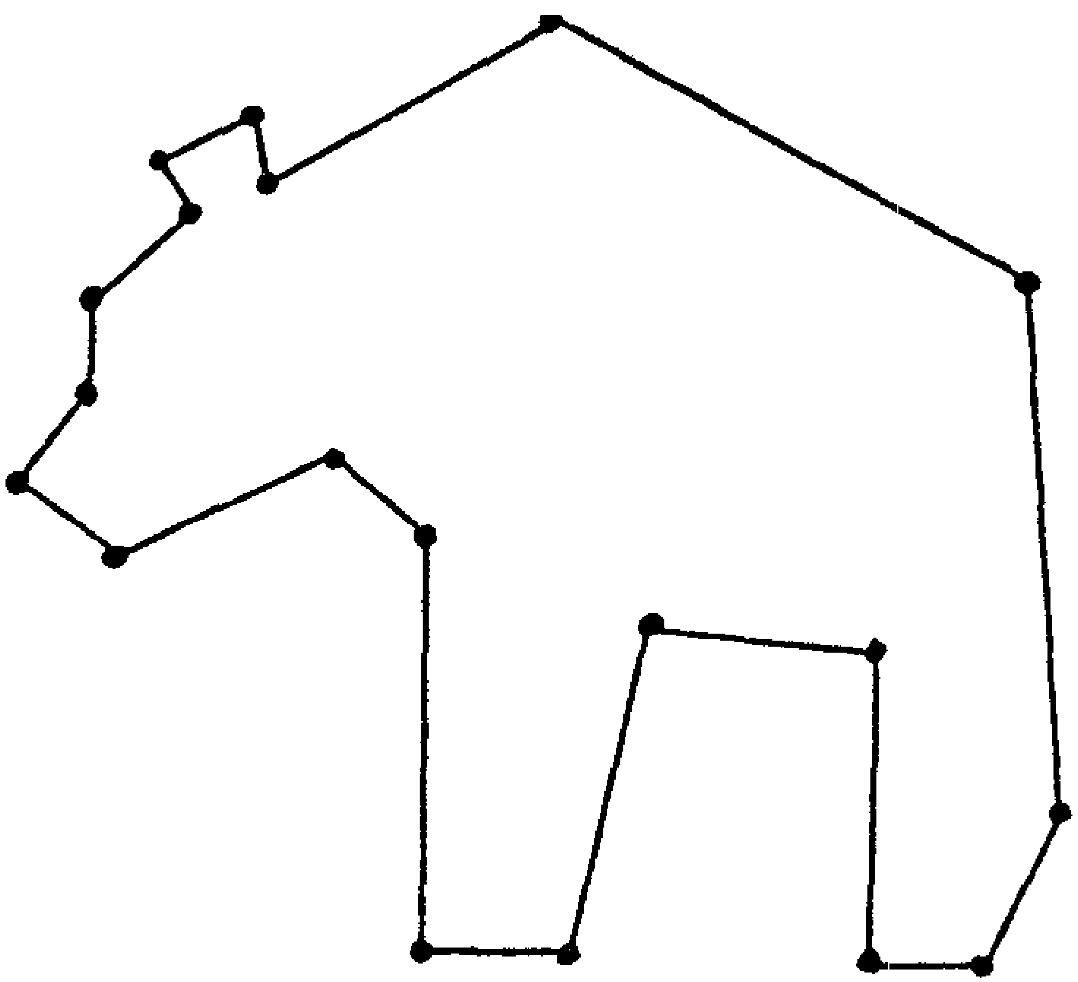
Visibility Graphs, Dismantlability, and the Cops-and-Robbers Game

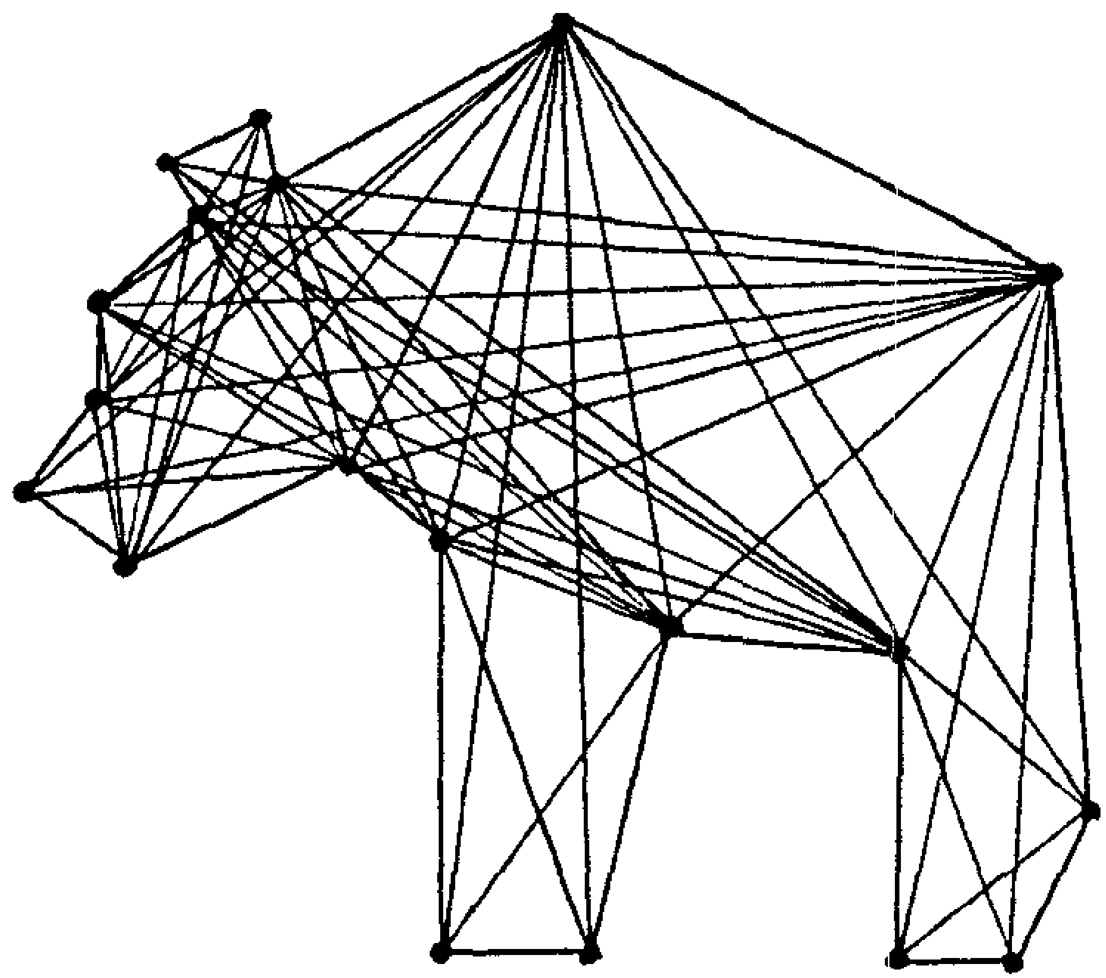
Anna Lubiw

University of Waterloo

Visibility Graphs

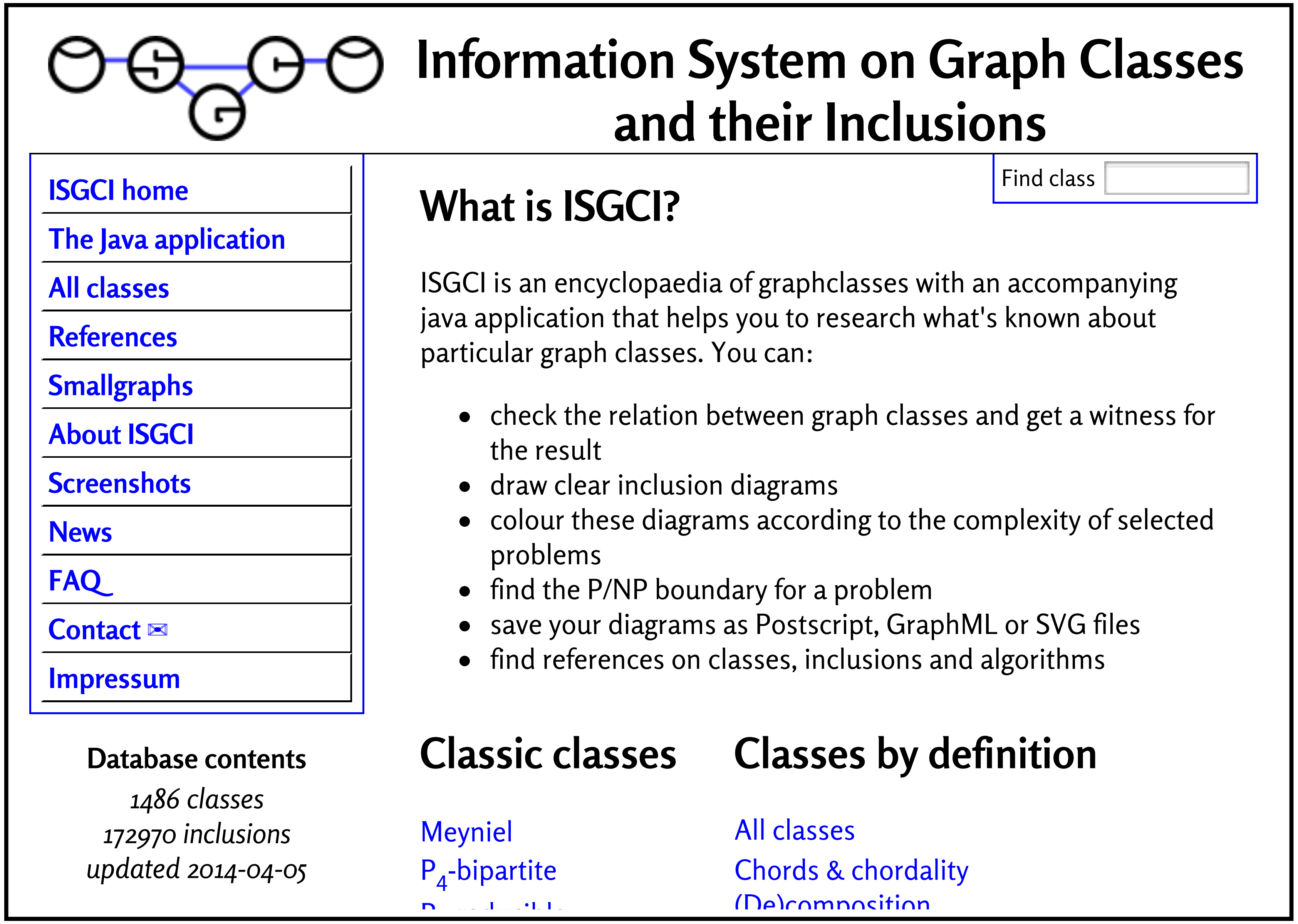


polygon



visibility graph

visibility graphs are a mystery with respect to other known graph classes


Information System on Graph Classes and their Inclusions

- ISGCI home
- The Java application
- All classes
- References
- Smallgraphs
- About ISGCI
- Screenshots
- News
- FAQ
- Contact ✉
- Impressum

Find class

What is ISGCI?

ISGCI is an encyclopaedia of graphclasses with an accompanying java application that helps you to research what's known about particular graph classes. You can:

- check the relation between graph classes and get a witness for the result
- draw clear inclusion diagrams
- colour these diagrams according to the complexity of selected problems
- find the P/NP boundary for a problem
- save your diagrams as Postscript, GraphML or SVG files
- find references on classes, inclusions and algorithms

Database contents

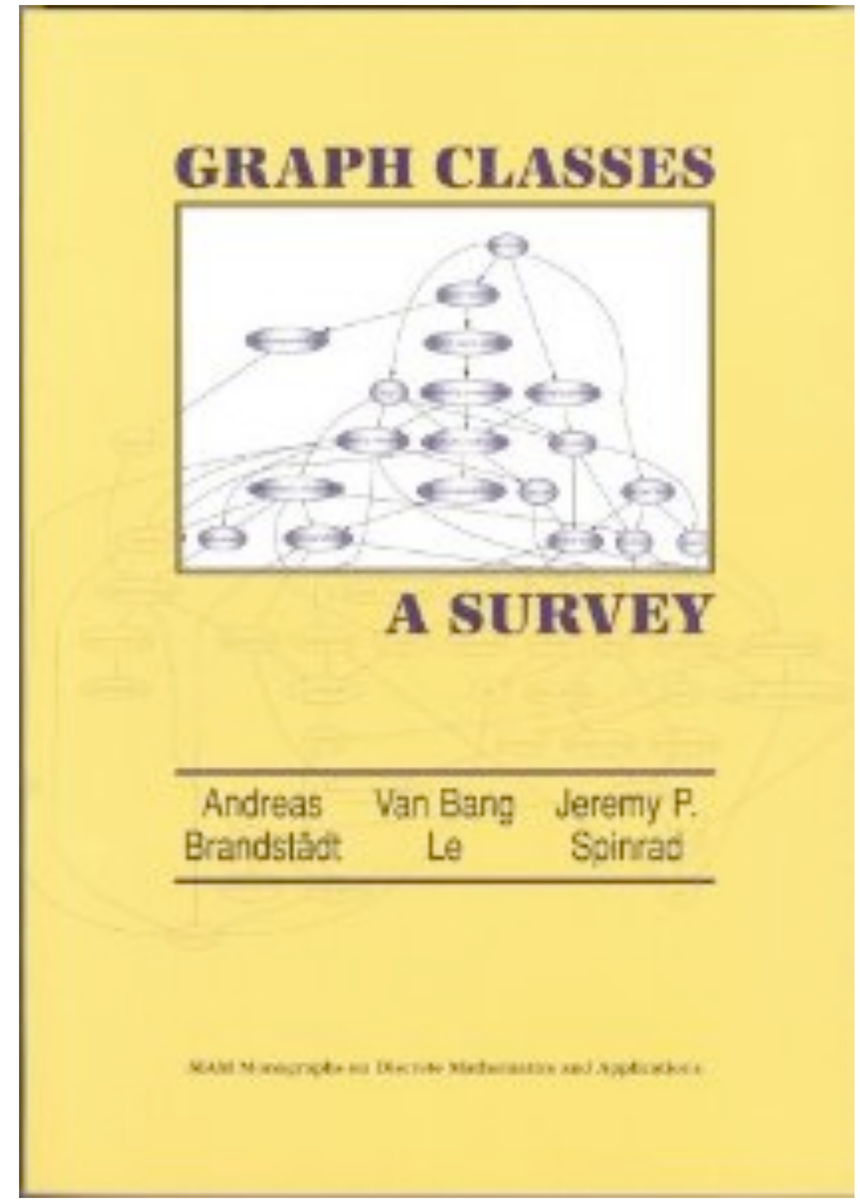
1486 classes
172970 inclusions
updated 2014-04-05

Classic classes

Meyniel
P_4-bipartite

Classes by definition

All classes
Chords & chordality
(De)composition



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Computational Geometry 5 (1995) 51–63

Computational Geometry
Theory and Applications

Negative results on characterizing visibility graphs

H. Everett [a,*], D. Corneil [b]

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JOURNAL OF ALGORITHMS 11, 1–26 (1990)

Recognizing Visibility Graphs of Spiral Polygons

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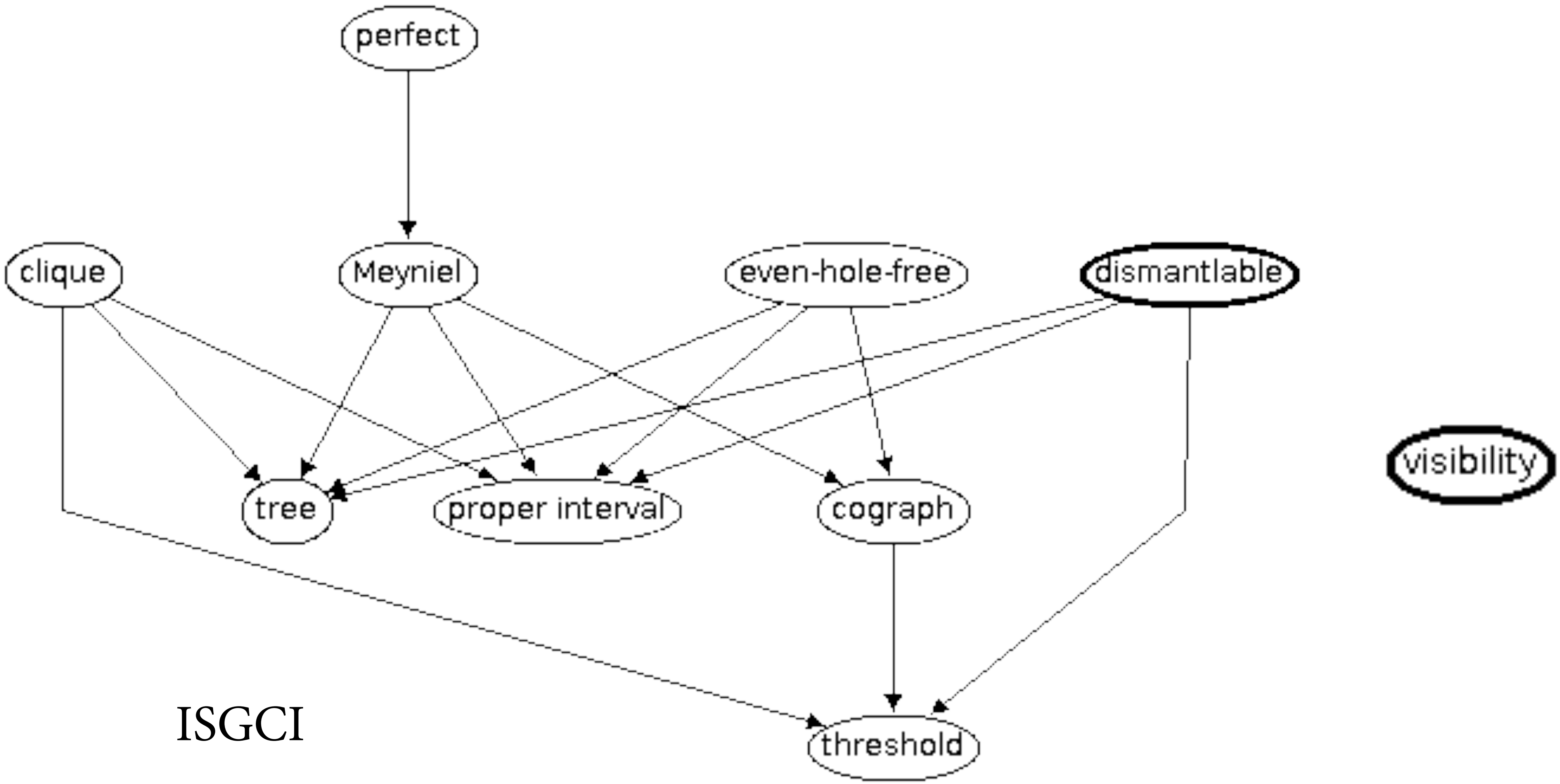

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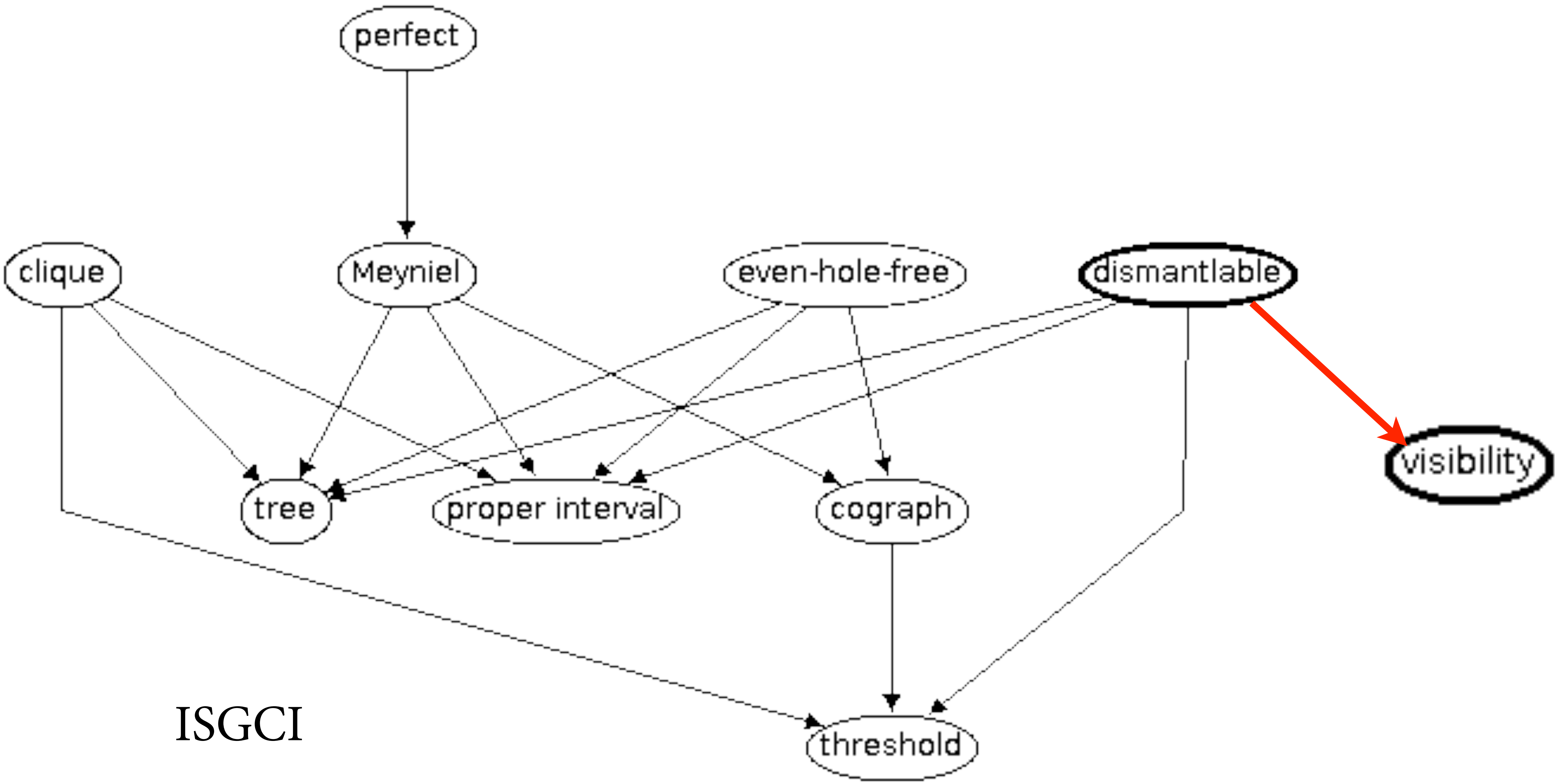

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Dismantlable Graphs

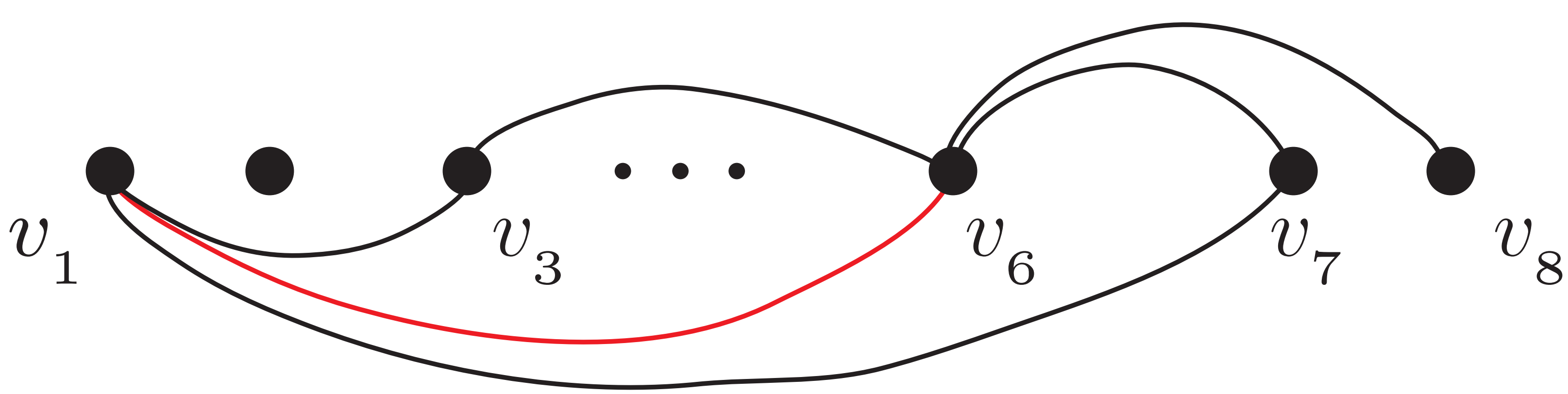
Theorem. [A.L. et al.] Visibility graphs are dismantlable.

