Financial Instability Contagion: a quantitative definition and mechanism

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Objective

1. Questions:
   - What is contagion?
   - Why and how does that happen?
   - Can it be prevented?
2. Answers
   - Quantitative definition of (instability) contagion
   - Mechanism
   - Hope so

Outline

1. Previous Result (w/ R. Douady, G. Castellacci)
   - Agent-based dynamical system of wealth
   - Early warning system: Market Instability Indicator
   - Extension to multiple economies
   - Quantitative definition of (instability) contagion
2. Main Result (w/ G. Castellacci)
   - Mechanism of contagion
3. Theory vs. Real Life
   - Working with data
4. Conjectures and wishes (by YC)

Dynamical System of Wealth I

- Divide an economy into n aggregates called *agents*
- $w_i(t)$ = Wealth of Agent i at time t
  - $w_i(t)$ = Equity + Debt = Cash(ables) + Invested Assets
  - $w_i(t) = E_i(t) + D_i(t) = L_i(t) + K_i(t)$
- $w_i(t+1) = w_i(t)$ + Internal Growth + Cash In - Cash Out
  - $w_i(t+1) = w_i(t) + F_{ii}(t) + \sum_{j\neq i}^{n} F_{ij}(t) - \sum_{k\neq i}^{n} F_{ki}(t)$
  - $F_{ij}(t)$ = fund transferred from j to i at t
- Wealth dynamical system $f : \bar{M} \subset \mathbb{R}^n \longrightarrow \bar{M}$,
  $(w_1(t), w_2(t), \ldots, w_n(t)) \longmapsto (w_1(t+1), w_2(t+1), \ldots, w_n(t+1))$

Feedback Loop via Flow of Funds

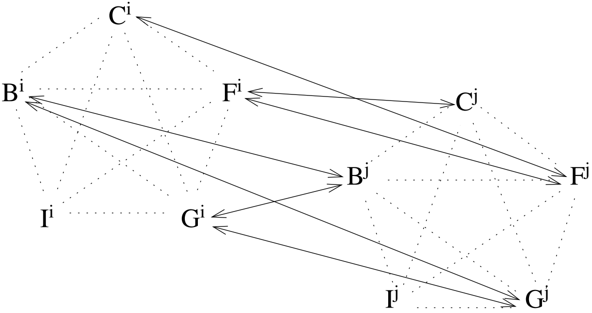


Figure : Feedback loop in a two-economy system. Five agents in each economy, Consumers, Firms, Banks, Government, and Investors, are interconnected by flow of funds

Dynamical System of Wealth II

- Stable equilibrium: persists perturbation
- Unstable equilibrium: perturbation propagates through feedback loop

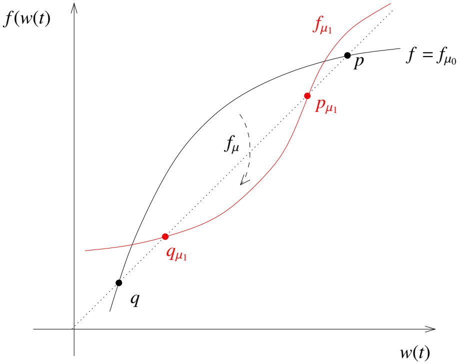
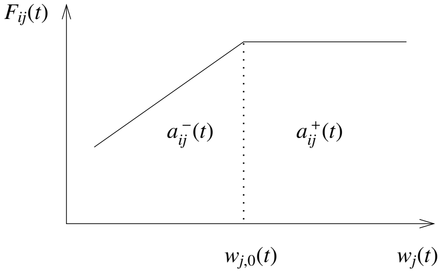


Figure : One dimensional illustration of stability change

Elasticity Coefficient I

- Elasticity Coefficient $a_{ij}(t) = \dfrac{\partial F_{ij}(t+1)}{\partial w_j(t)}$
- Different sign of $\Delta w_j(t)$ yields different reaction of $F_{ij}(t)$:
  - Post-Crisis Banks: credit reduction vs. hoarding cash
  - Post-Crisis Firms: layoff vs. hire freeze



Elasticity Coefficient II

- Elasticities vs. Jacobian $df(w(t)) = B(w(t))$:
  - $b_{ii} = 1 + a_{ii} - \sum_{k \neq i}^{n} a_{ki}$
  - $b_{ij} = a_{ij}$ for $i \neq j$
- Canonical embedding of local elasticities and Jacobians

$$A(t) = \begin{pmatrix} A^{(1)}(t) & A^{(12)}(t) & \dots & A^{(1s)}(t) \\ A^{(21)}(t) & A^{(2)}(t) & & \\ \vdots & & \ddots & \\ A^{(s1)}(t) & \dots & & A^{(s)}(t) \end{pmatrix} \quad B(t) = \begin{pmatrix} B^{(1)}(t) & A^{(12)}(t) & \dots & A^{(1s)}(t) \\ A^{(21)}(t) & B^{(2)}(t) & & \\ \vdots & & \ddots & \\ A^{(s1)}(t) & \dots & & B^{(s)}(t) \end{pmatrix}.$$

Market Instability Indicator and Contagion

- *Market Instability Indicator (MII)*

$$I(t) = \text{Spectral Radius of } B(w(t)) = \rho\left(B(w(t))\right)$$

  - $I(t) < 1$: perturbations of the system tend to be absorbed
  - $I(t) > 1$: small perturbations tend to increase when propagating

- We say that *instability contagion* occurs if given two times $0 < t_0 < t_1$,

  1. At $t < t_0$, $\max_k \rho\left(B^{(k)}(t)\right) < 1$ and $\rho\left(B(t)\right) < 1$
  2. At $t \in (t_0, t_1)$, $\max_k \rho\left(B^{(k)}(t)\right) > 1$ and $\rho\left(B(t)\right) < 1$
  3. At time $t > t_1$ $B(t) \neq \oplus_{k=1}^{s} B^{(k)}(t)$ and $\rho\left(B(t)\right) > 1$.

