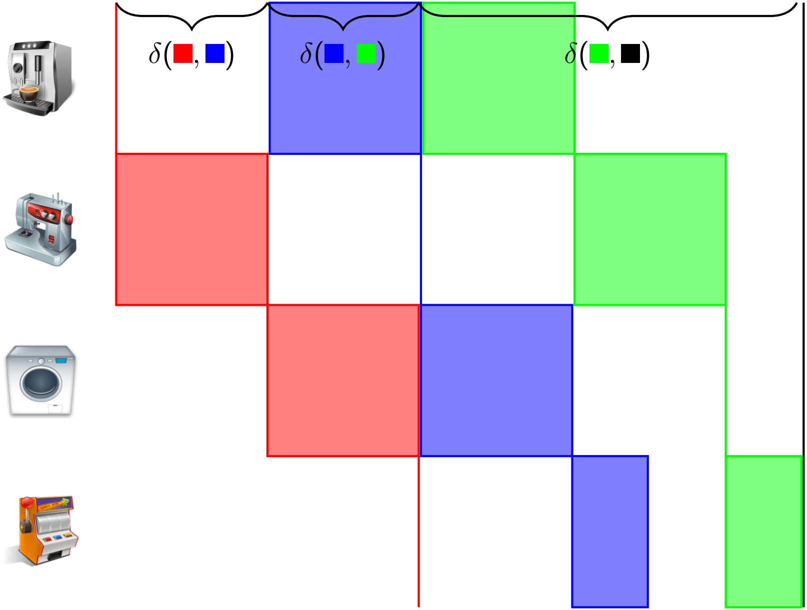


No-Wait Flowshop as ATSP



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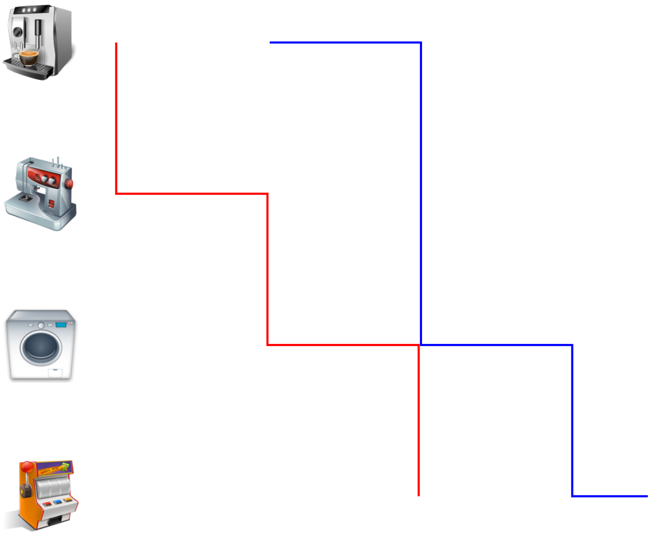
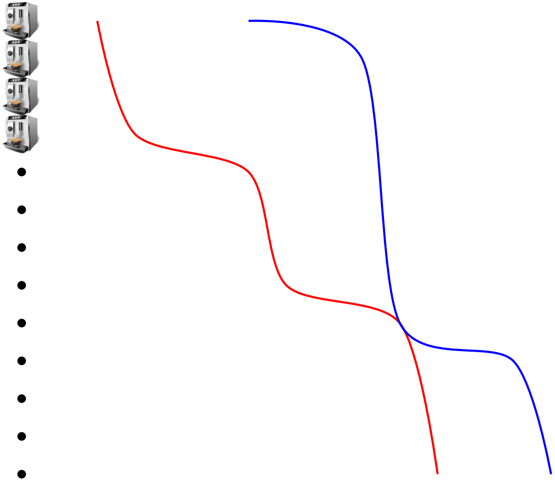