Definition. u *dominates* v if $N[u] \supseteq N[v]$.

Definition. A graph G is *dismantlable* if it has a vertex ordering $\{v_1, v_2, \ldots, v_n\}$ such that

for each $i < n$, there is a vertex v_j, $j > i$ that *dominates* v_i in the graph induced by $\{v_i, \ldots, v_n\}$.

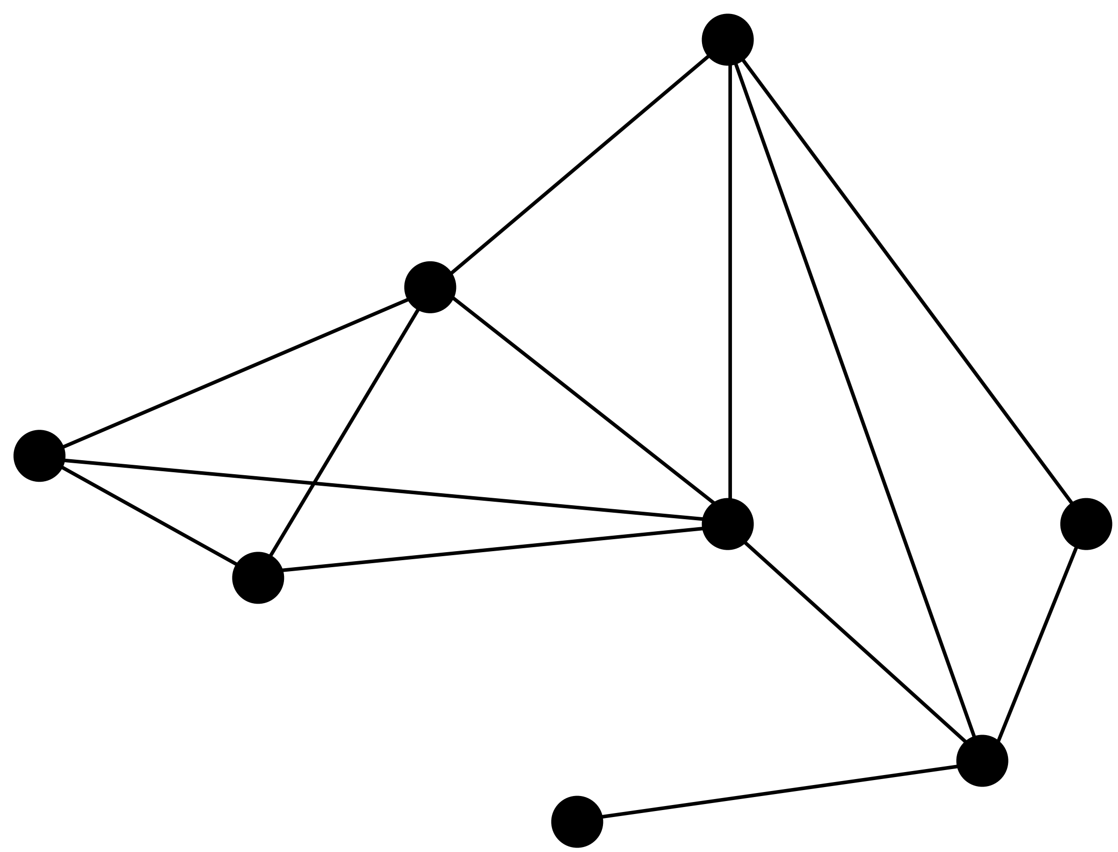
Equivalently: v_1 is dominated by another vertex and $G - v_1$ is dismatlable.

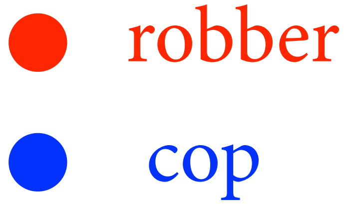


Dismantlable Graphs

Theorem. [Nowakowski and Winkler, 1983. Quilliot, 1983]
Graph G is dismantlable iff it is cop-win in the cops and robbers game.

Cops and Robbers Game:





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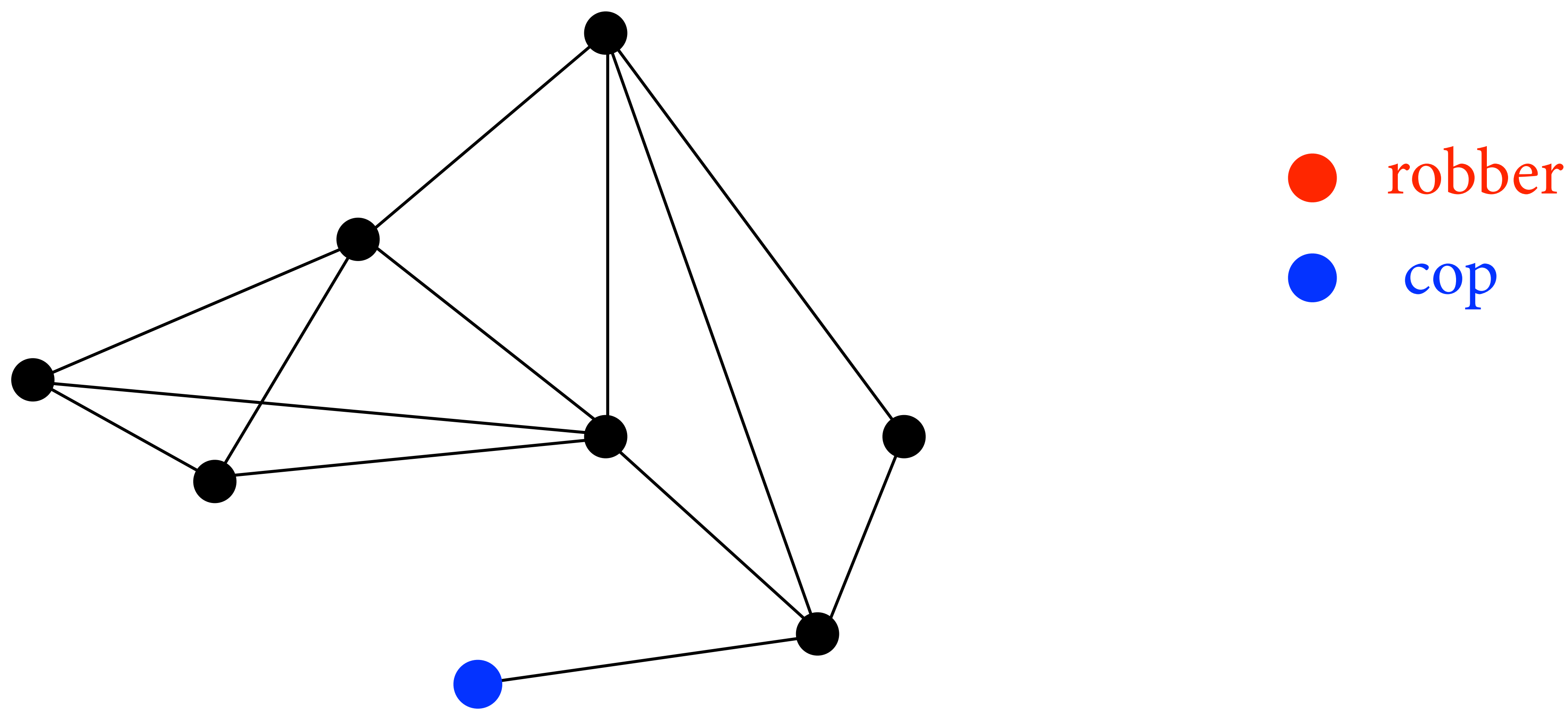

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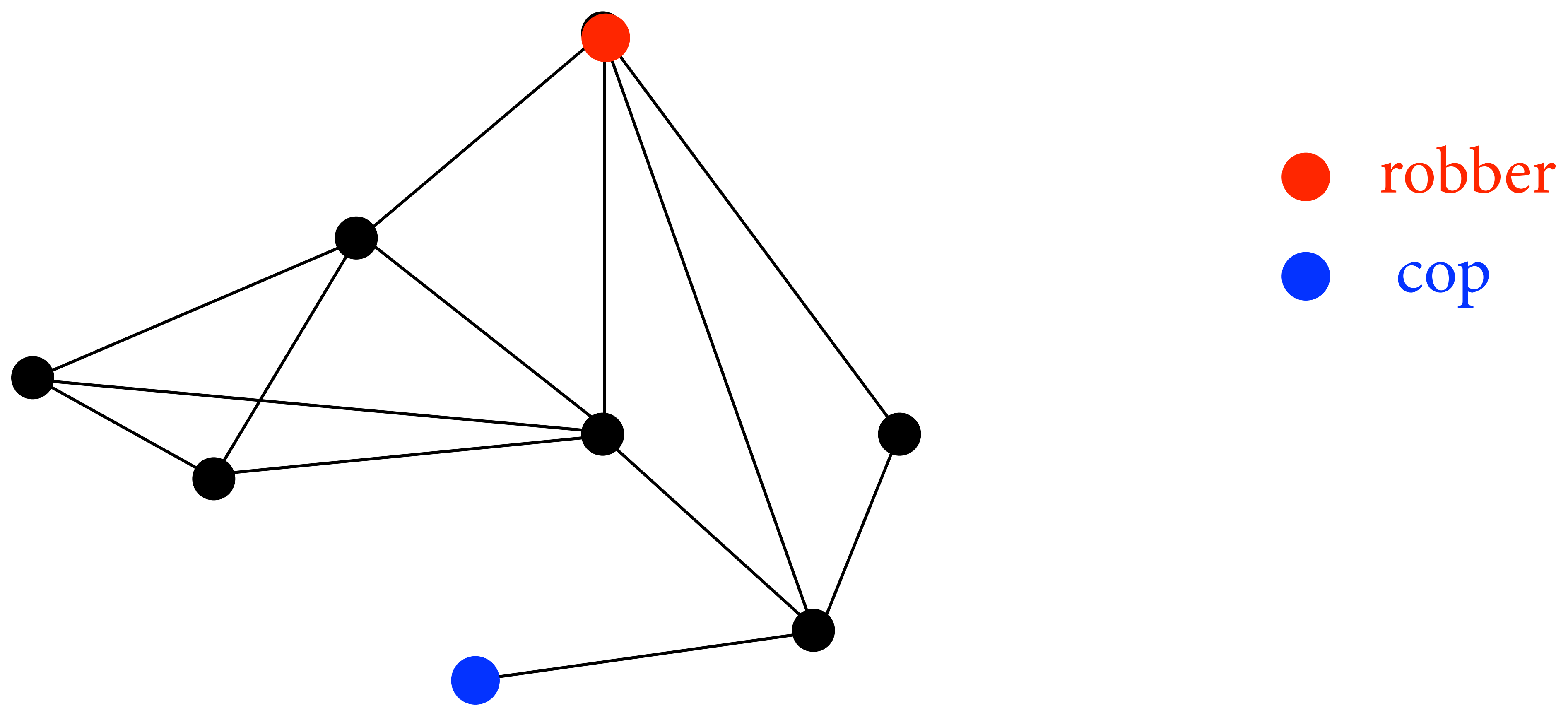

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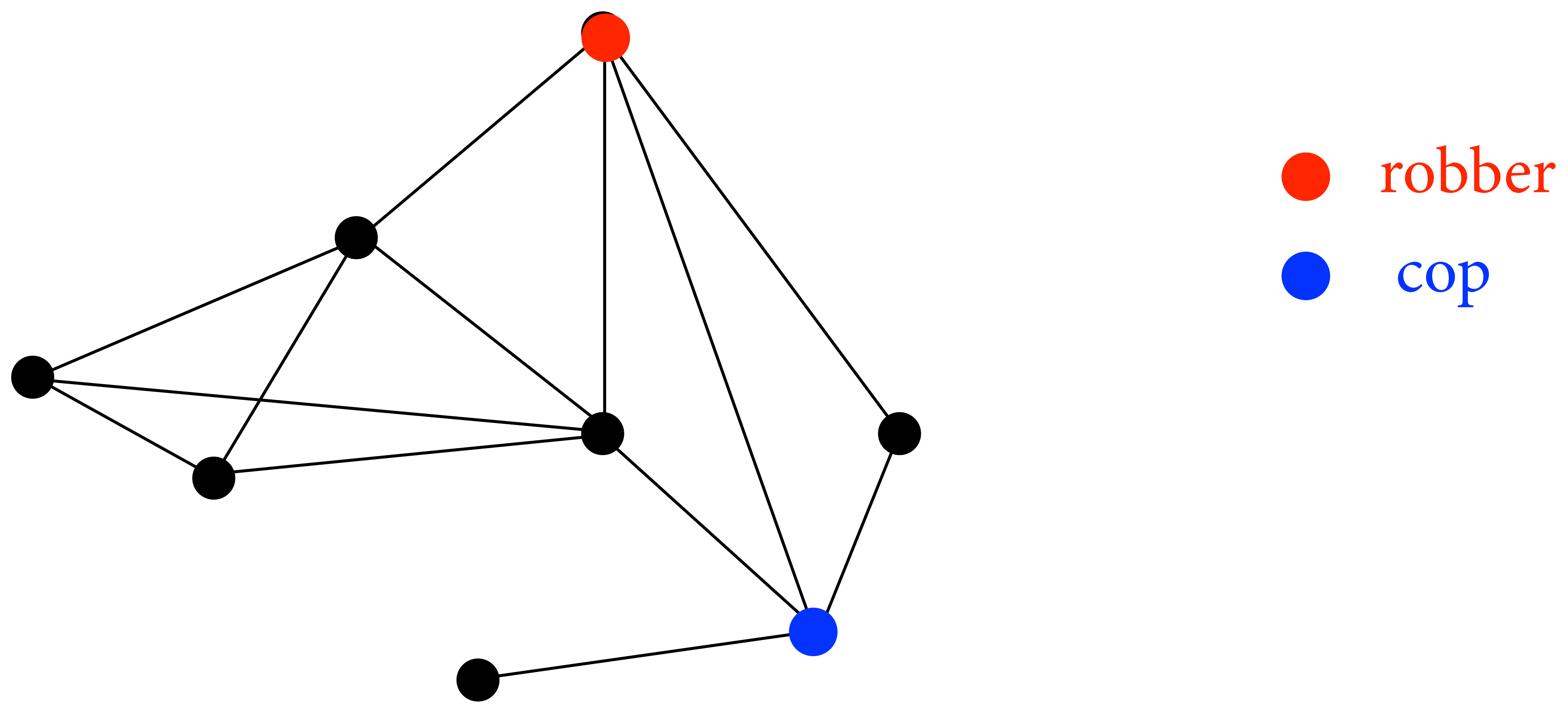

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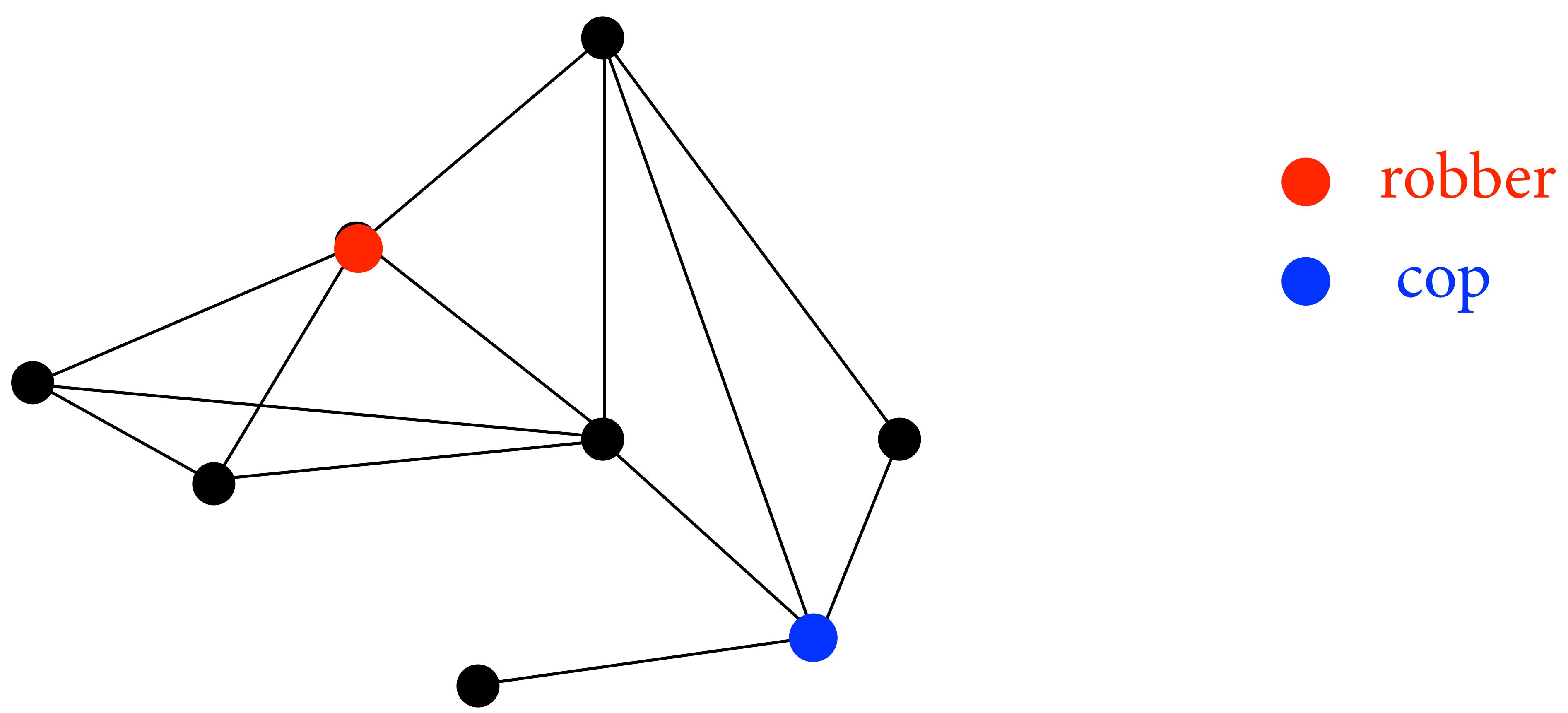

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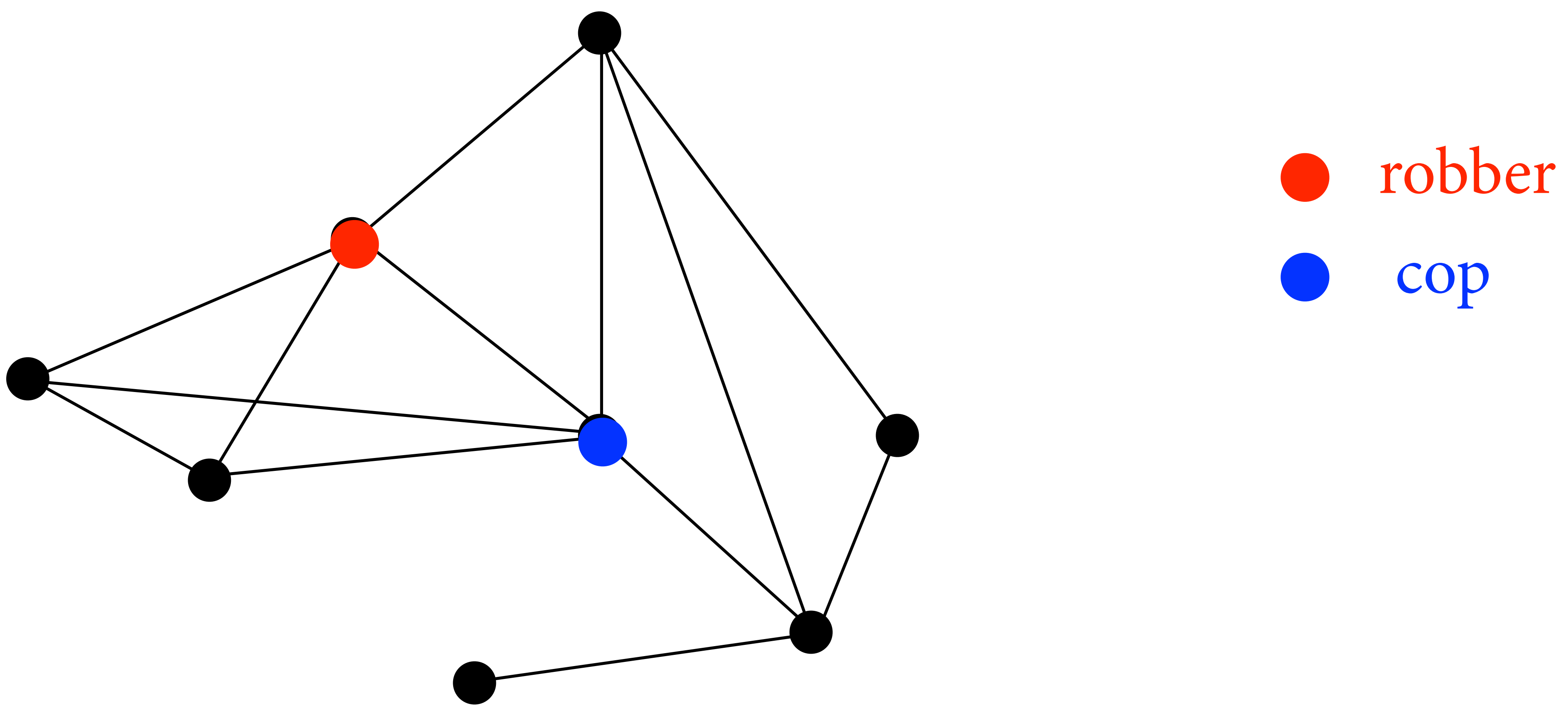

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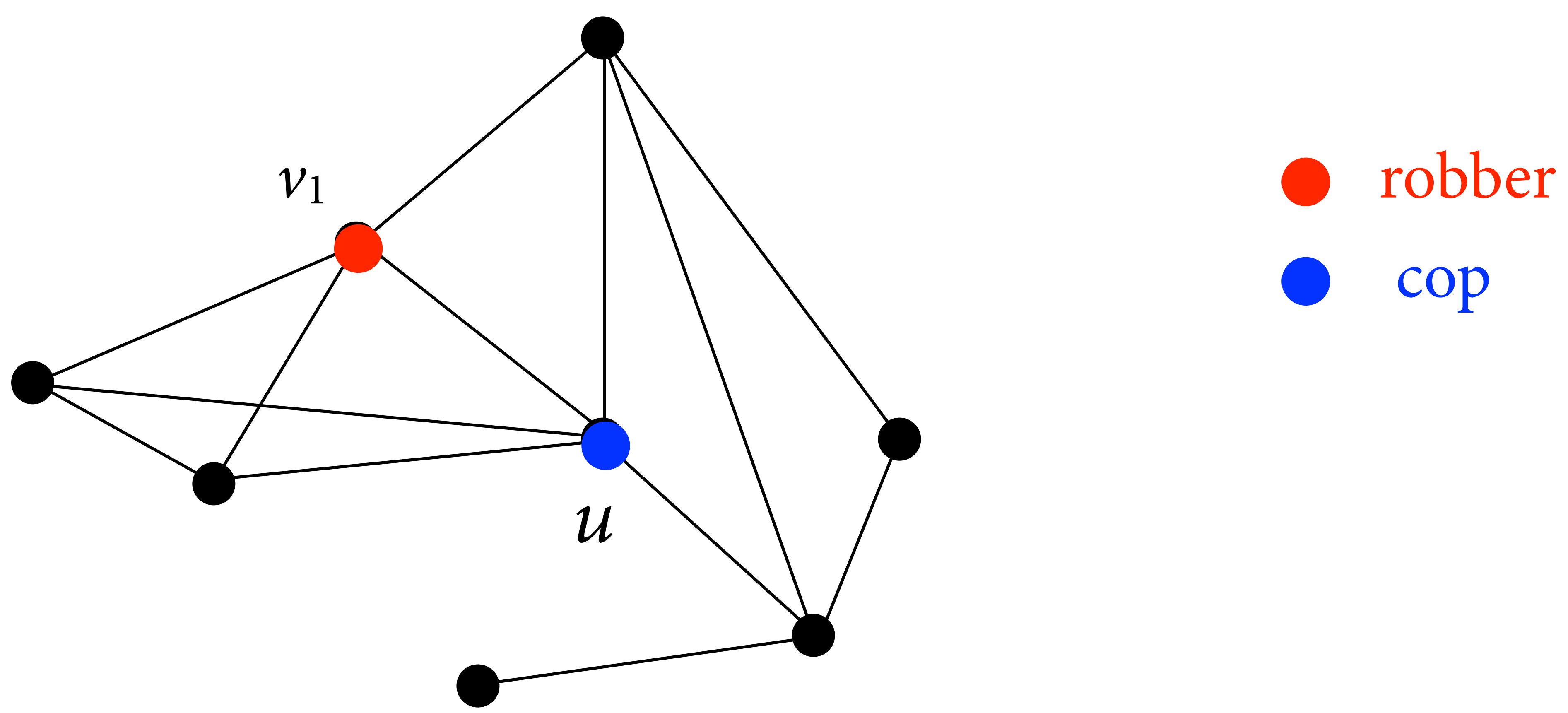