- Condition 3 implies $A^{(ij)}(t) \neq 0$ $(i \neq j)$

  $\Longrightarrow$ causal nature of contagion

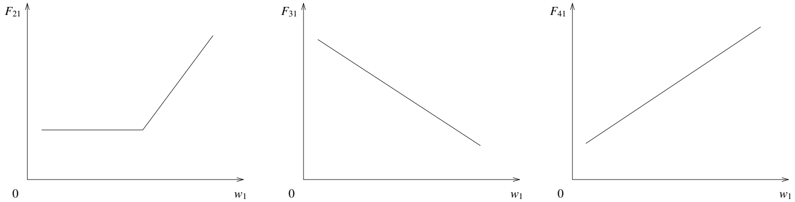
Mechanism of Contagion

- Lower bound of MII
  - $\frac{|\mathrm{tr}\,(B)|}{n} \leq \max_{\lambda_i \in \sigma(B)} |\lambda_i| = \rho(B)$
  - $b_{ii} = 1 + a_{ii} - \sum_{k \neq i}^{n} a_{ik}$
- $w_i(t+1) = w_i(t) + F_{ii}(t) + \sum_{j \neq i}^{n} F_{ij}(t) - \sum_{k \neq i}^{n} F_{ki}(t)$
  - If $w_i(t)$ is strictly decreasing in t, then $b_{ii}(t) = \frac{\partial w_i(t+1)}{\partial w_i(t)} > 0$
  - If $w_i(t)$ is strictly decreasing in t and concave, then $b_{ii}(t) > 1$
  - When w_i decreases, $\sum_{j \neq i}^{n} F_{ij}$ tends to drop and $\sum_{k \neq i}^{n} F_{ki}$ goes up
  - When agent i hits liquidity and solvency constraints, F_{ki} becomes obliged
  - These push up b_{ii}, thus $\rho(B)$ as well

Contagion within an Economy I

6-agent model with consumers (1), firms (2), banks (3), government (4), investors (5), and rest of the world (6).

- Assume consumers are highly leveraged, have little L_1
- K_1 and w_1 are decreasing and concave $\Longrightarrow a_{11} > 0$
- As $w_1 \downarrow$, eventually $a_{21} \sim 0$, $a_{31} < 0$, $a_{41} > 0$, $a_{51} \sim 0$, and $a_{61} \sim 0$
- $b_{11} \approx 1 + a_{11} - a_{31} - a_{41} \gg 1$



Contagion within an Economy II

- Split the economy into C-B (1) and F-G-I-R (2) partitions
  - $b_{11} \gg 1 \Rightarrow$ likely $\rho(B^{(1)}) \geq \frac{|b_{11}+b_{33}|}{2} > 1$ while $\rho(B(t)) < 1$
  - There is financial instability in Partition (1)
  - Stage (ii) of instability contagion
- If C defaults on payment, then $w_3(t)$ decreases
  - Out of panick, w_3 would decrease with acceleration, so $a_{33} > 0$
  - Due to mass-withdrawal, $a_{i3} < 0$ for $i = 1, 2, 5, 6$, risking a bank run
  - $b_{33} = 1 + a_{33} - \sum_{k\neq 3}^{6} a_{k3}$ jumps, driving $\rho(B(t)) \geq \frac{\sum_{i=1}^{6} |b_{ii}|}{6} > 1$
  - Stage (iii) of instability contagion
- Instability contagion has taken place
- In reality: US authorities immediately intervened

Contagion across Two Economies

11-agent model with C1, F1, B1, G1, I1, C2, F2, B2, G2, I2, and R (1 - 11).

- Assume G1 has difficulty paying back B2
  - w_i is has been decreasing and $a_{ii} > 0$ for $1 \le i \le 5$
  - $b_{44} = 1 + a_{44} - \sum_{k \neq 4}^{11} a_{k4} \approx 1 + a_{44} - a_{34} - a_{54} - a_{84} \gg 1$
  - High probability that $\rho(B^{(1)}(t)) > 1$ while $\rho(B(t)) < 1$
  - Stage (ii) of Contagion
- If G1 defaults on payment, then B2 is hit and $w_8(t)$ goes down
  - Due to panick, $b_{88} \gg 1$, possibly driving $\rho(B(t)) \ge \frac{\sum_{i=1}^{11} |b_{ii}|}{11} > 1$
  - Stage (iii) of instability contagion
- Instability contagion has taken place
- In reality: ECB "...whatever it takes..." (in Stage (ii))

Work in Progress and Current Issues

- Data!!
  - Federal Reserve Board, Bureau of Economic Analysis, (FDIC)
  - Not enough details and frequency
  - Selection of Agents: Nonprofit Organizations, Fed
- QE distorted market: we are riding a saddle
  - Regrouping agents are necessary to measure the true effect of QE
- Printed monies: seeds of the next (mega) bubbles?
- Detecting bubbles (after defining them) using MII

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