Many machines



many machines

small operations

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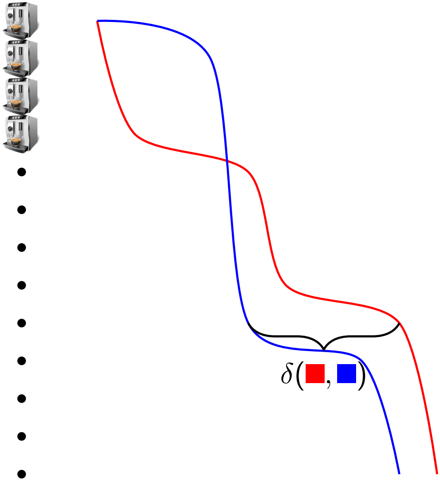

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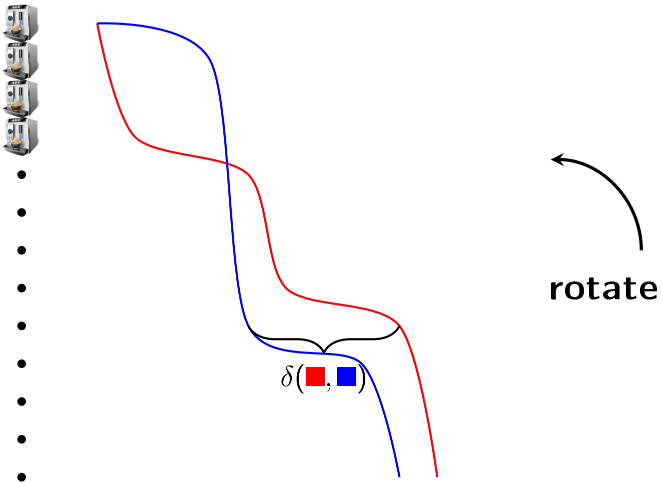

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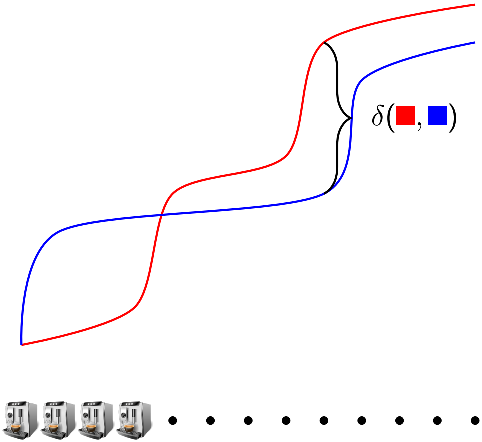

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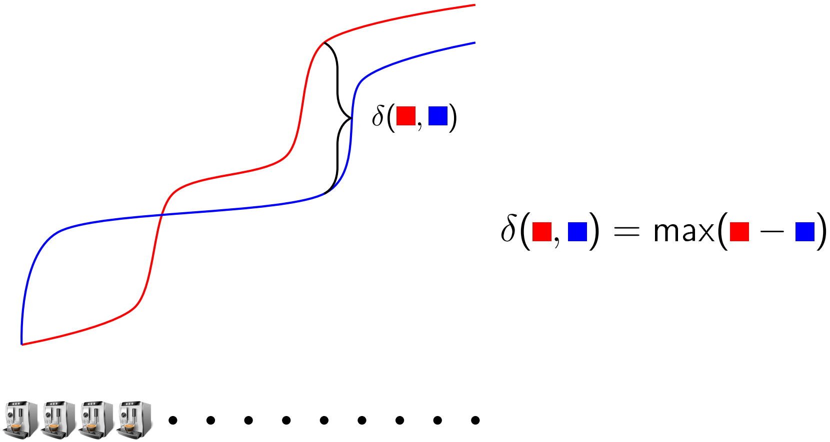

No-Wait Flowshop distance is hard!

Fact

Any n-point semi-metric (V, d) embeds isometrically into the semi-metric $(\mathbb{R}^n, \delta)$ with

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- The total length of each job is $\Omega(nW) >> OPT$.