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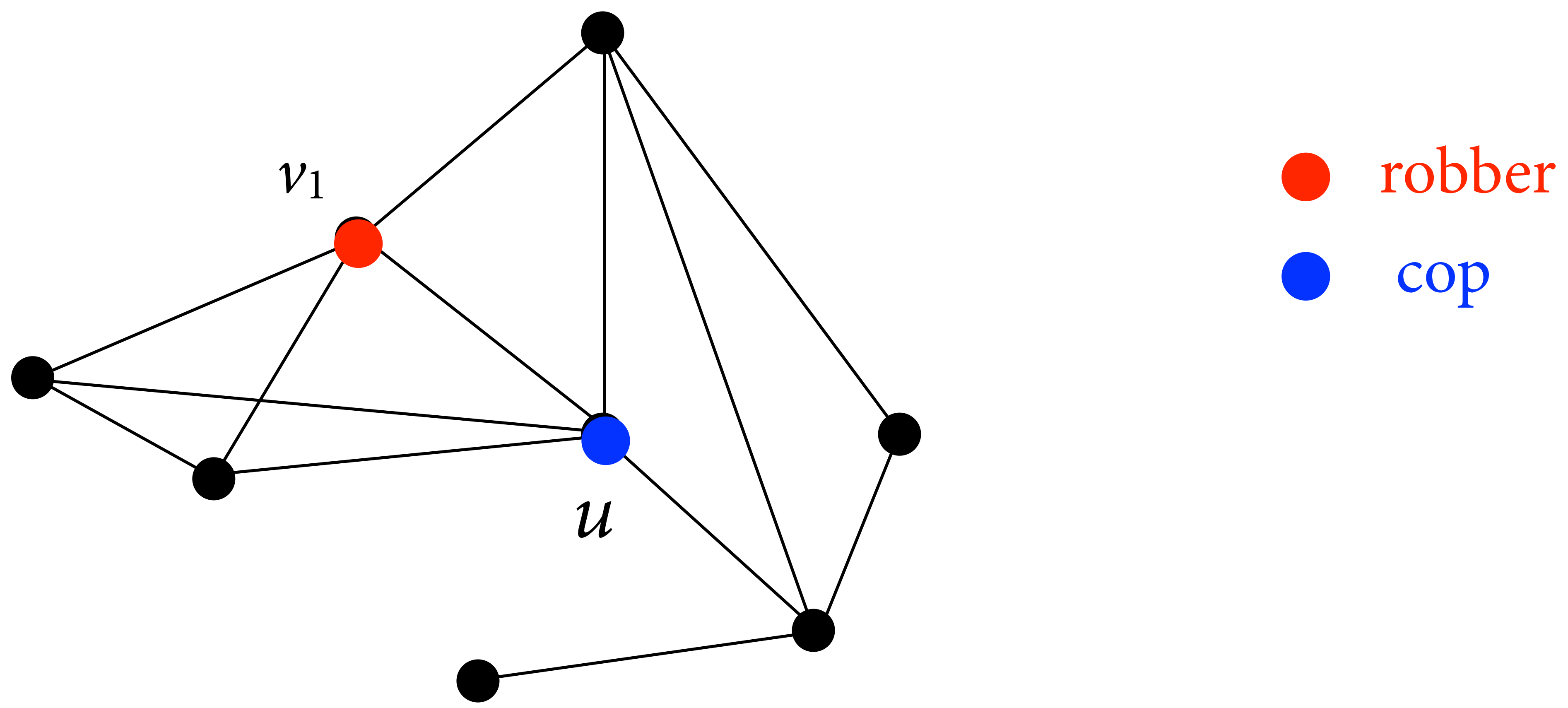


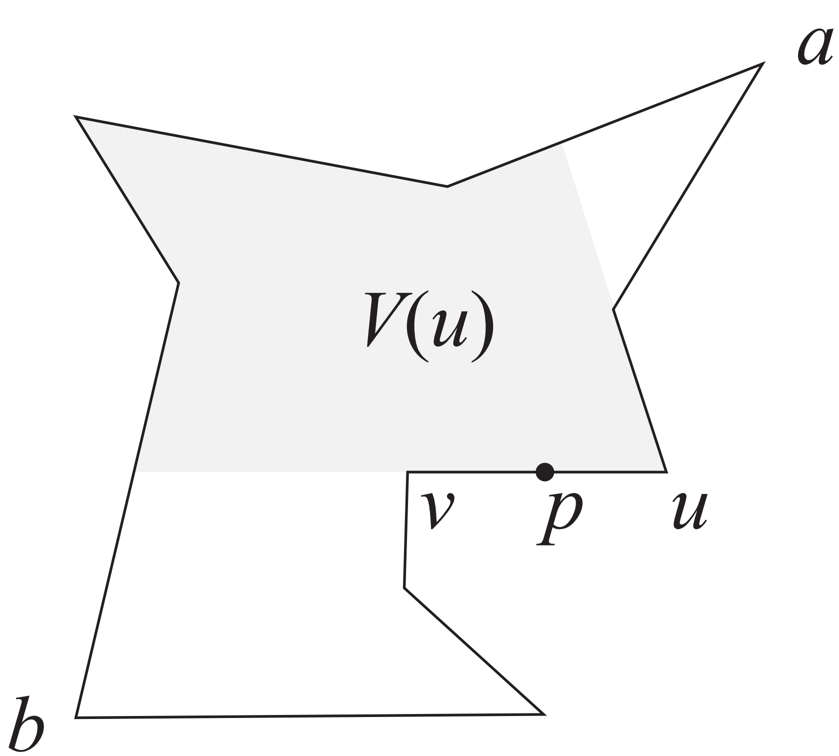
a cop-win graph has a vertex v_1 dominated by another vertex

Dismantlable Graphs

Theorem. [A.L. et al.] Visibility graphs are dismantlable.

Proof. Every polygon has a *visibility-increasing edge*:
an edge (u,v) such that for every point p along the edge (u,v),

$$V(u) \subseteq V(p) \subseteq V(v).$$



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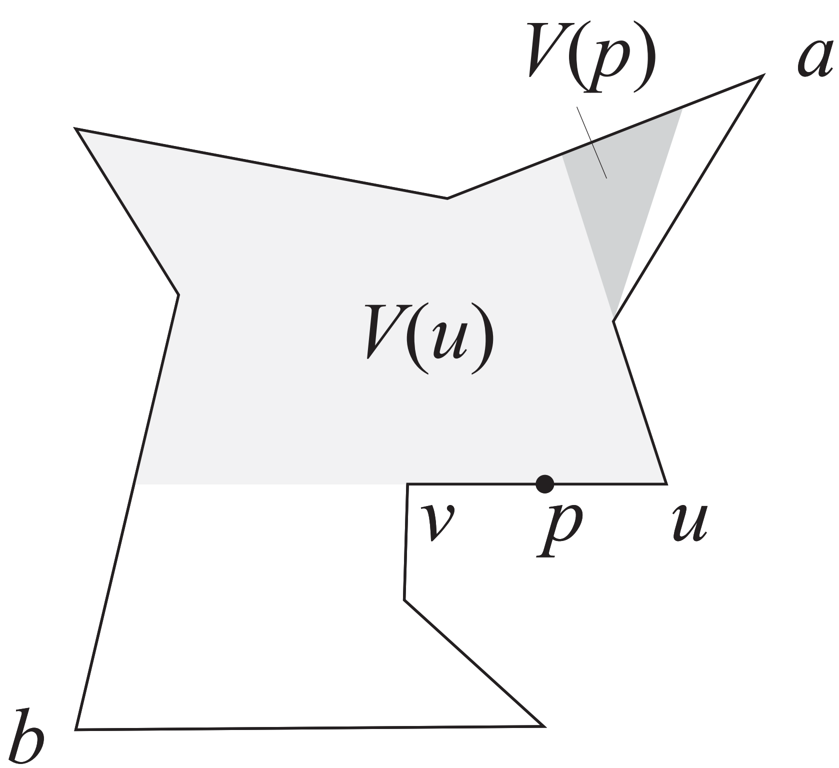

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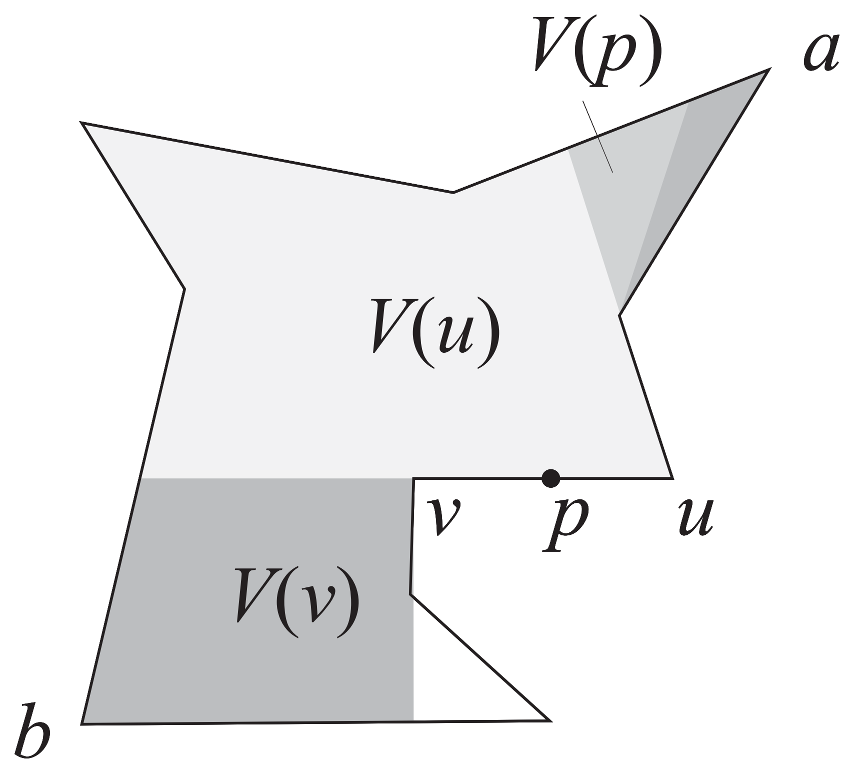

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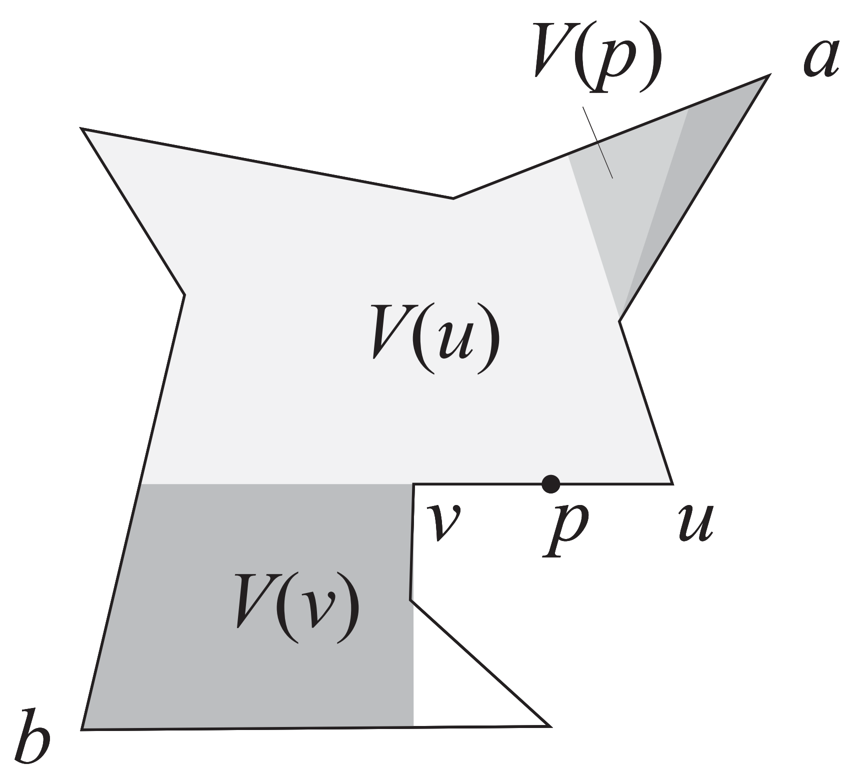
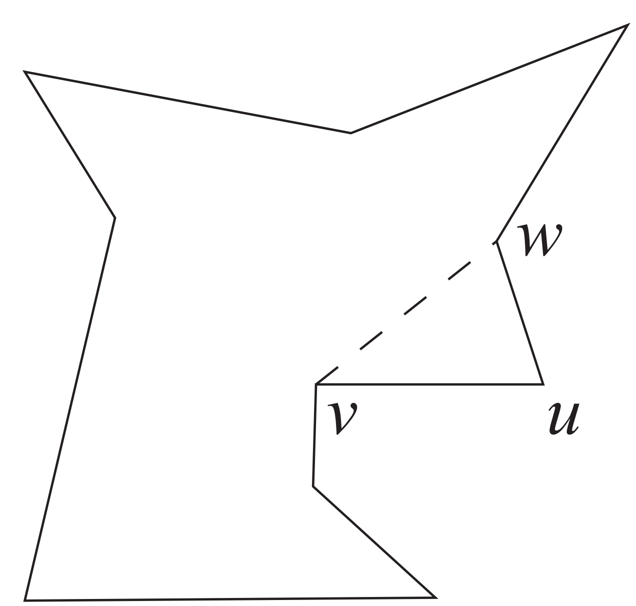
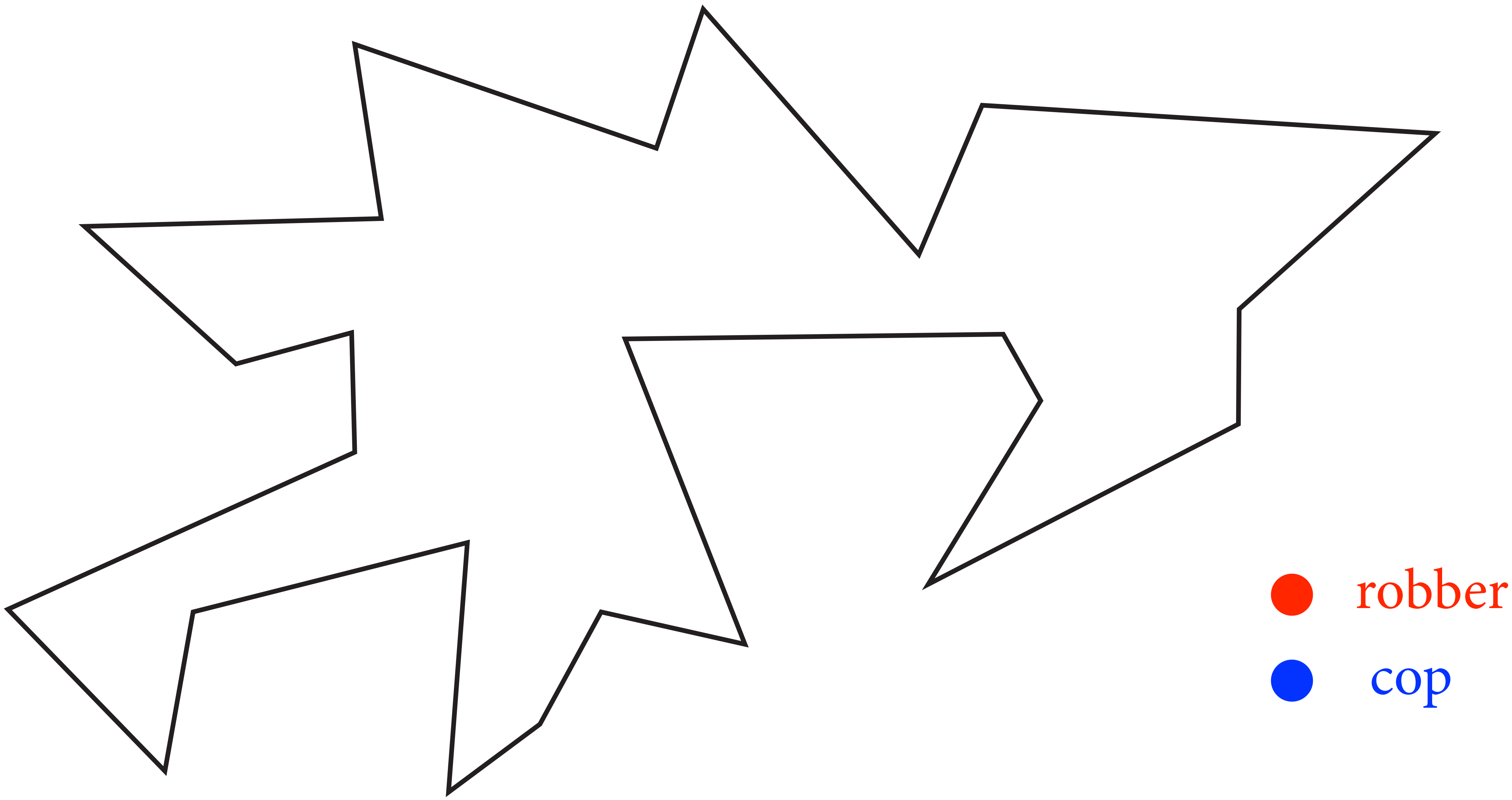


Then v dominates u and removing u gives smaller polygon.



Cops and Robbers in a Polygon



The cop wins the cops and robbers game on the visibility graph of a polygon

. . . because the graph is dismantlable

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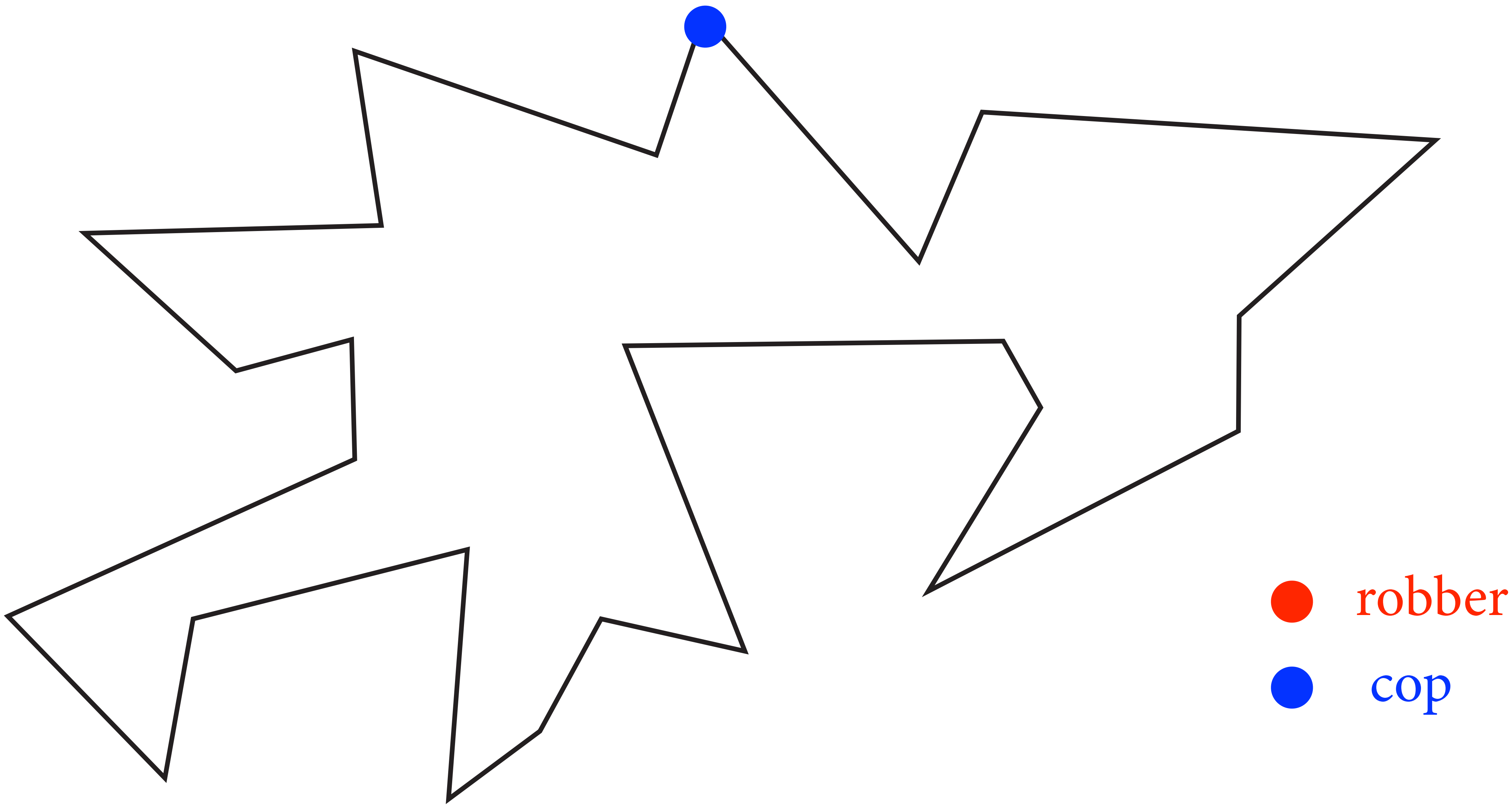

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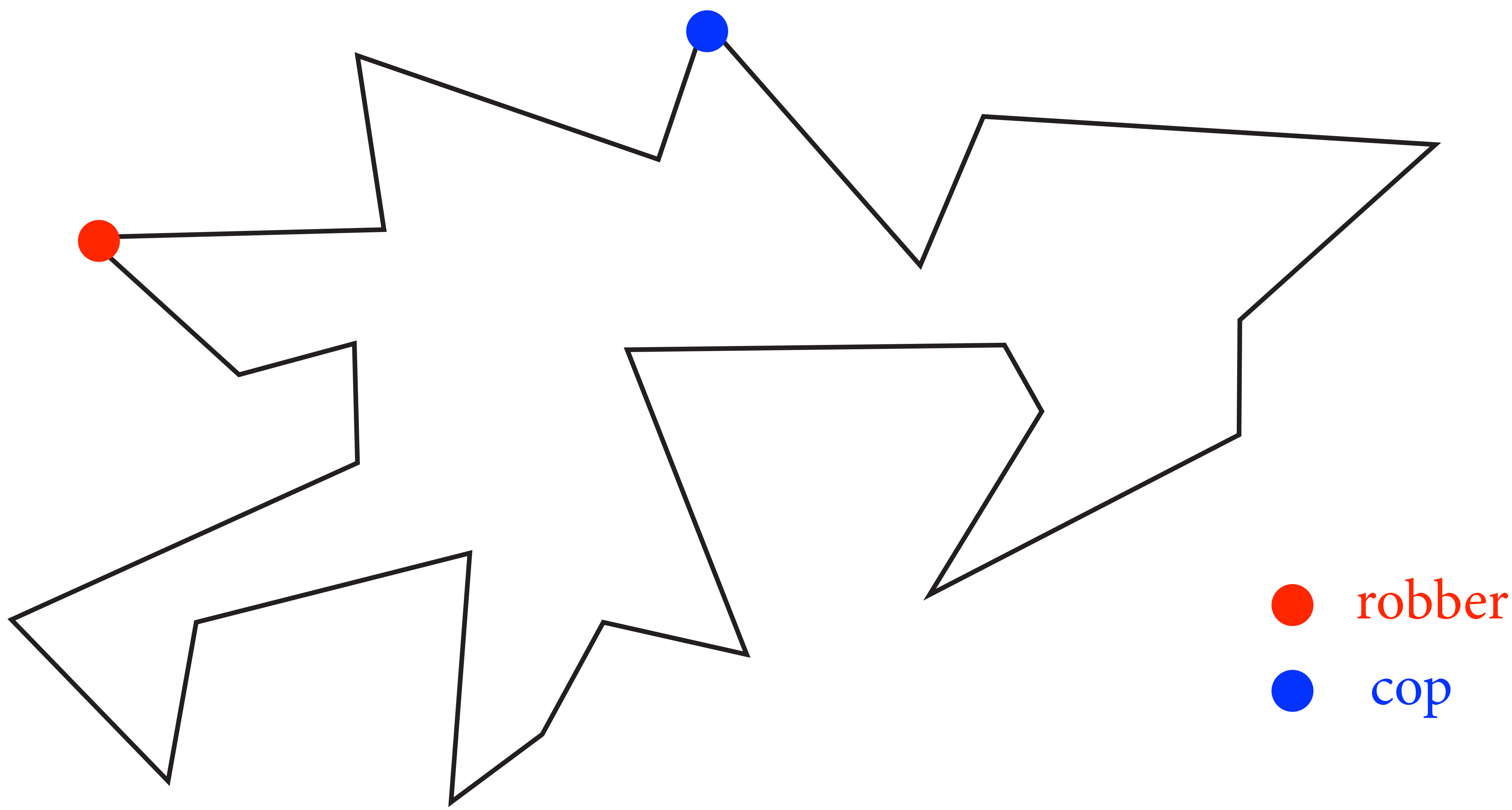

