A Lower Bound for Parametric Global Min Cut?

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Parametric global min cut

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- **Big question:** How many segments can this curve have?

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- By contrast, for Global Min Cut, Aissi, Mahjoub, Mc show that there are only $O(n^3)$ segments.

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- Challenge: construct a family of instances with, say, $\Omega(n^2)$ global min cuts.