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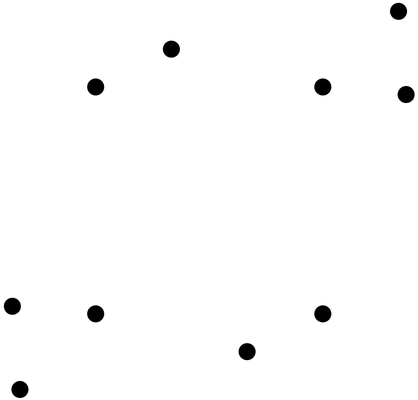

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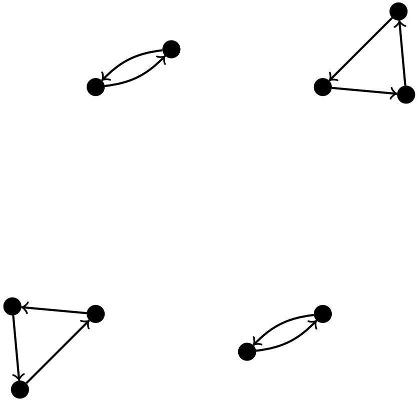

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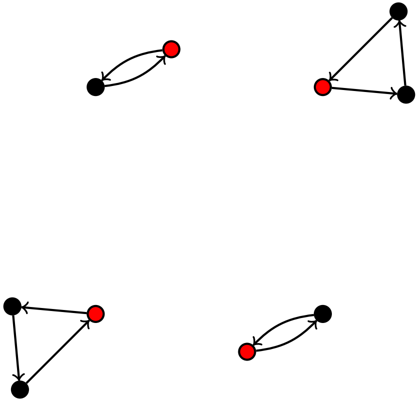

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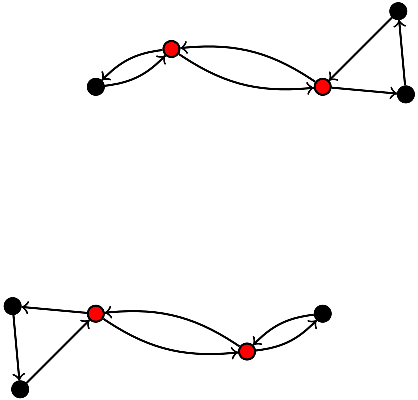

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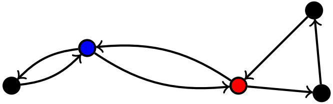

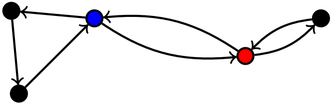

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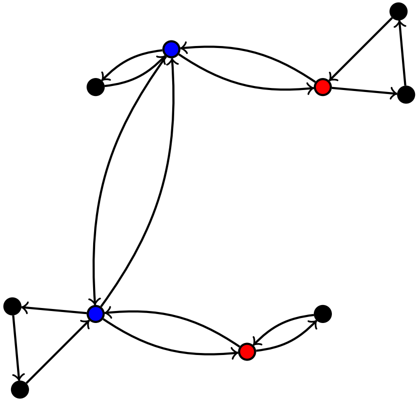

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Theorem

This algorithm is a $O(\log m)$-approximation.

$O(\log m)$-approximation

Proof.

Let $r_1, \ldots, r_k$ be the representative vertices after $\log m$ rounds and $L(r_1), \ldots, L(r_k)$ their lengths.

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This gives $OPT \geq \sum_i L(r_i)$.

The Hamiltonian cycle we add has at most this length. $\square$

Summary and Final Remarks

Gilmore, Gomory (1964) $O(n \log n)$ algorithm for $m = 2$.

Papadimitriou, Kanellakis (1980) Strongly NP-hard for $m \geq 4$. $\star$

Röck, Schmidt (1983) $\lceil \frac{m}{2} \rceil$-approximation. $\star$

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Sviridenko (2003) PTAS for $m = O(1)$.

Asadpour et al.(2010) $O(\log n / \log \log n)$ approximation via ATSP.

New result ATSP-hardness (for $m = polyn(n)$).

New result $O(\log m)$-approximation.

Open problems

Problem (1)

Bridge the gap betweeen a PTAS and $O(\log m)$.
$O(1)$*-approximation for some range of* m*?*

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Problem (2)

Can you get $O(\log m / \log\log m)$-approximation?

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Problem (2)

Can you get $O(\log m / \log \log m)$*-approximation?*

Problem (3)

Can you get $O(\log n / \log \log n)$*-approximation for ATSP à la Frieze, Galbiati, Maffioli?*

Thanks!