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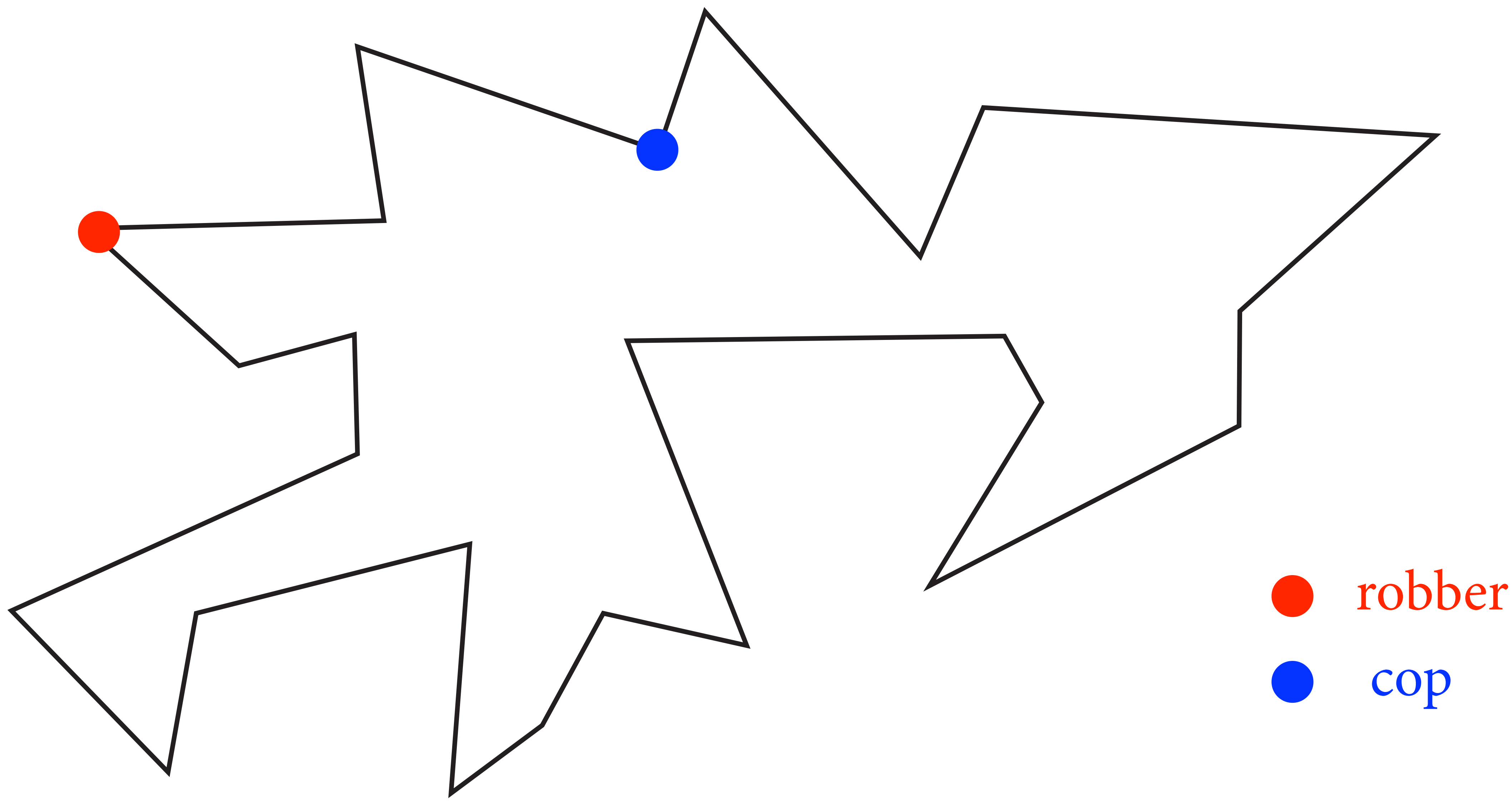

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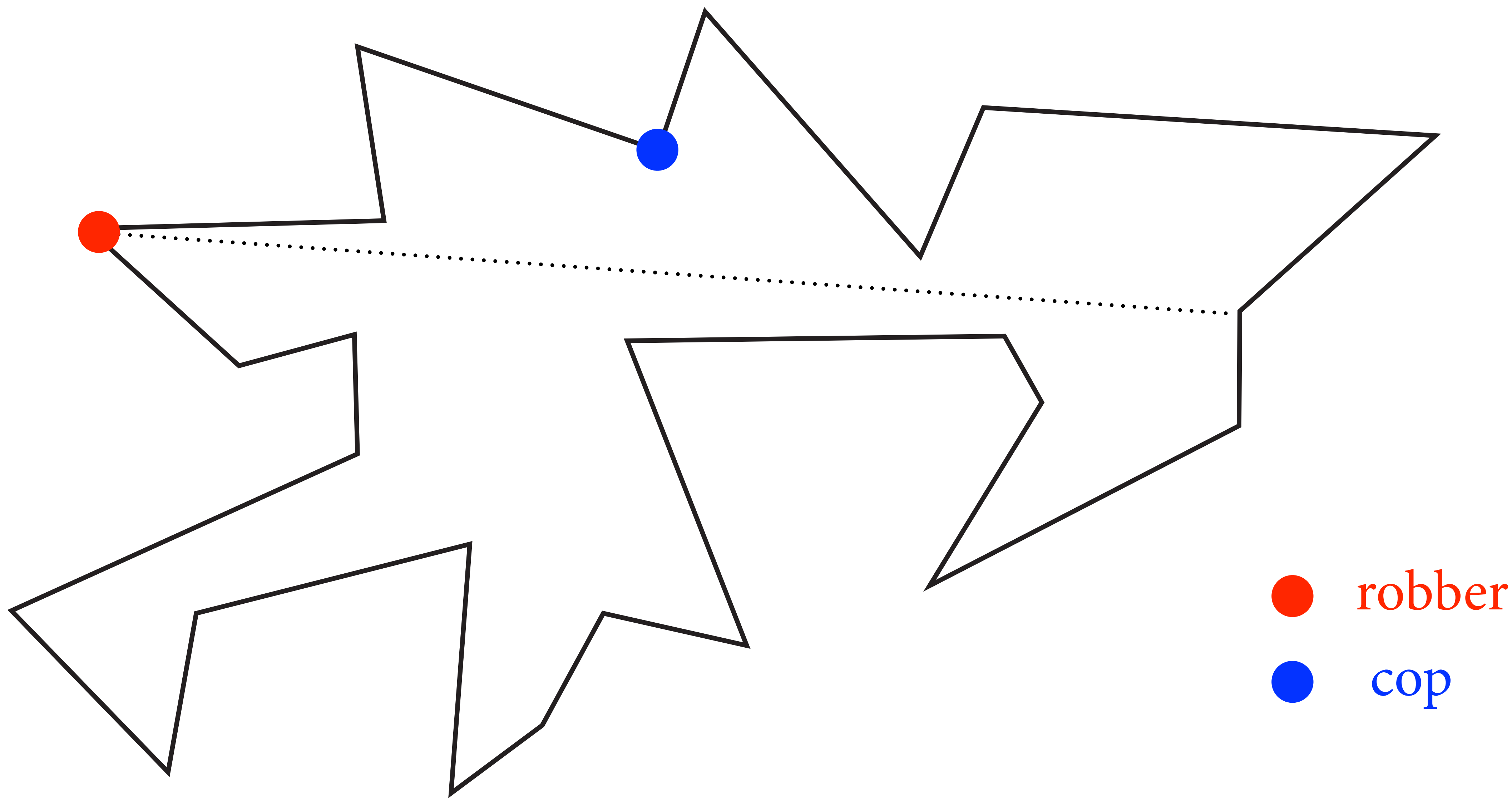

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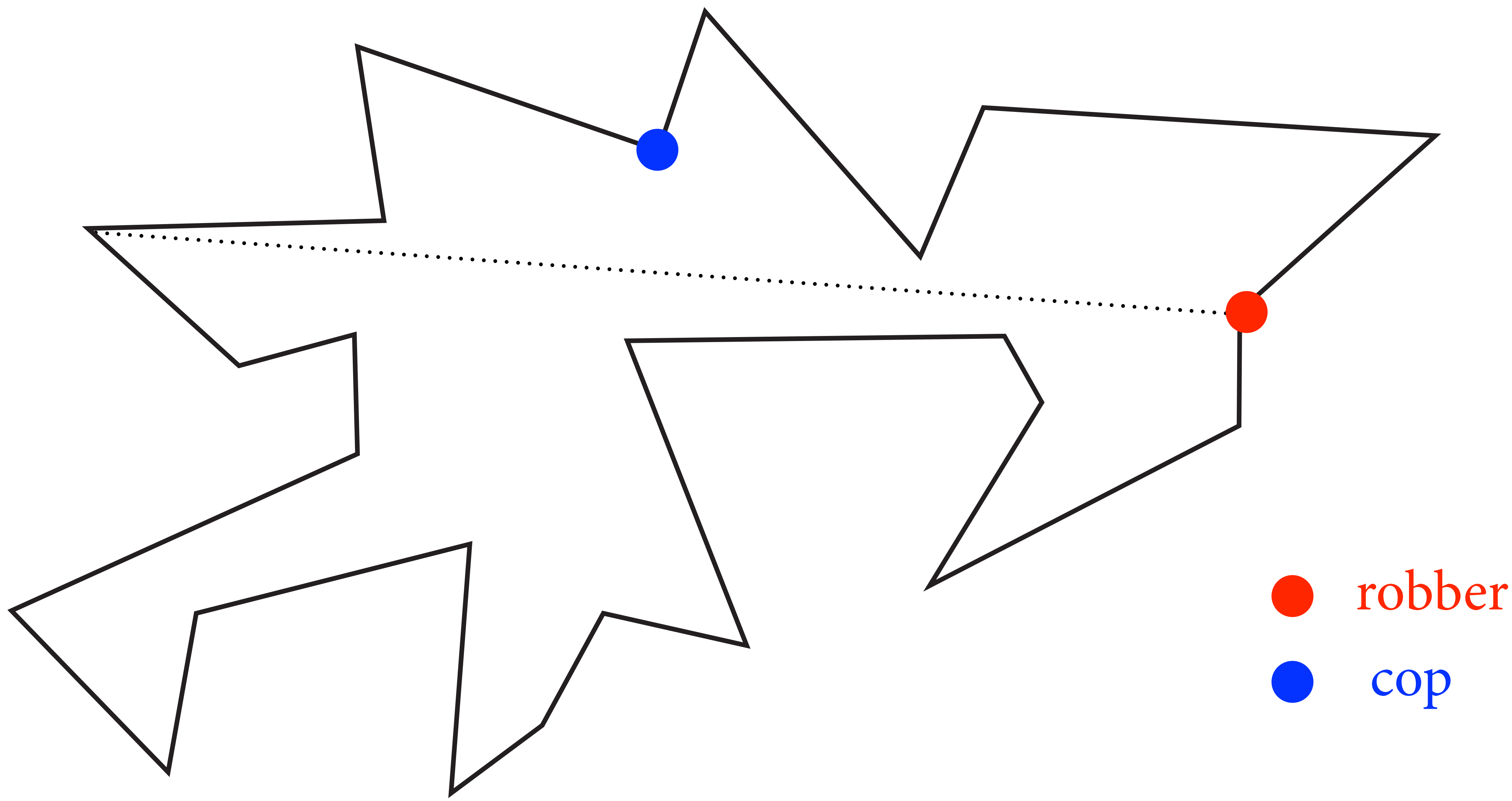

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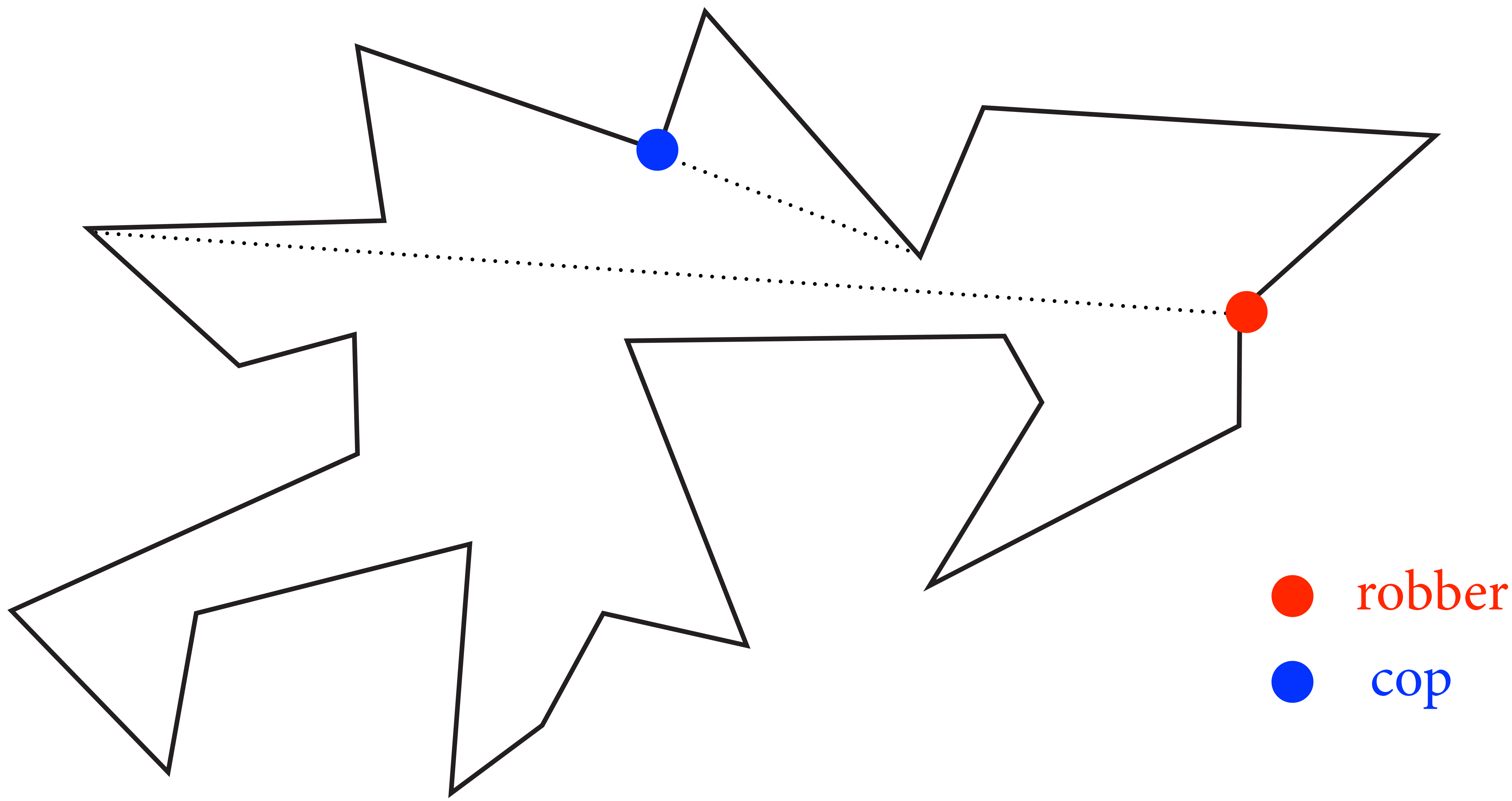

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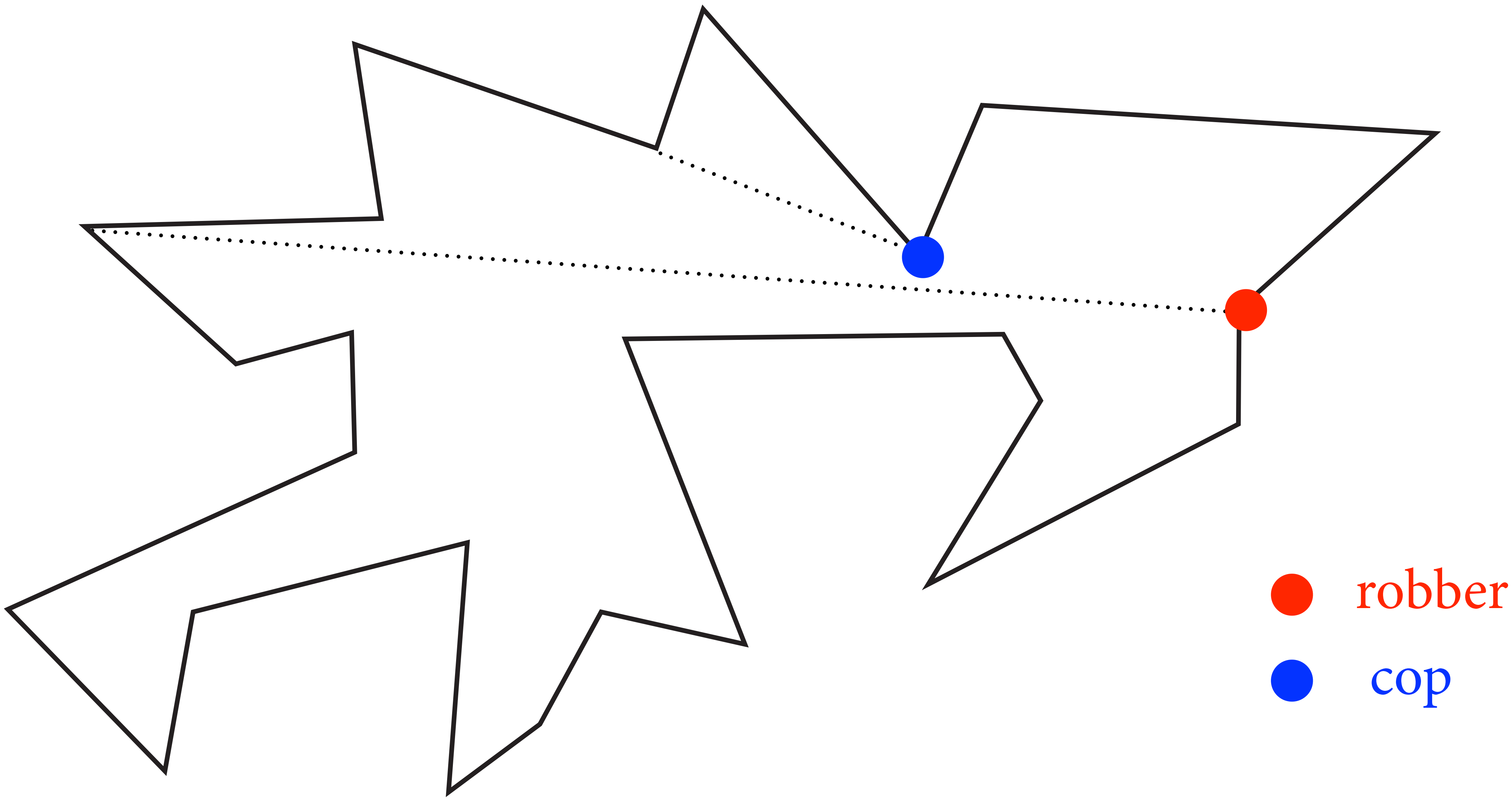

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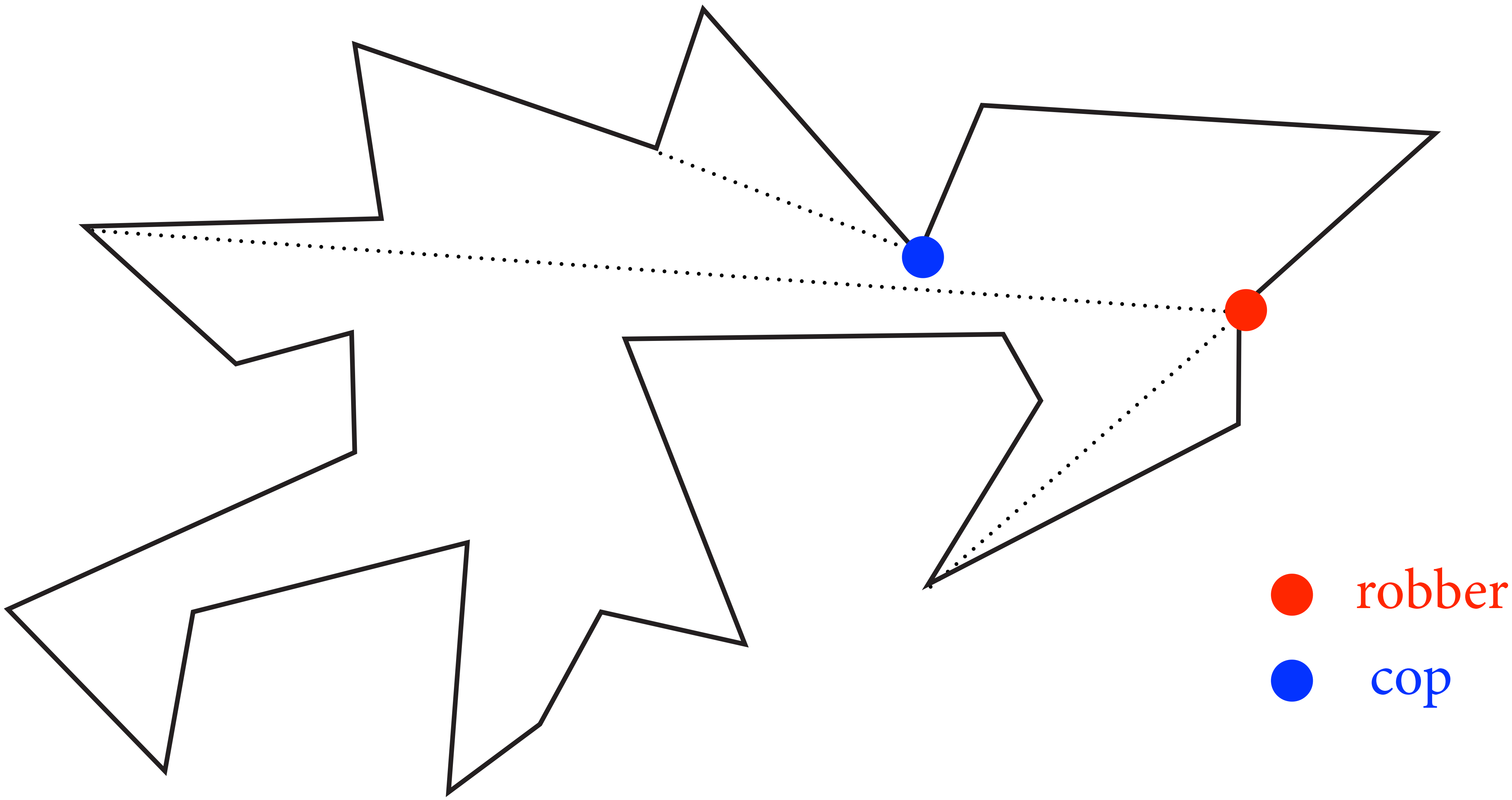

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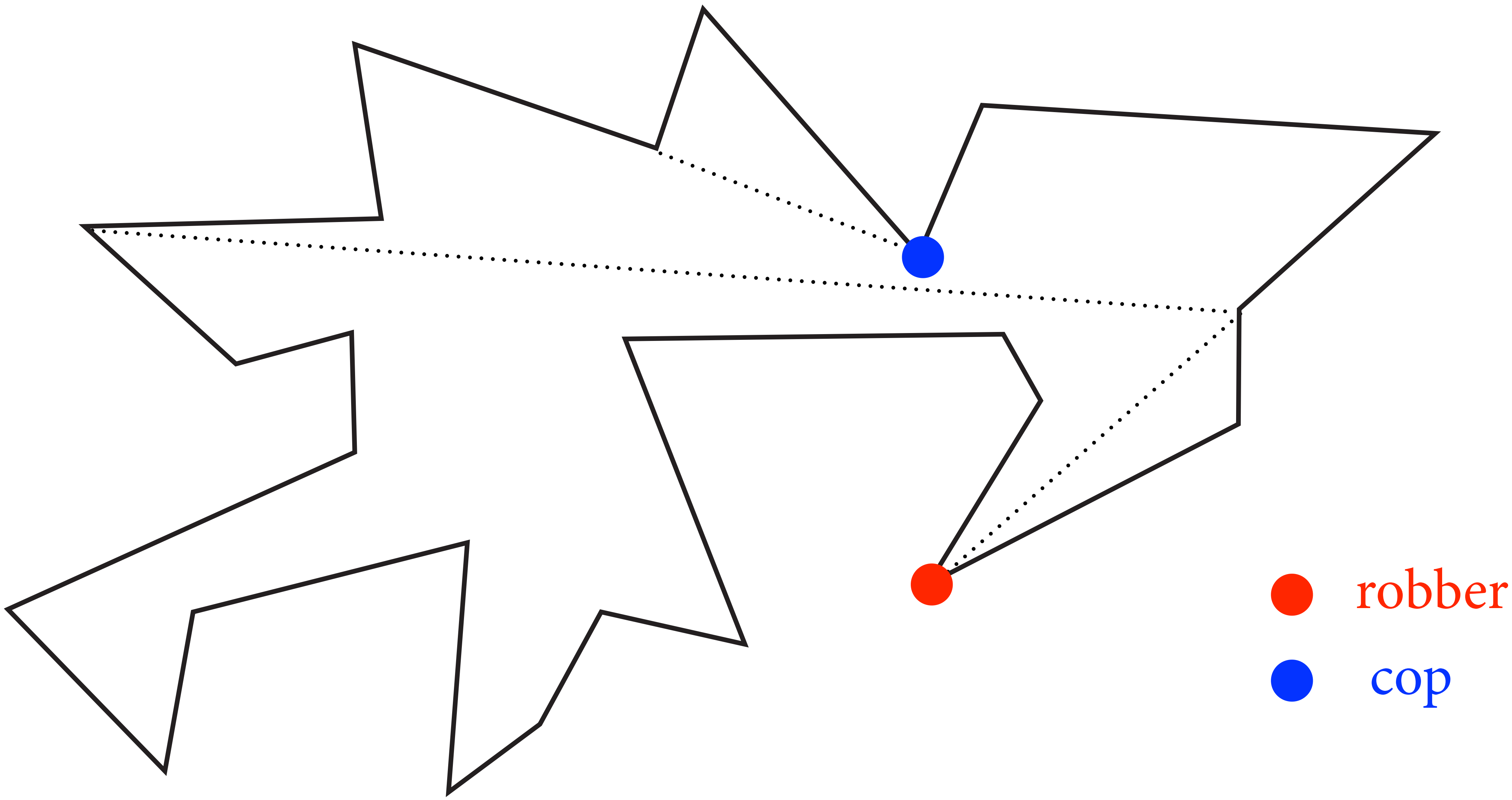

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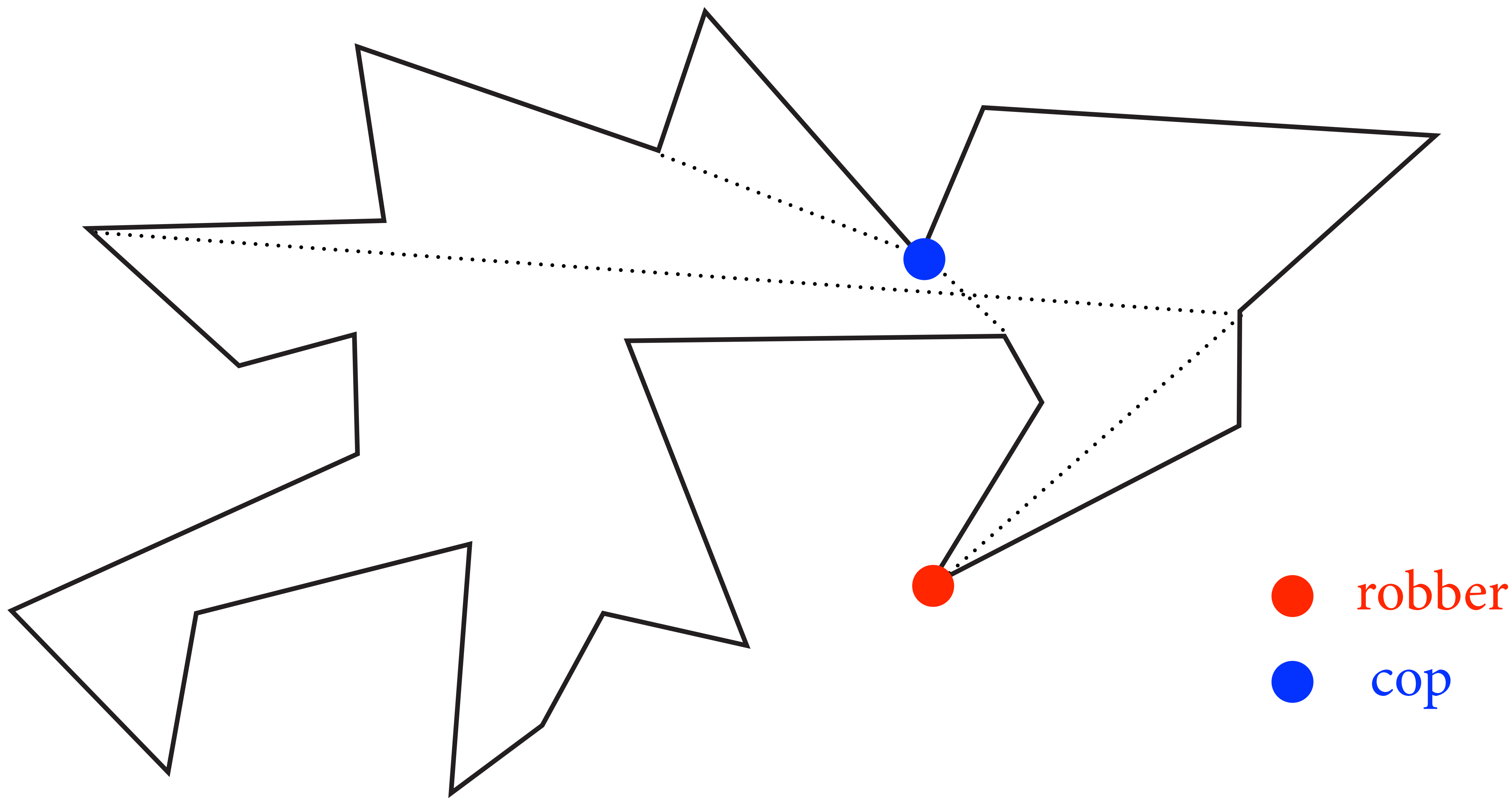

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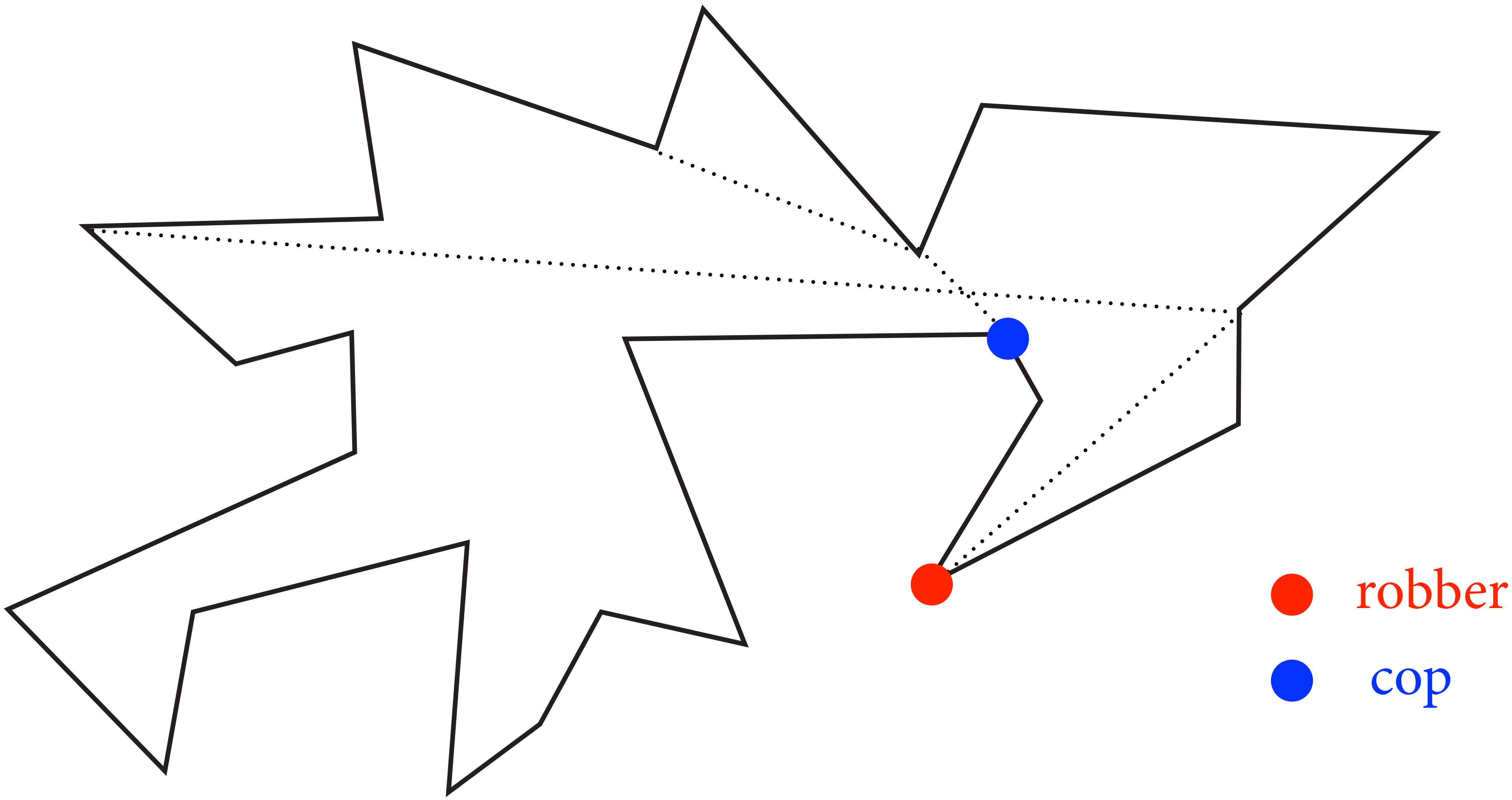

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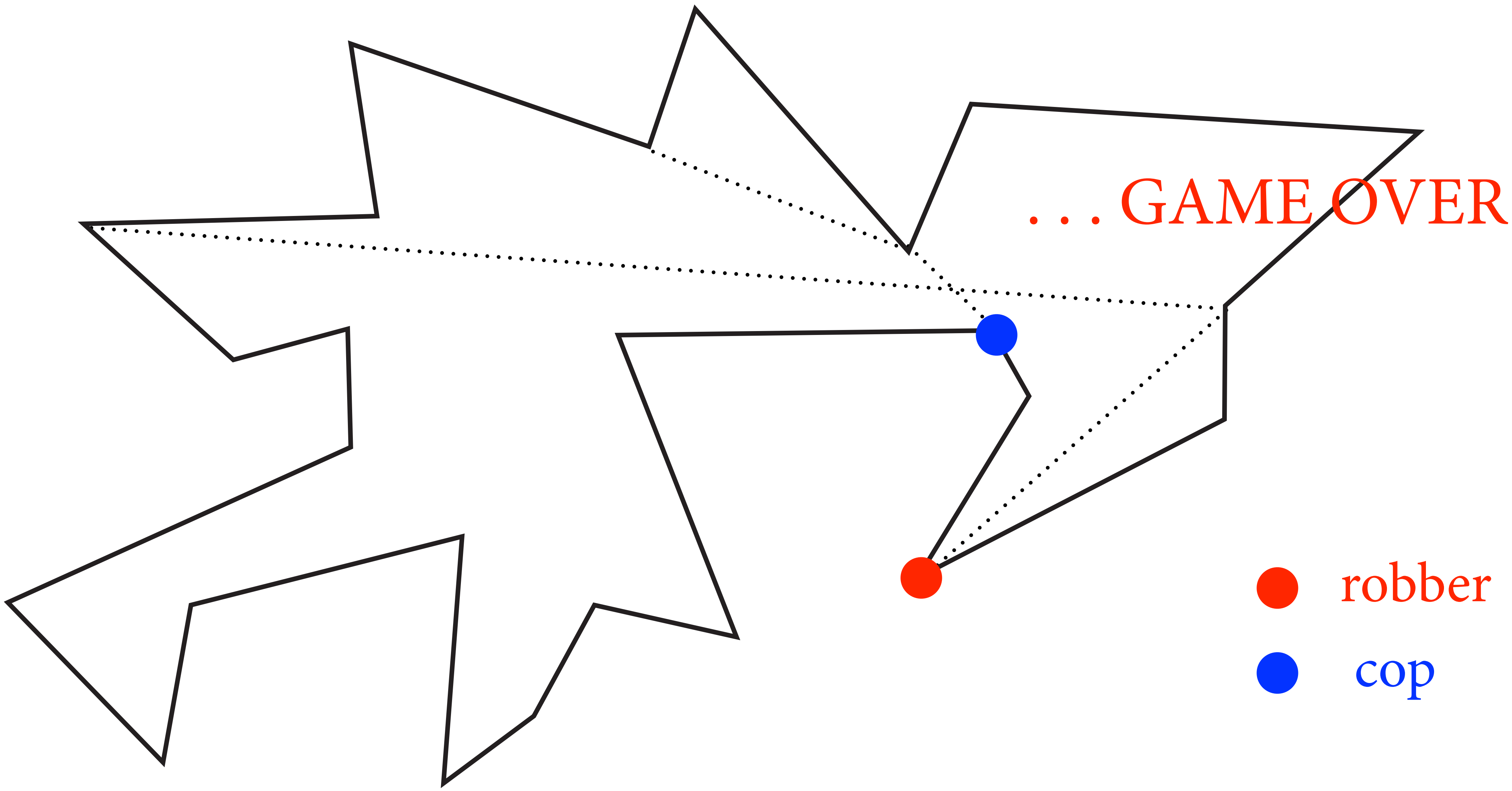

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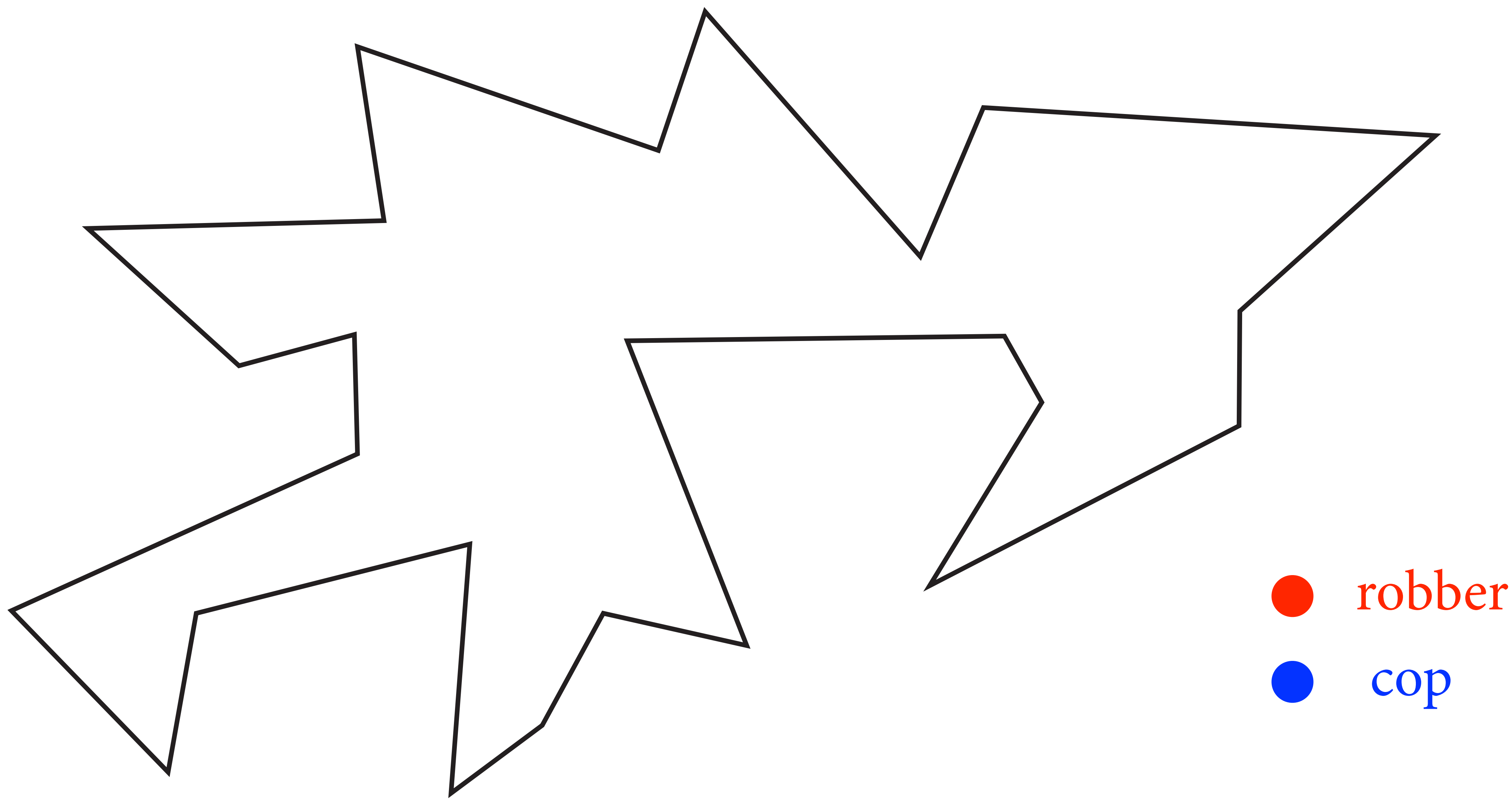
Cops and Robbers in the Interior of a Polygon



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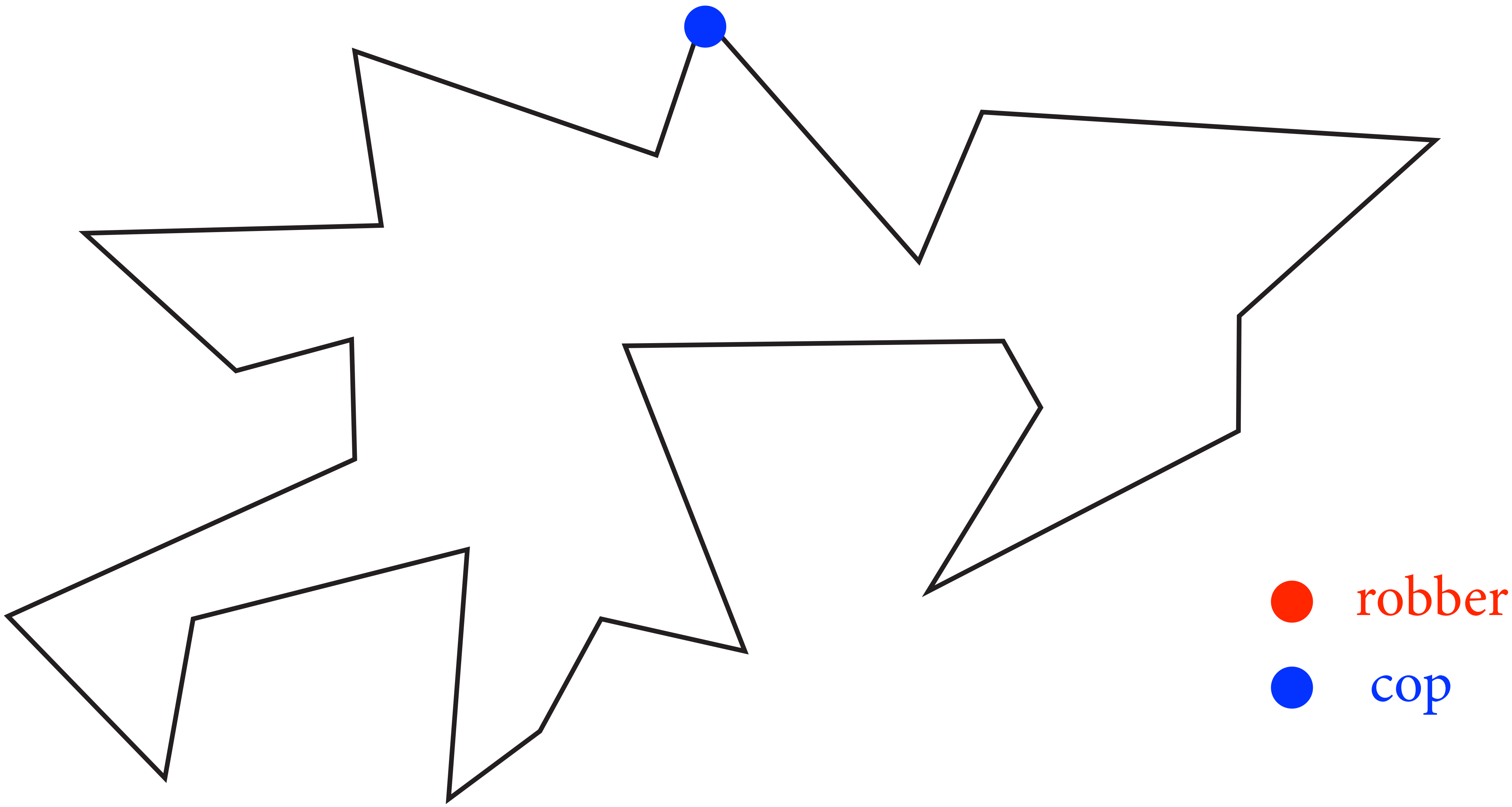

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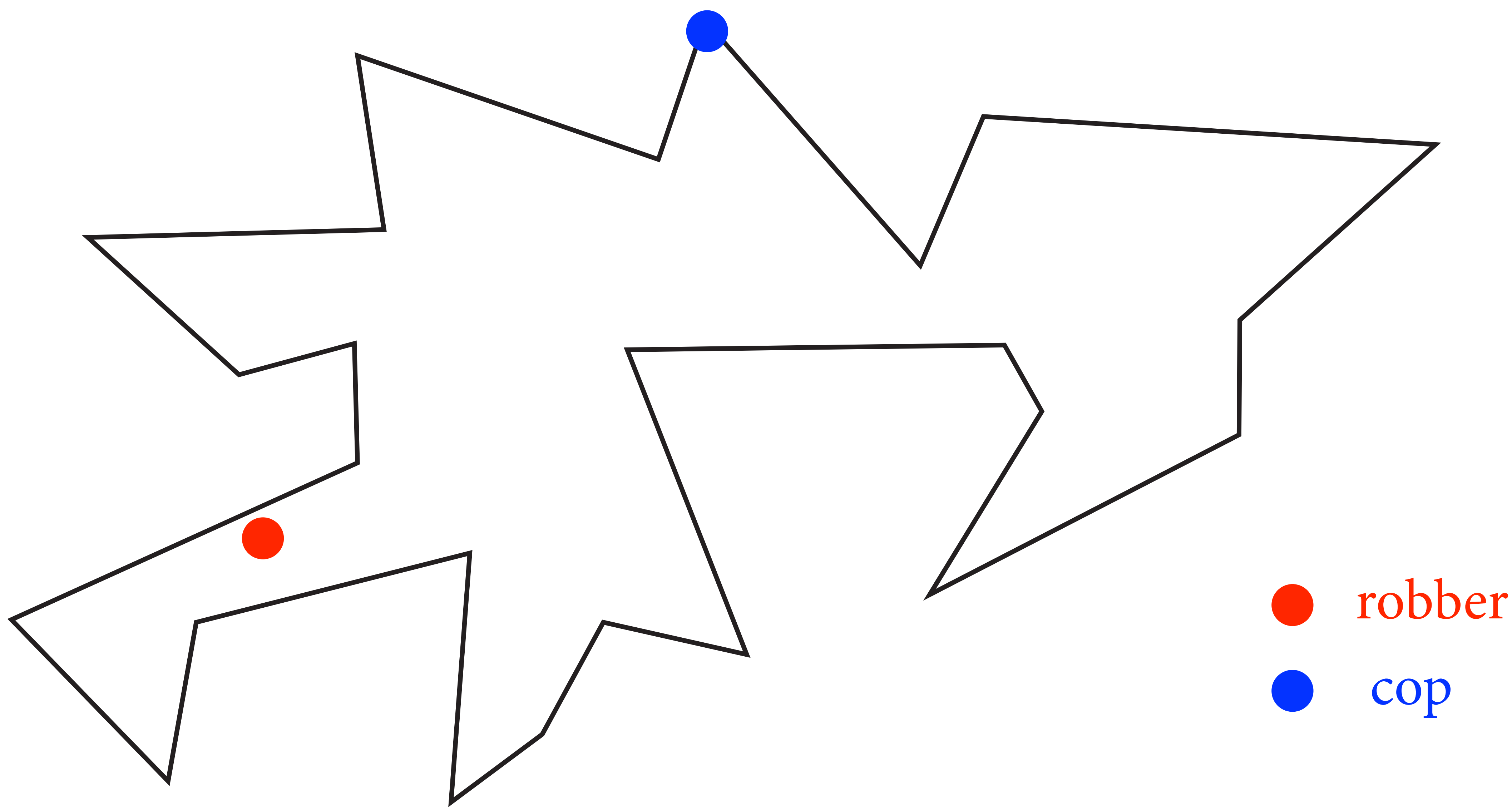

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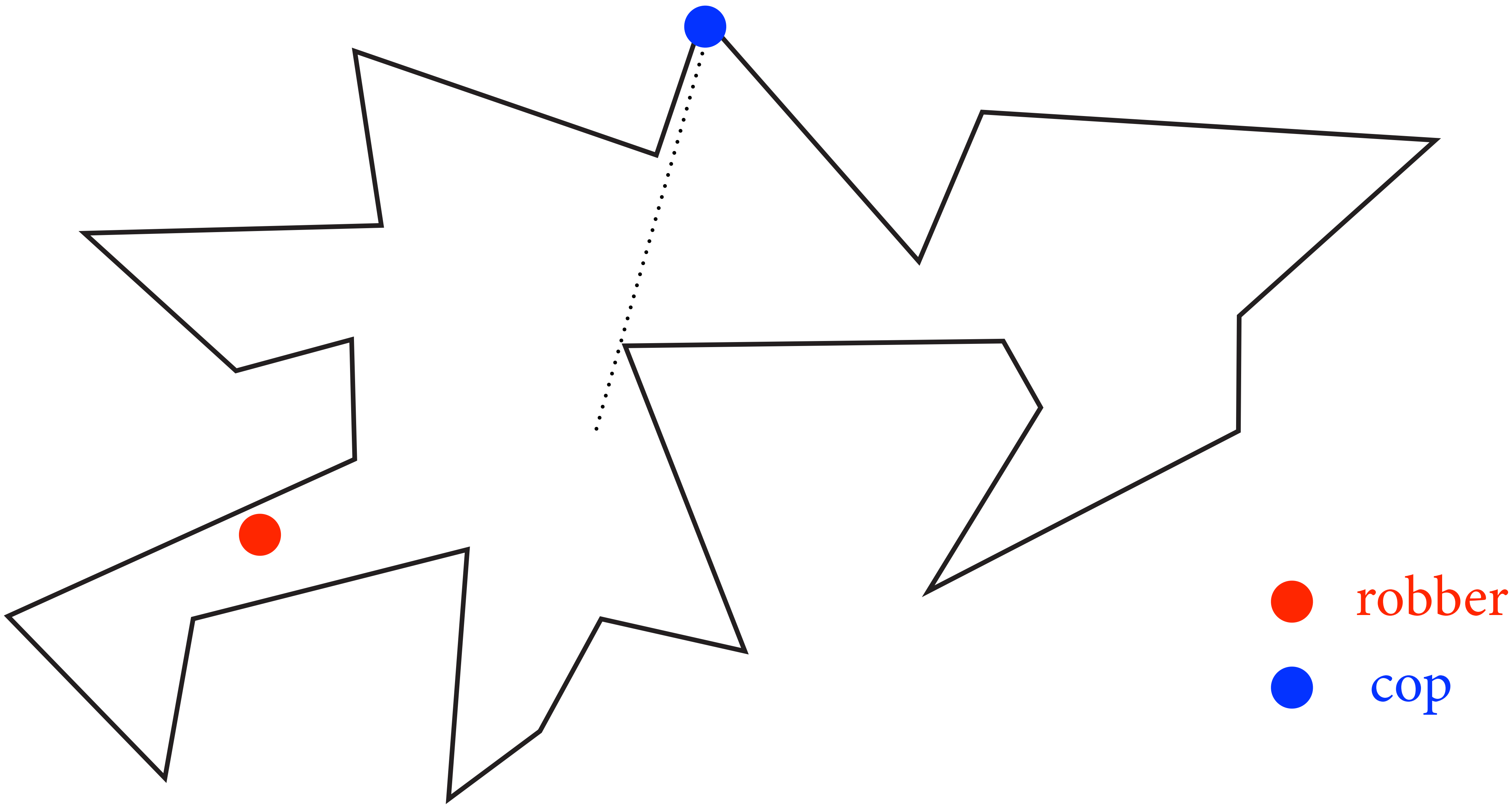

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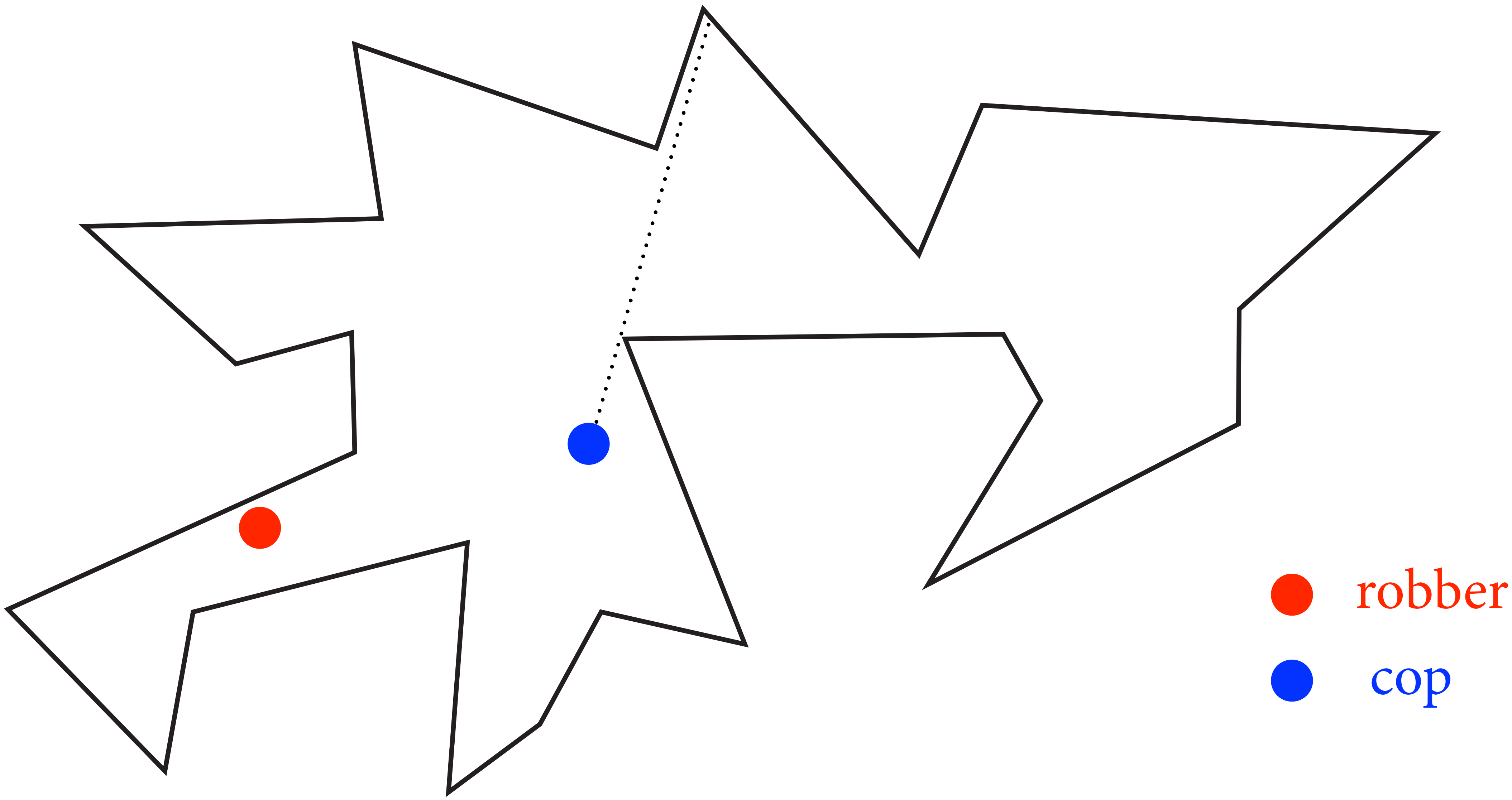

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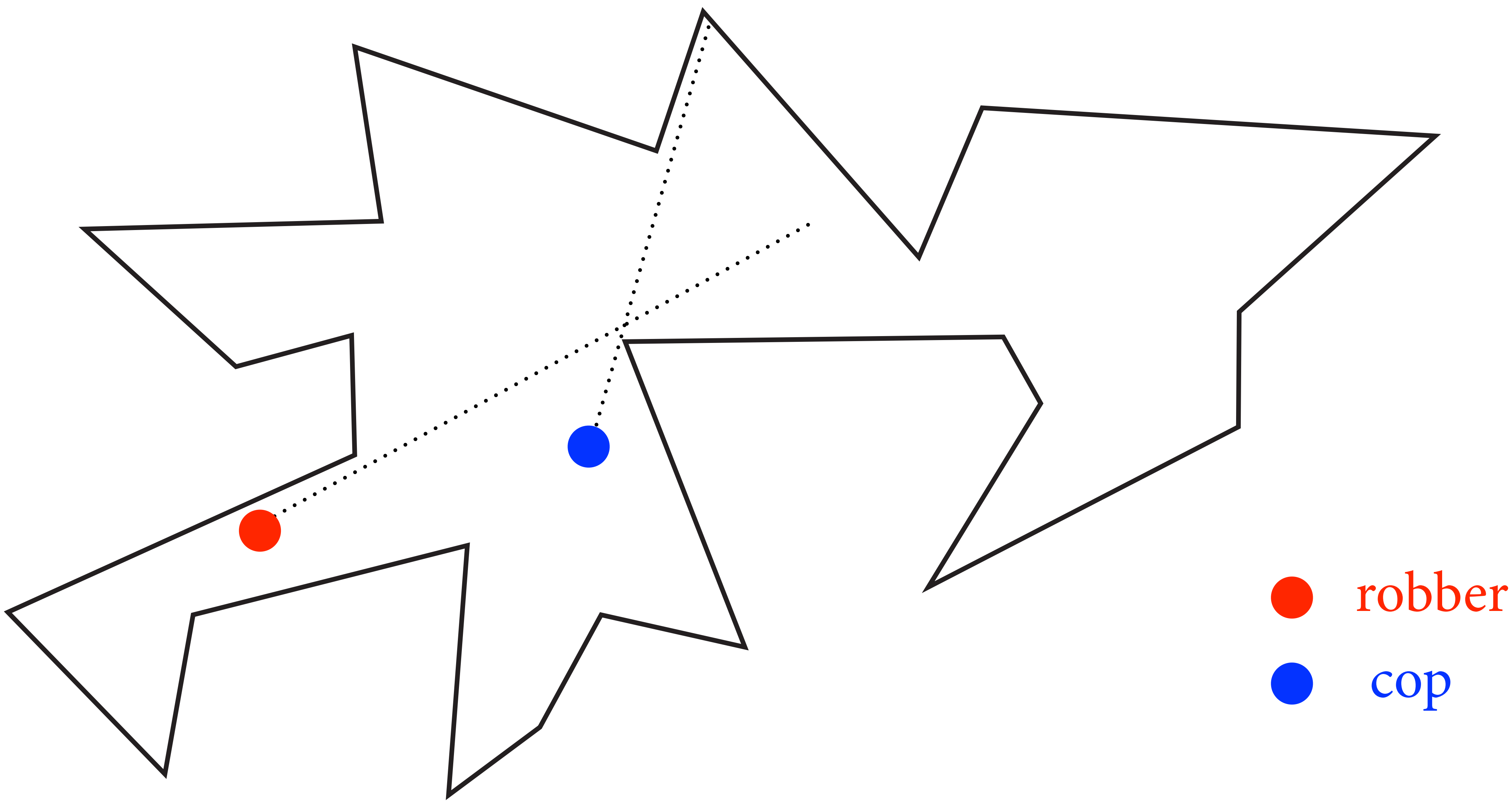

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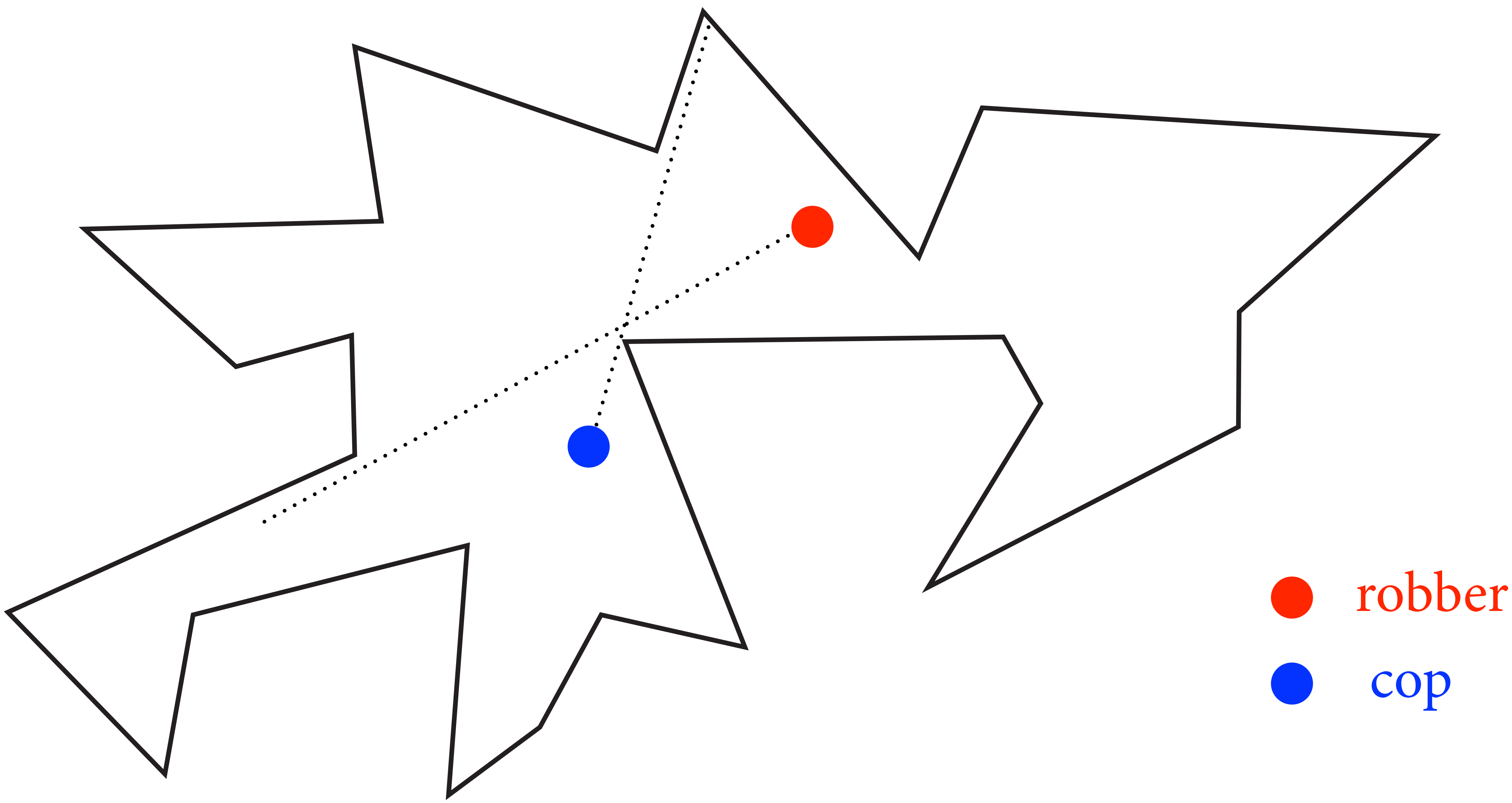

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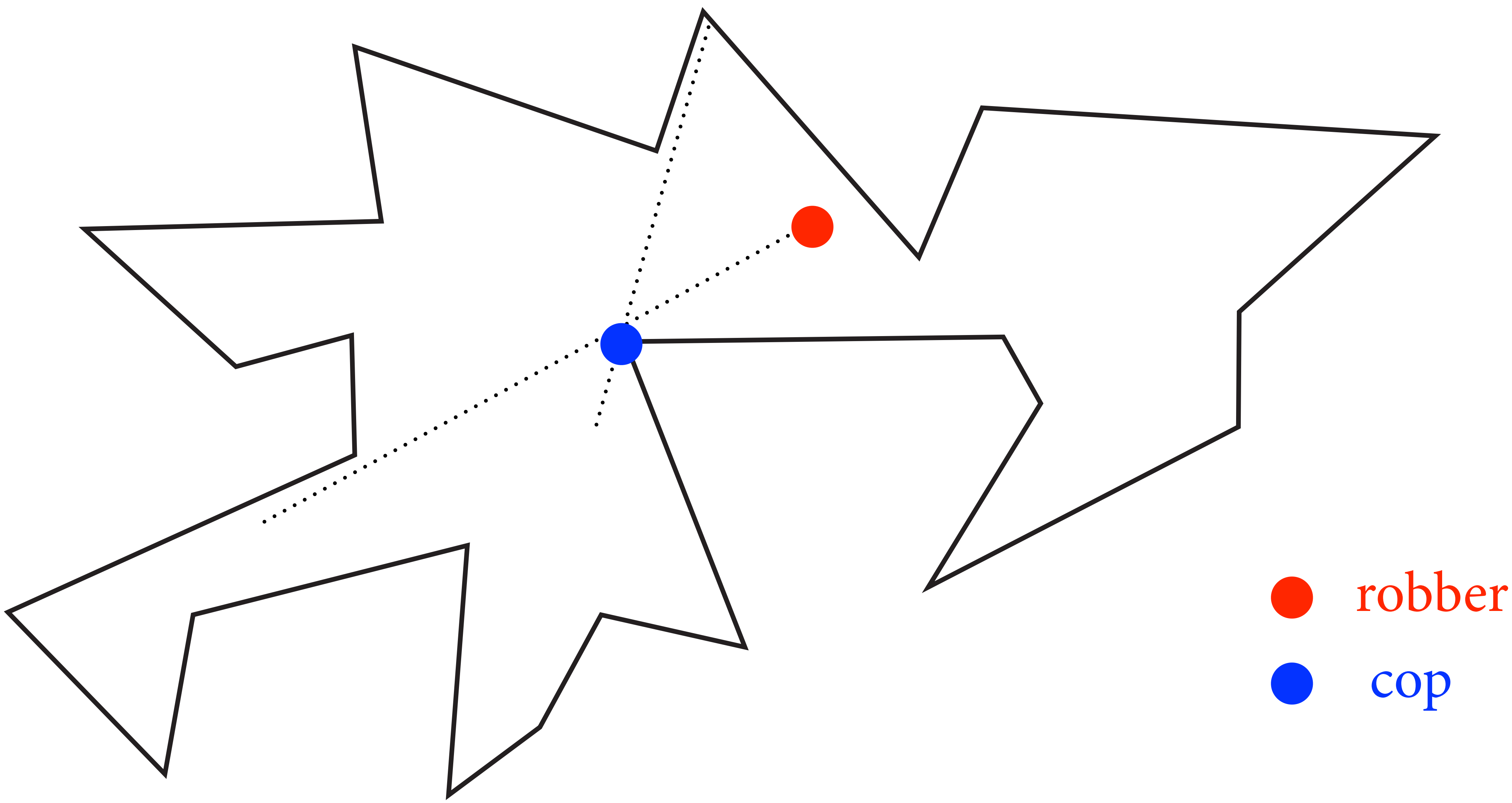

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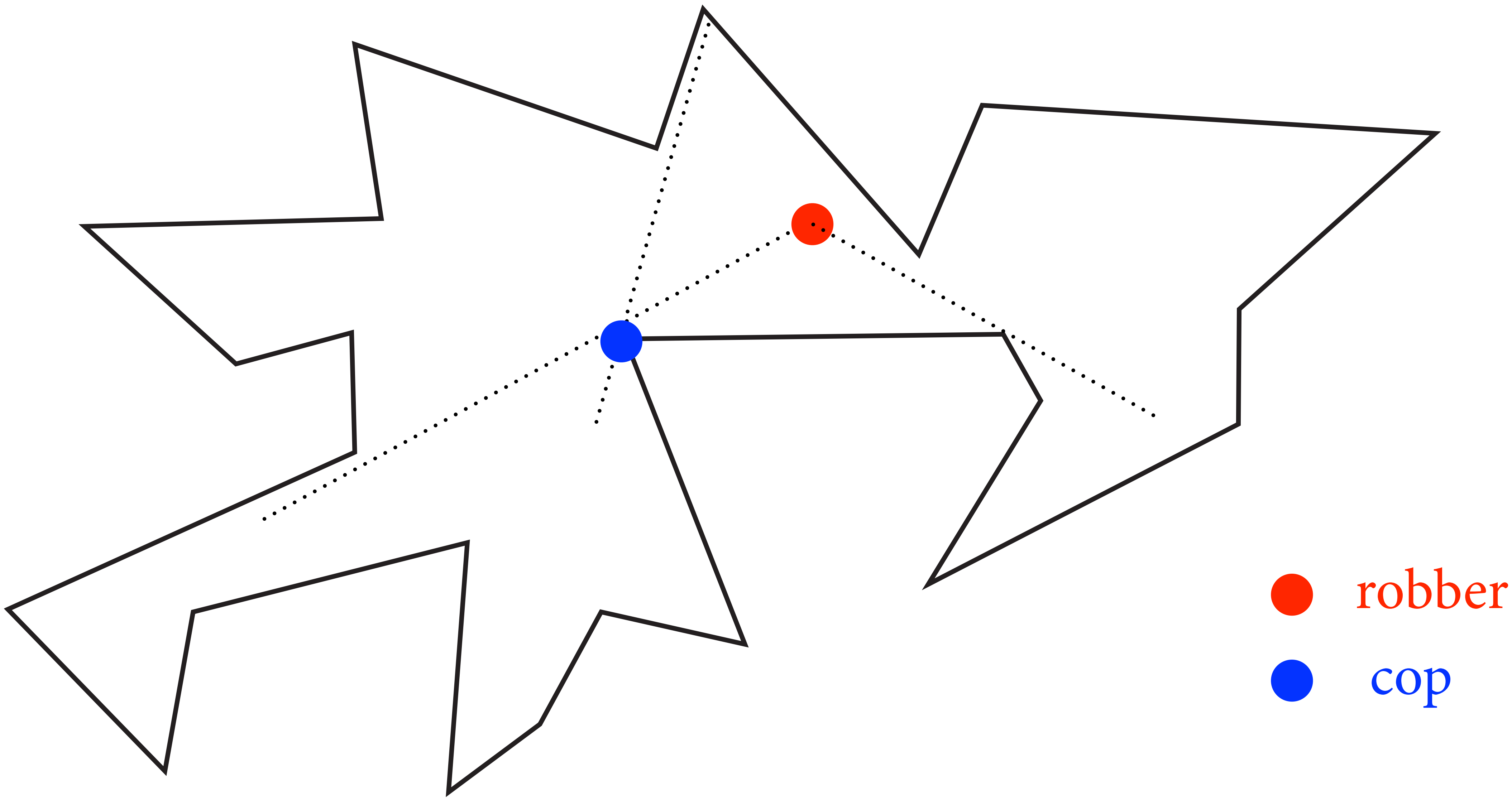

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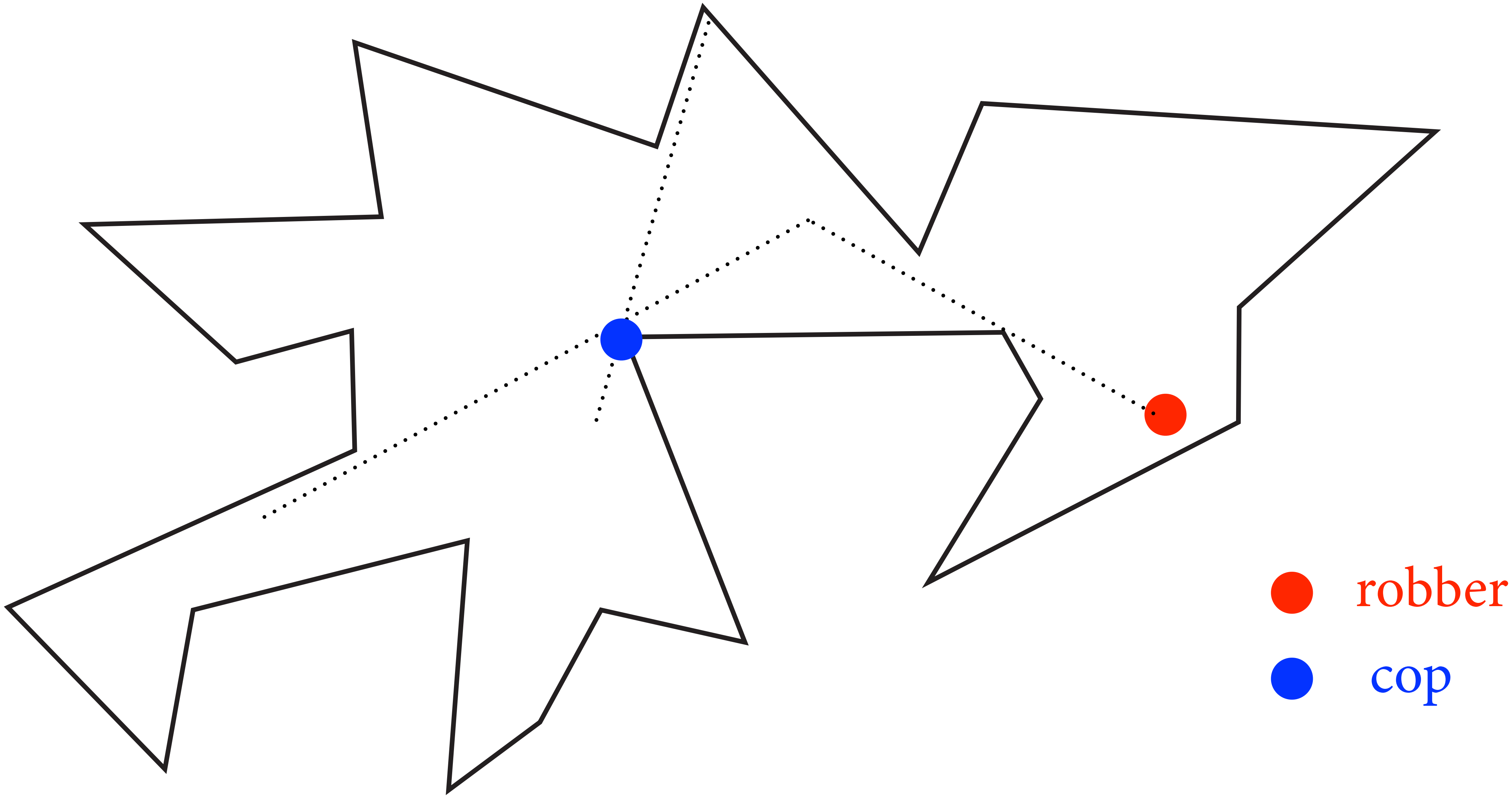

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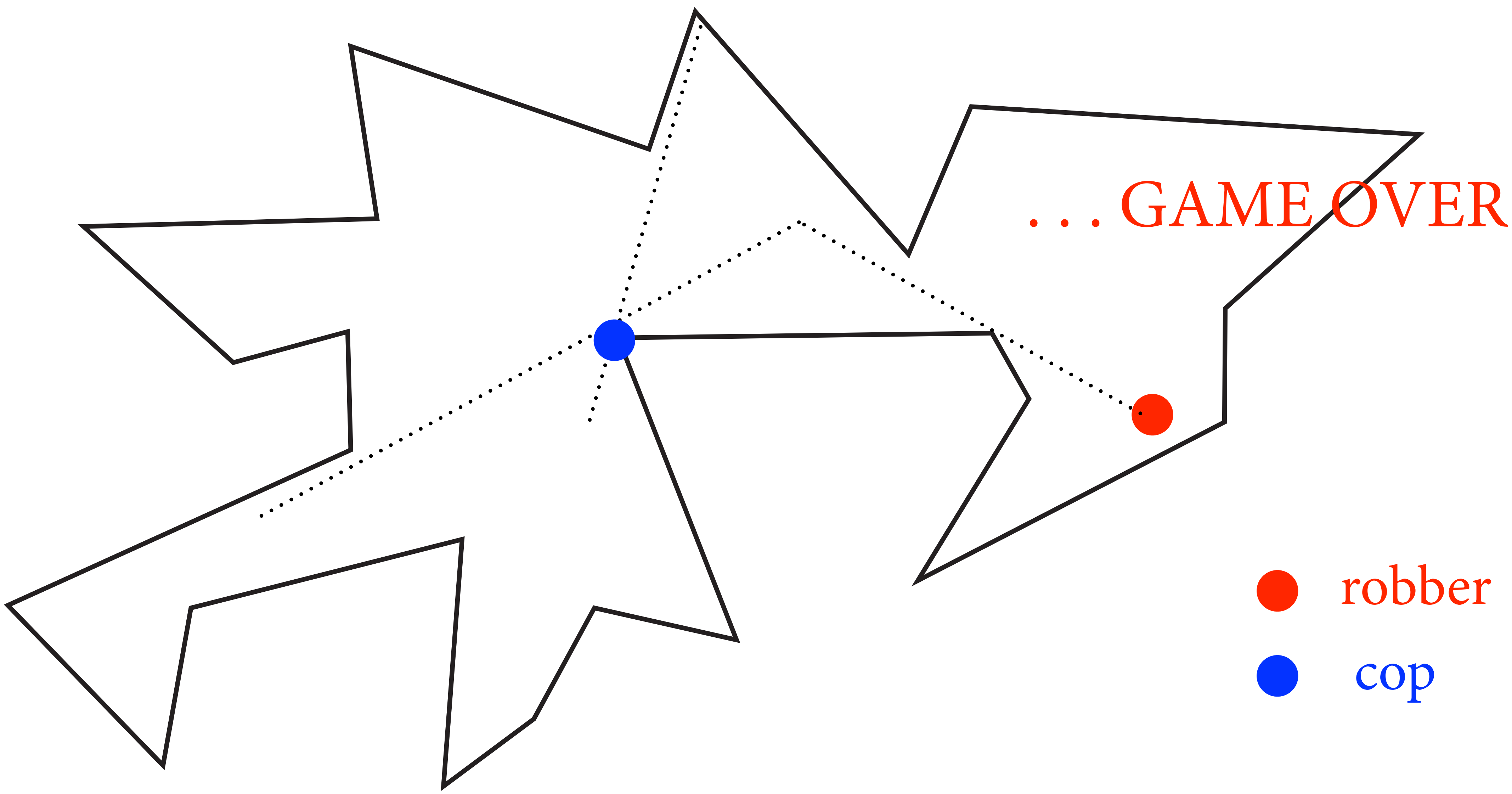

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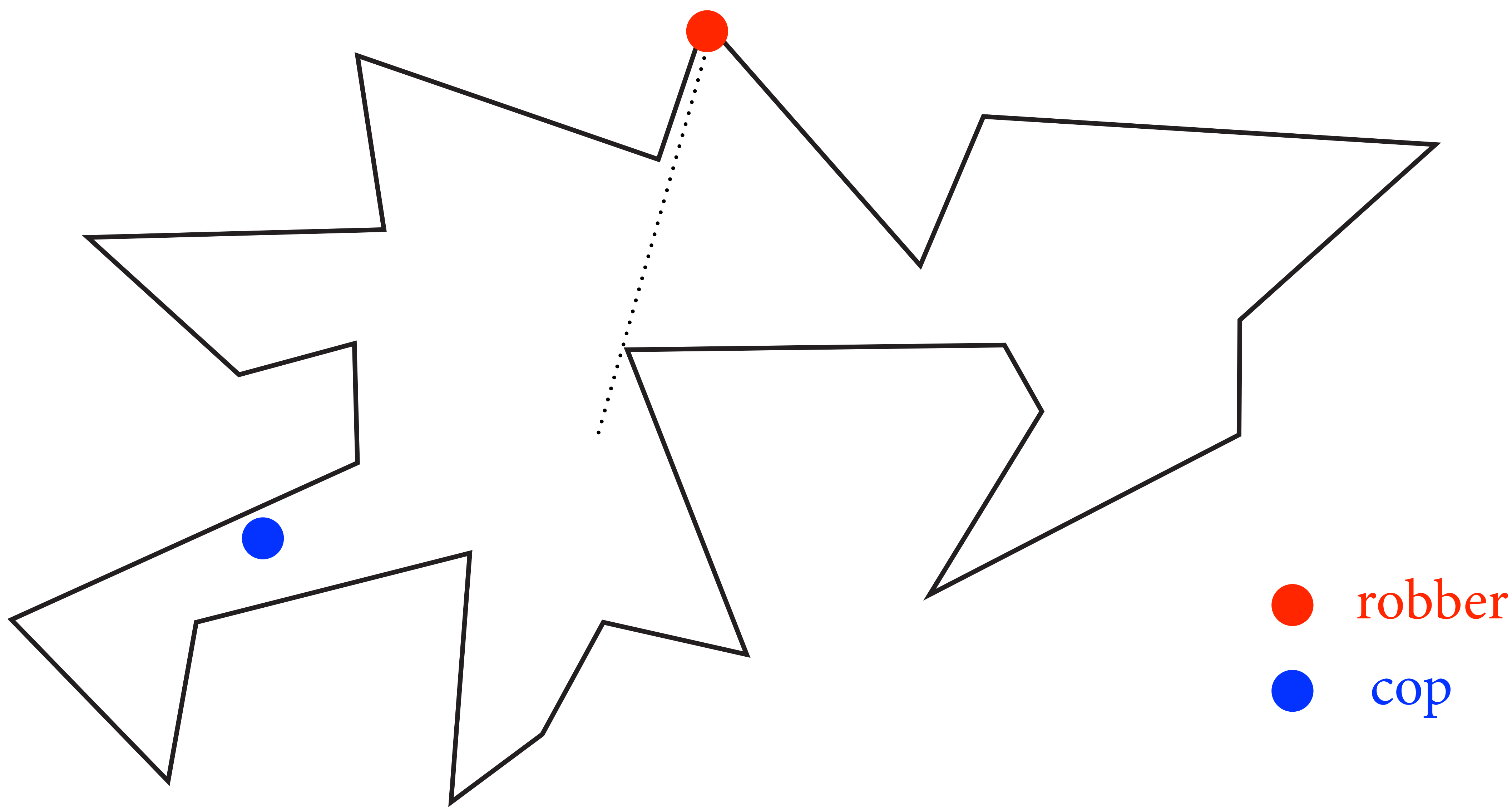

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In fact, the cop can win by playing on the reflex vertices.

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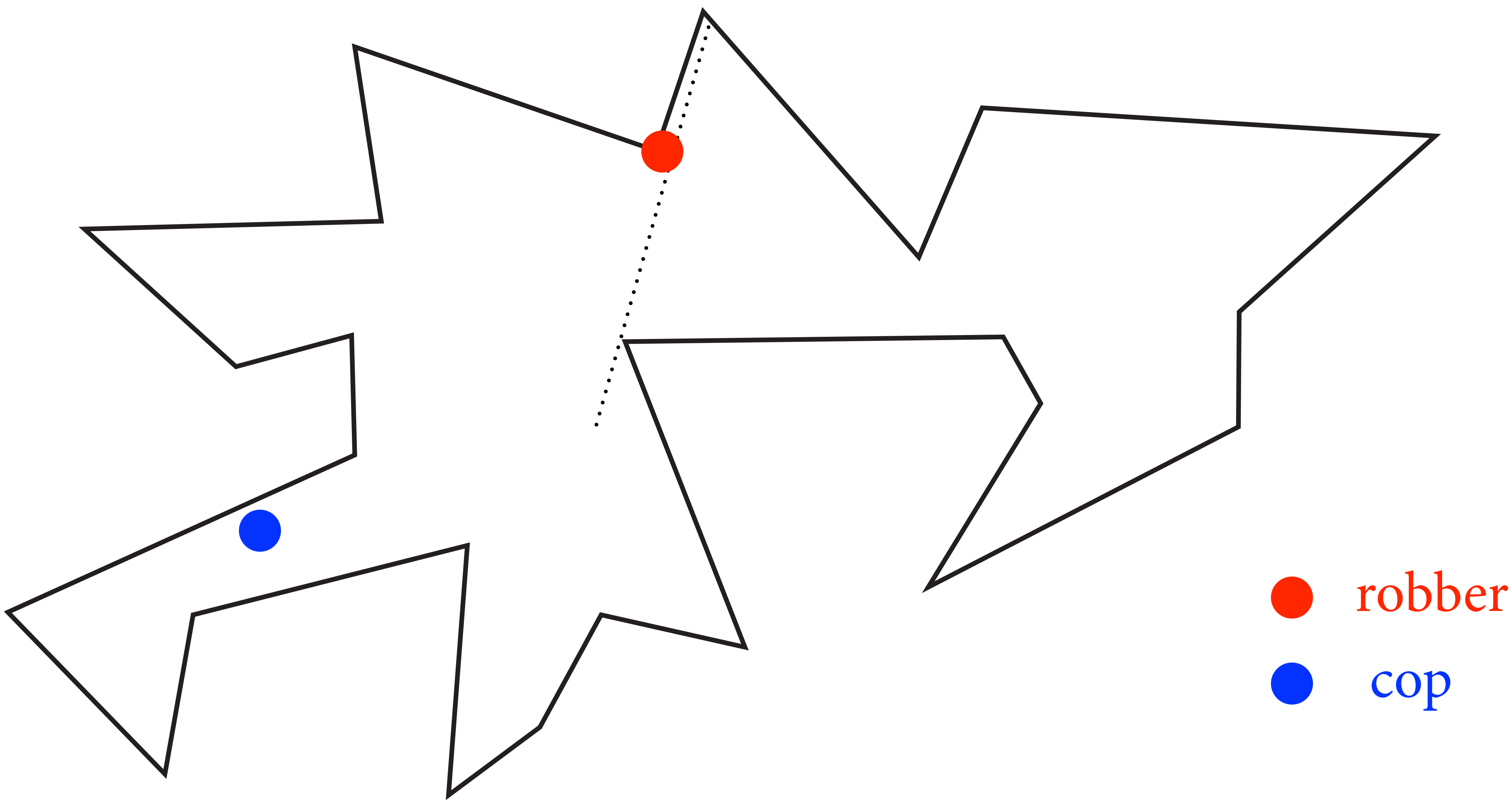

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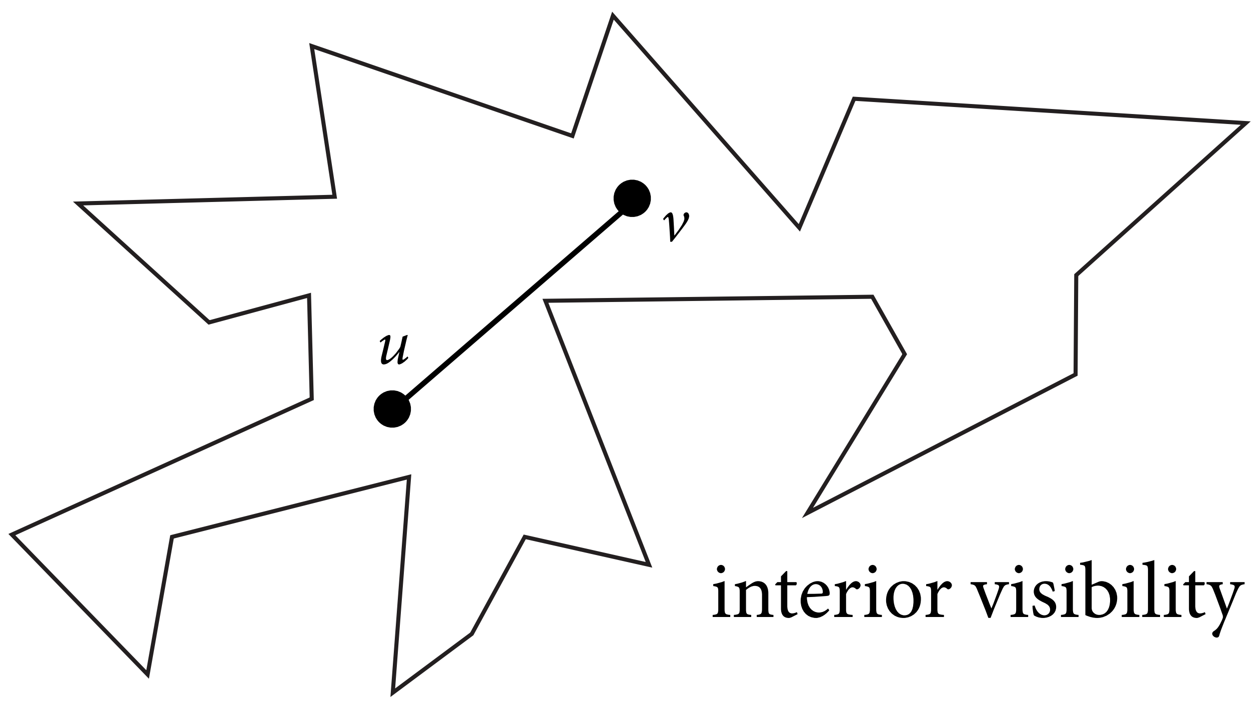
Theorem. [A.L., H. Vosoughpour] The cop wins the cops and robbers game in the interior of a polygon.

Consequence: interior visibility graphs are a natural class of infinite cop-win graphs.



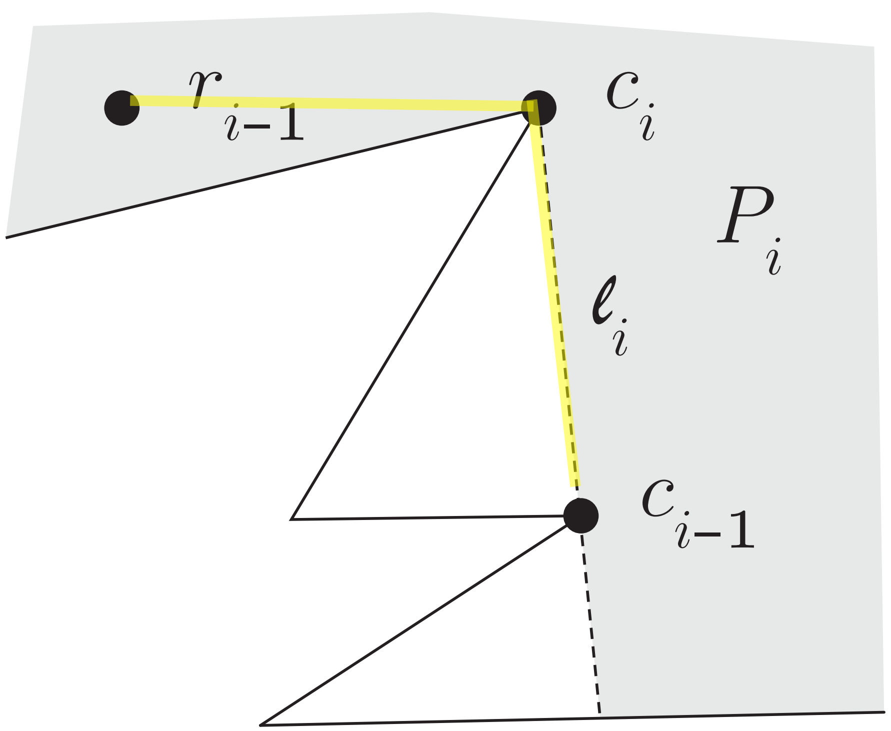
interior visibility graph

Geňa Hahn [Cops, Robbers and Graphs, 2007]: "As of this writing, no interesting classes of infinite cop-win graphs have been described."

Cops and Robbers in the Interior of a Polygon

Theorem. [A.L., H. Vosoughpour] The cop wins the cops and robbers game in the interior of a polygon.

Proof. The cop wins by taking the first step of the shortest path to the robber. In particular, the cop can play on the reflex vertices. The robber is trapped in an ever-shrinking region.

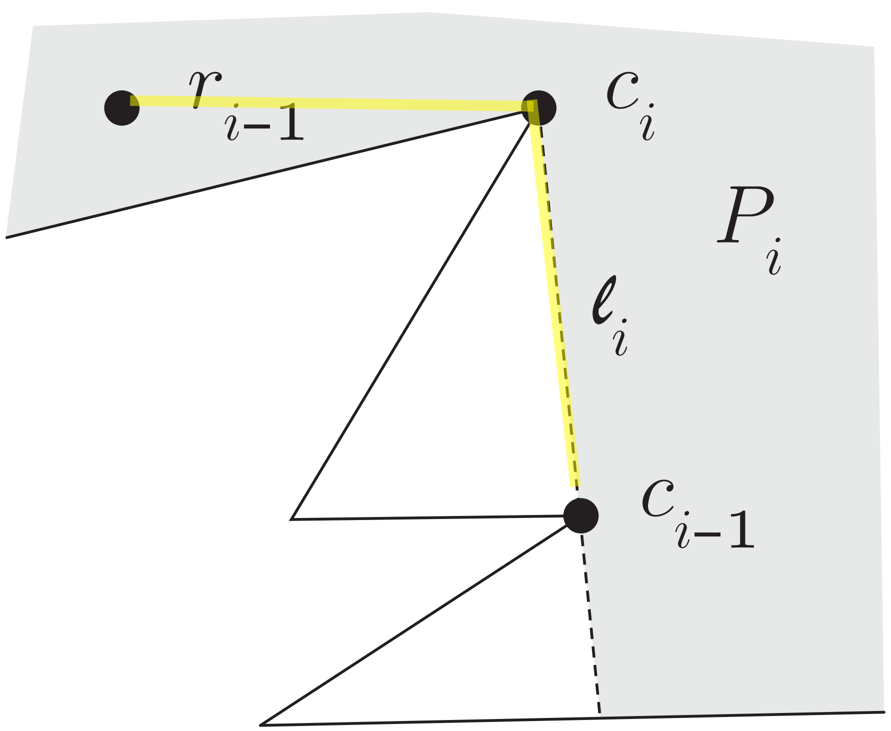


the robber can't leave P_i

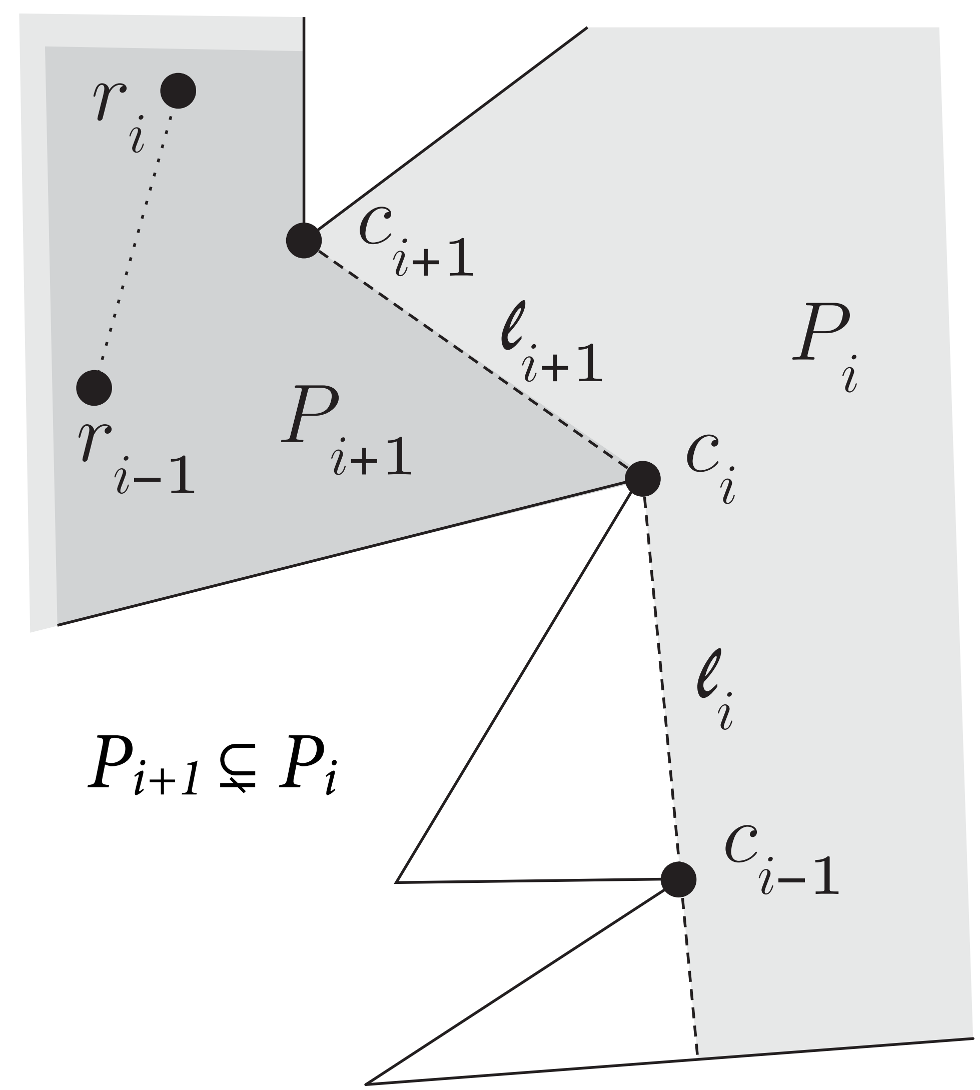
Cops and Robbers in the Interior of a Polygon

Theorem. [A.L., H. Vosoughpour] The cop wins the cops and robbers game in the interior of a polygon.

Proof. The cop wins by taking the first step of the shortest path to the robber. In particular, the cop can play on the reflex vertices. The robber is trapped in an ever-shrinking region.



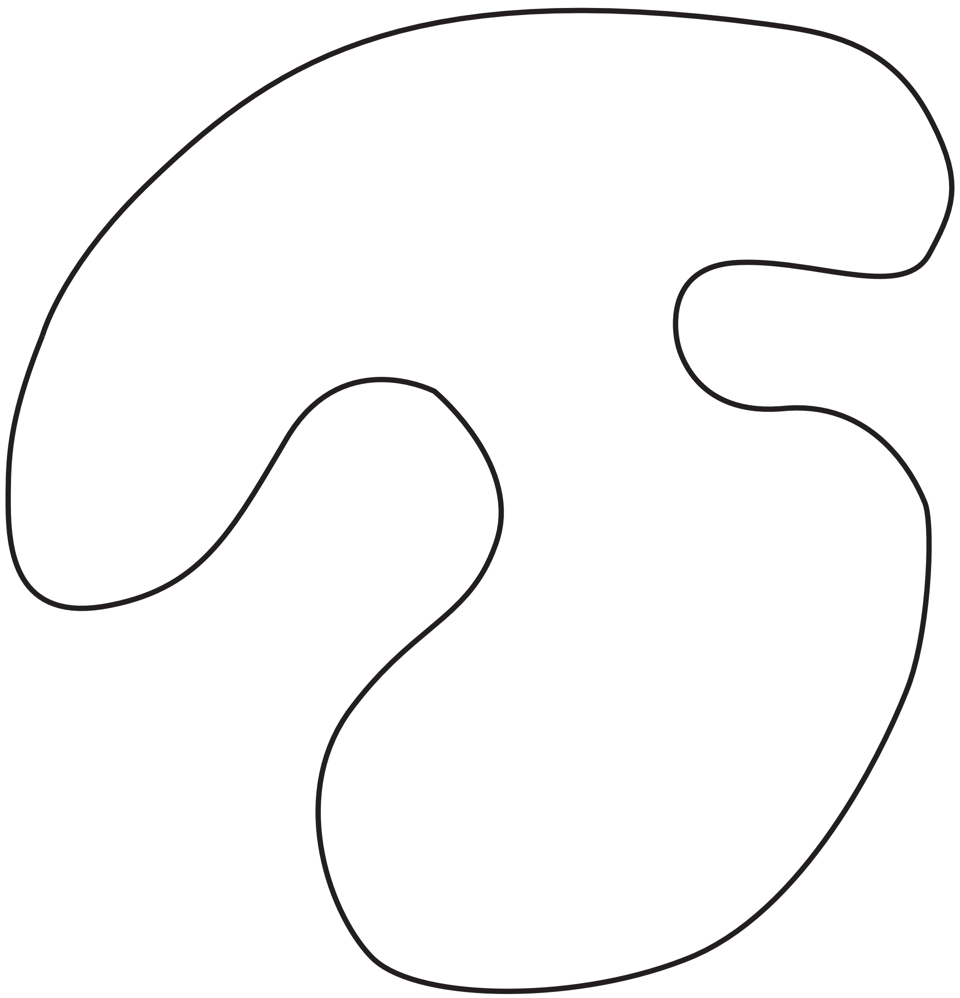
the robber can't leave P_i



$P_{i+1} \subsetneq P_i$

Cops and Robbers in Curved Regions

Theorem. [A.L., H. Vosoughpour] The cop wins the cops and robbers game in the interior of a closed region (with a reasonable boundary).



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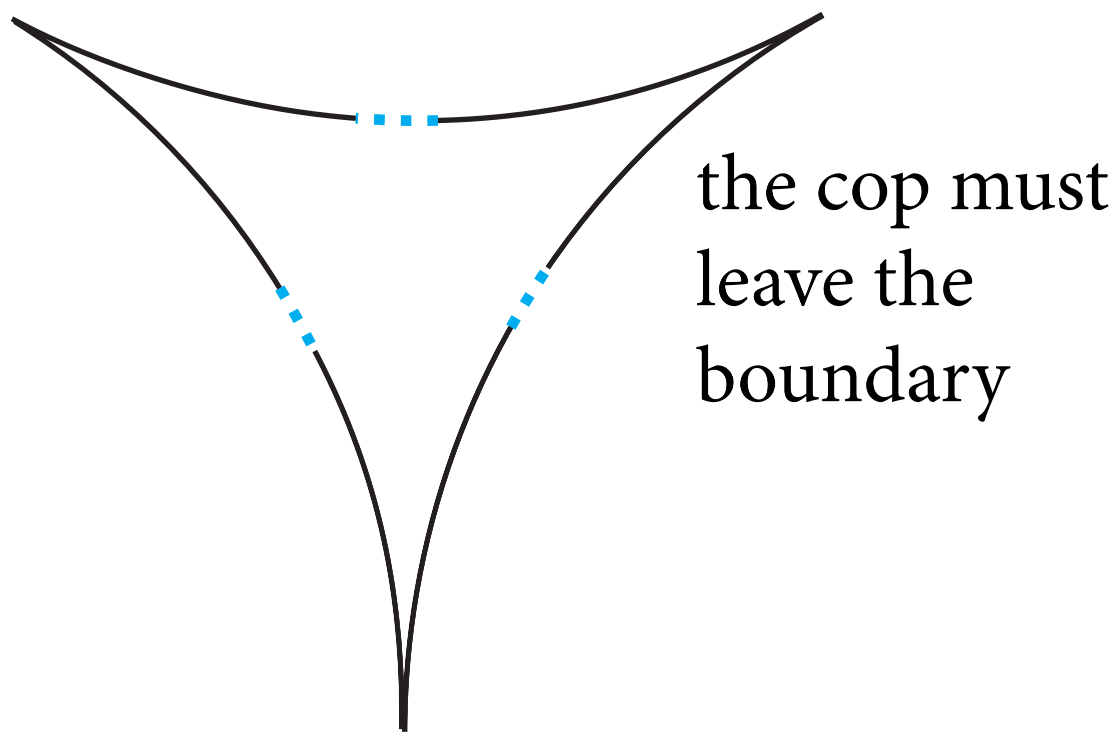

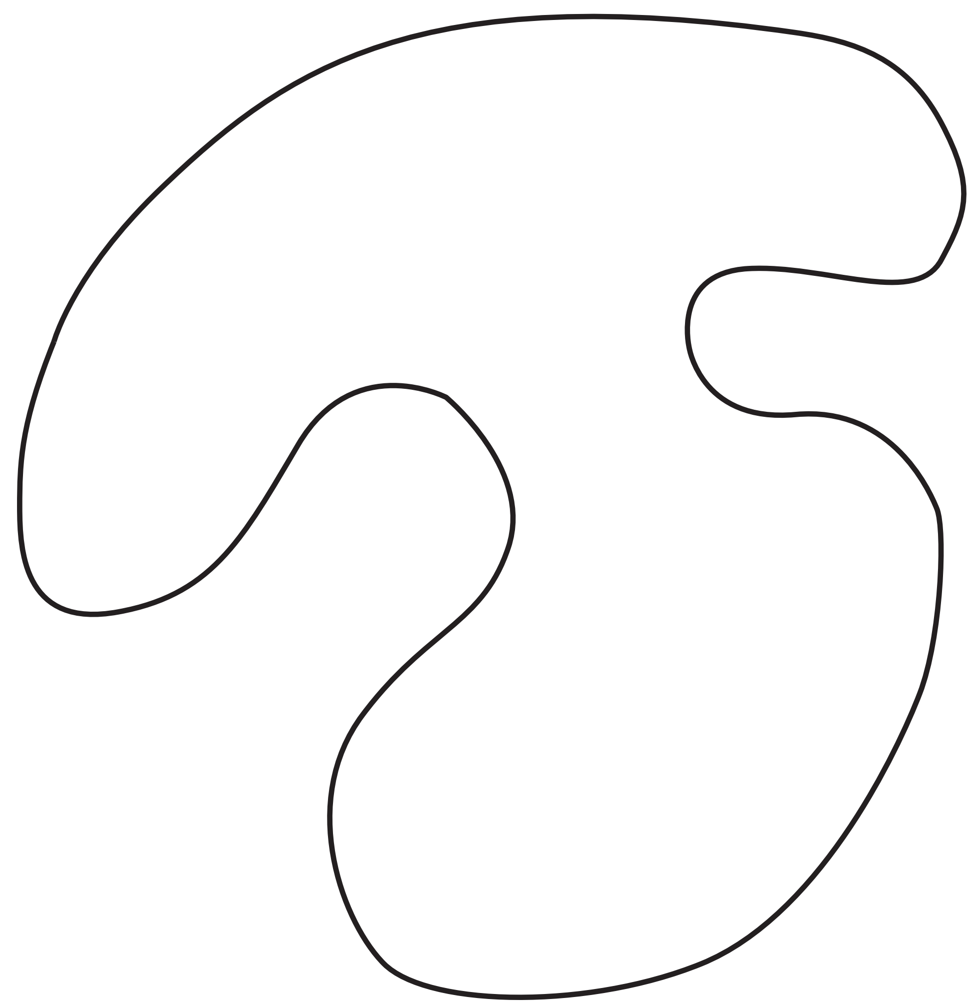

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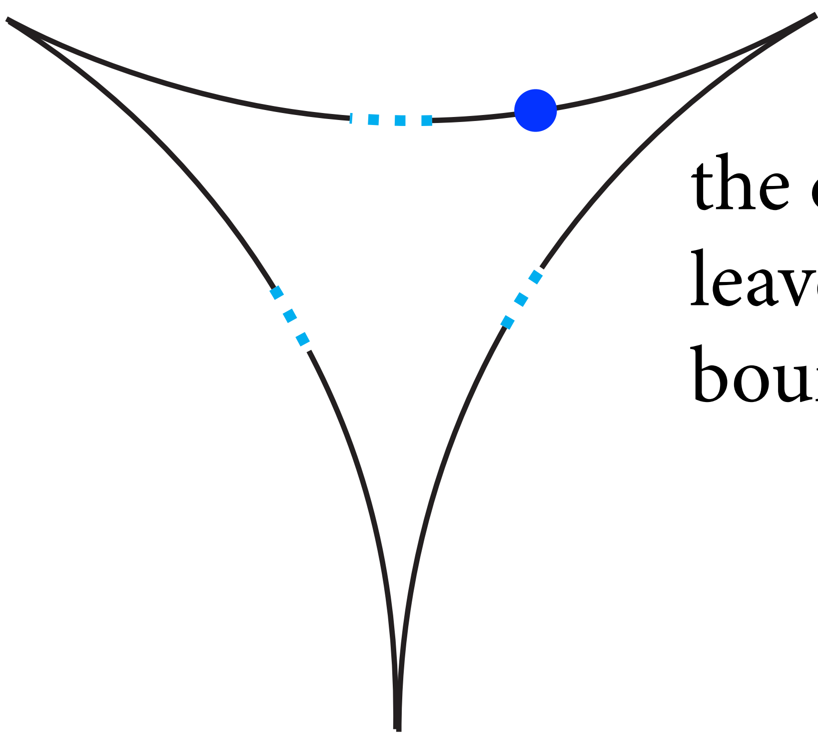

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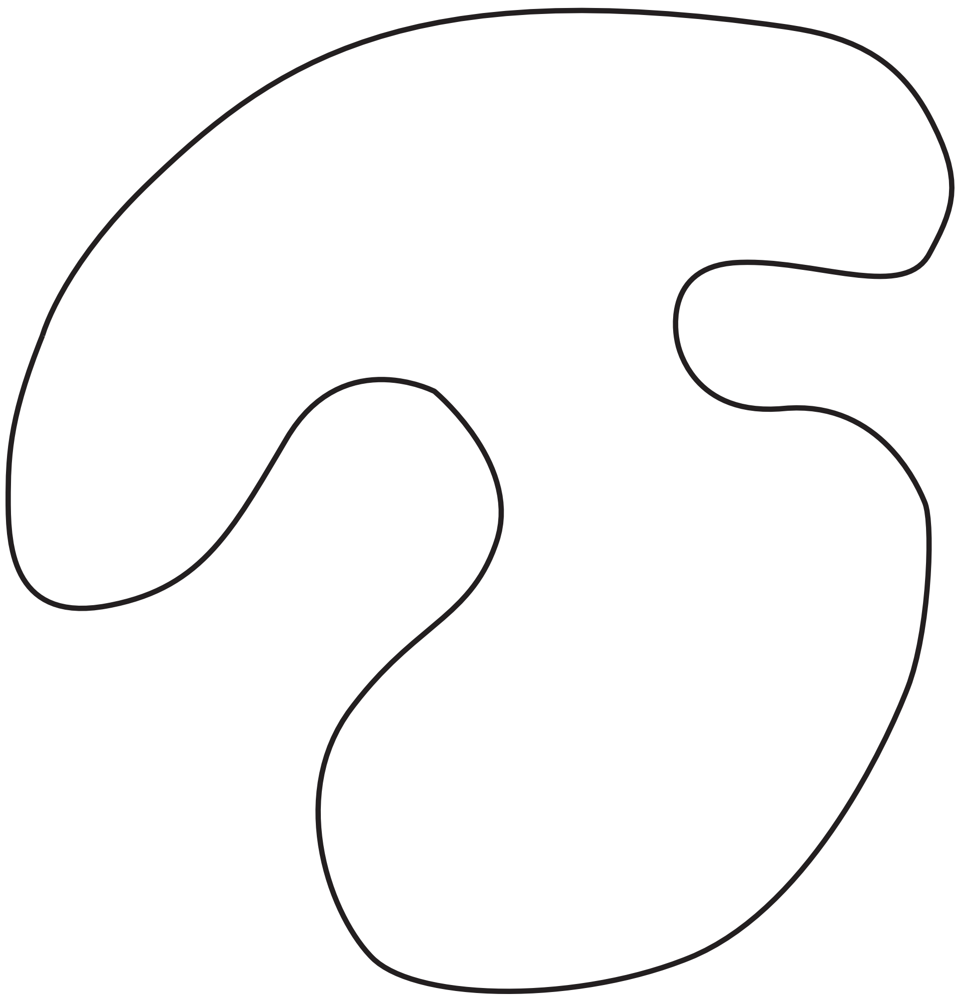

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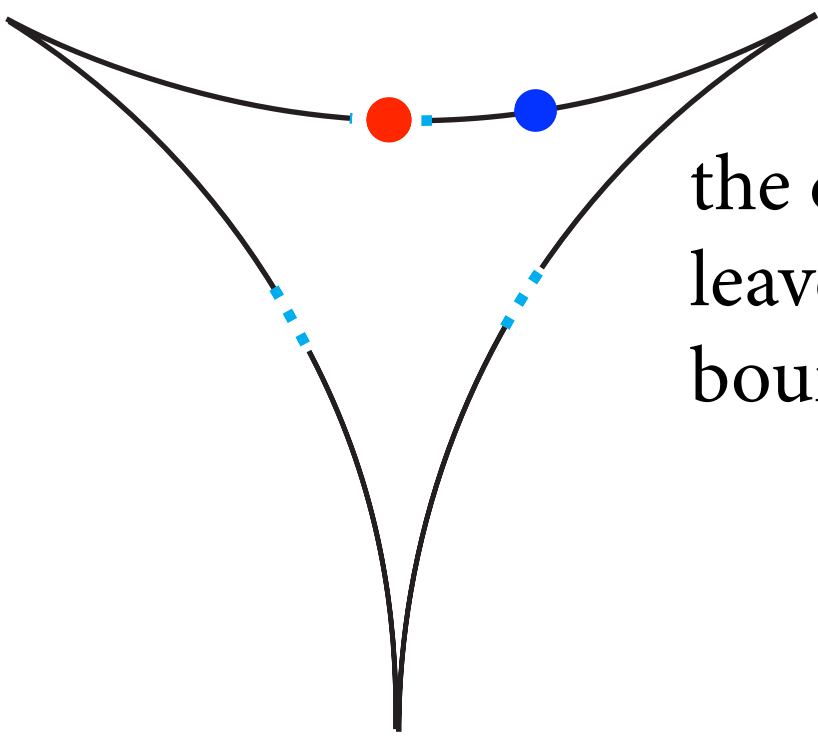

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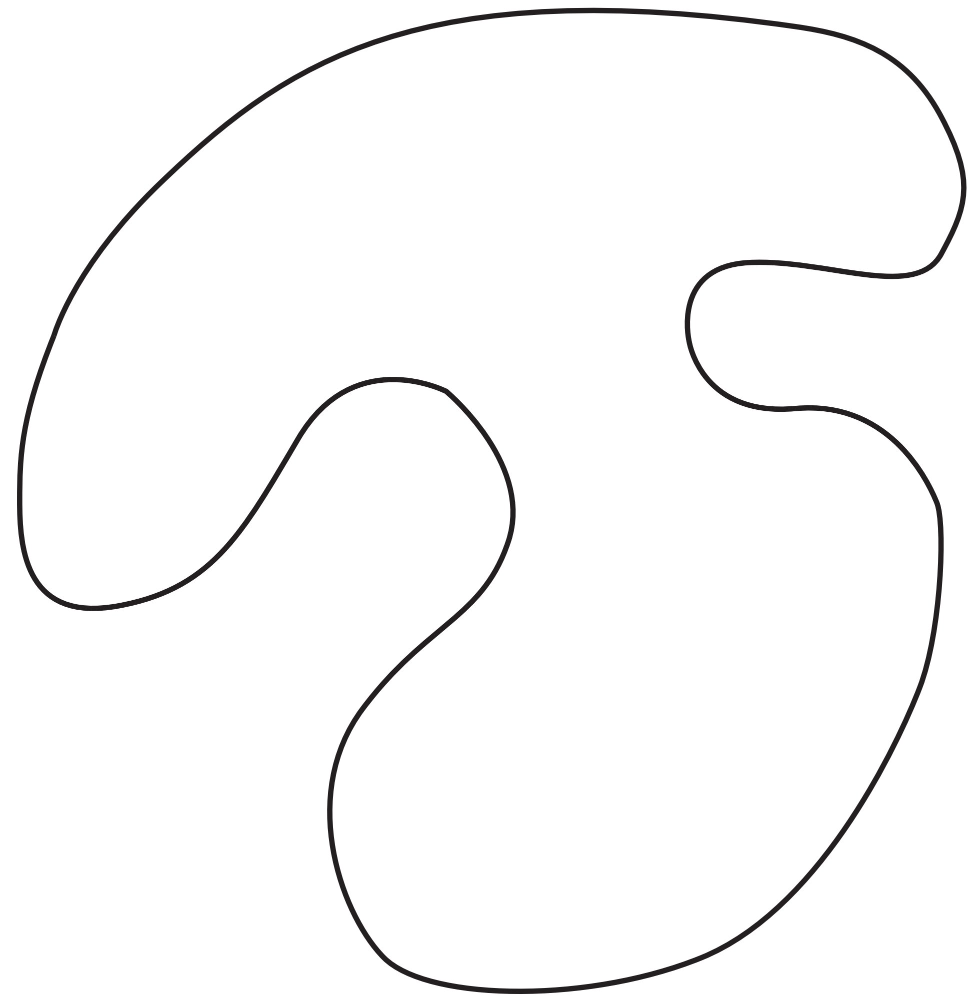

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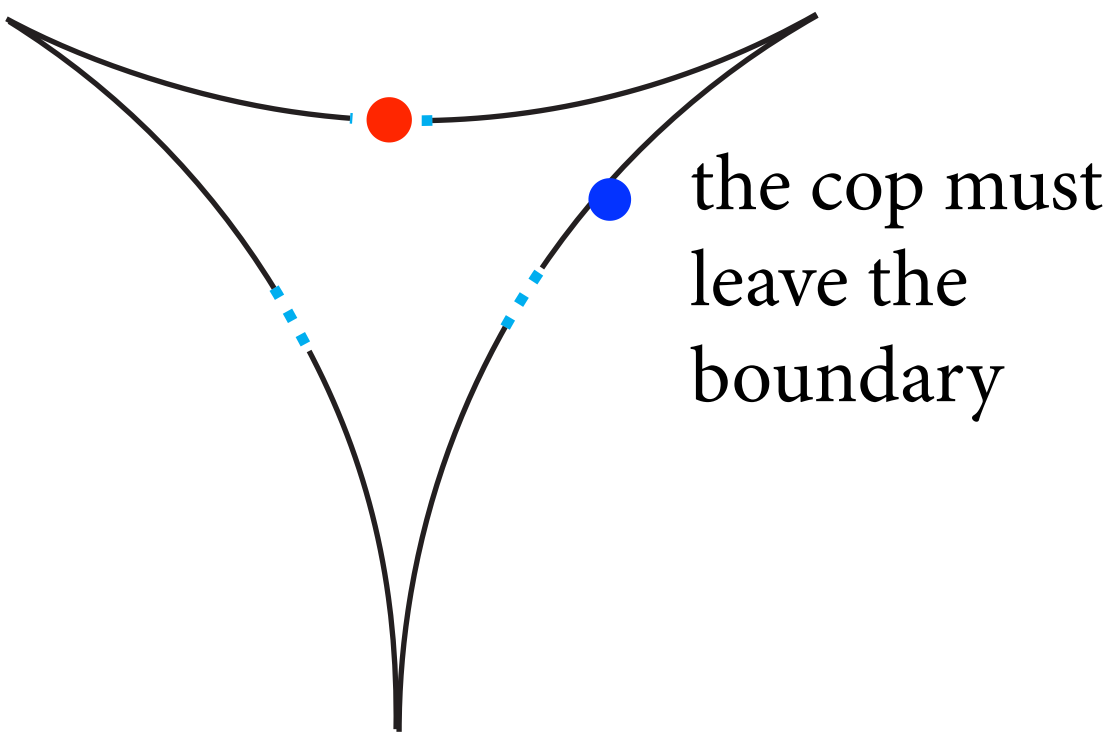

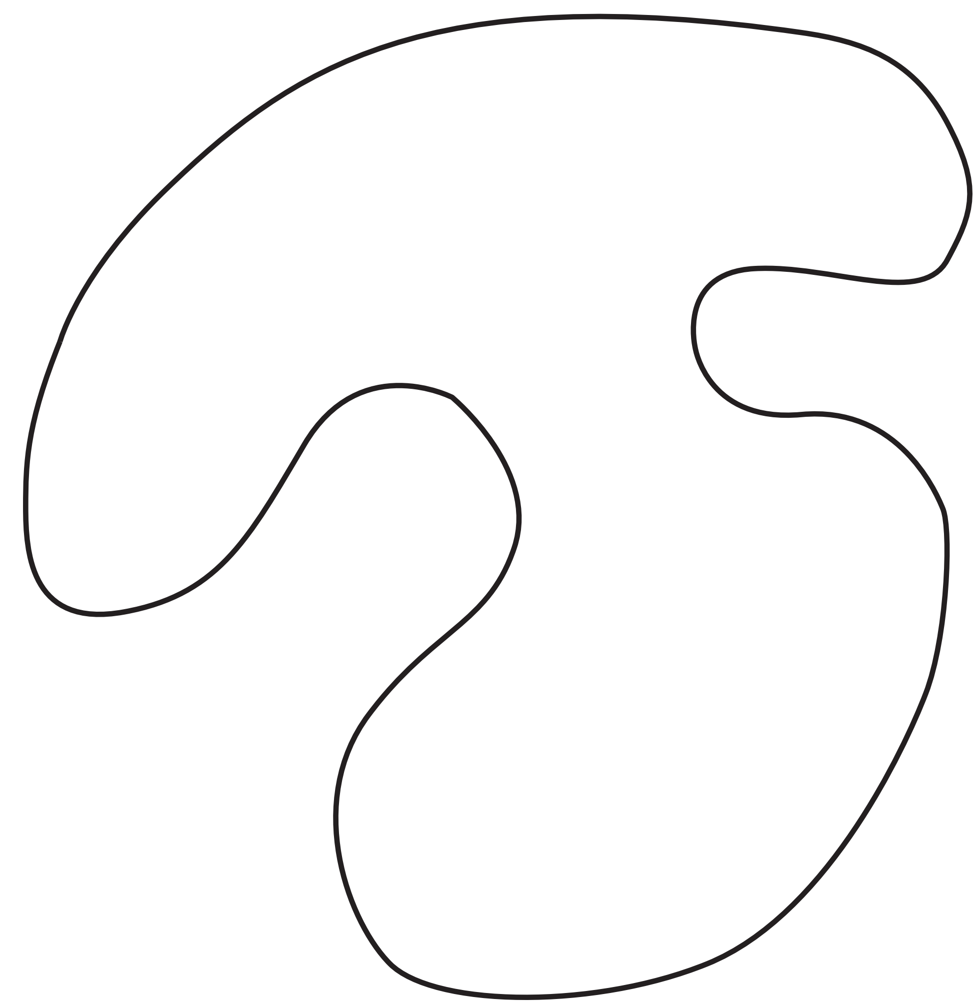

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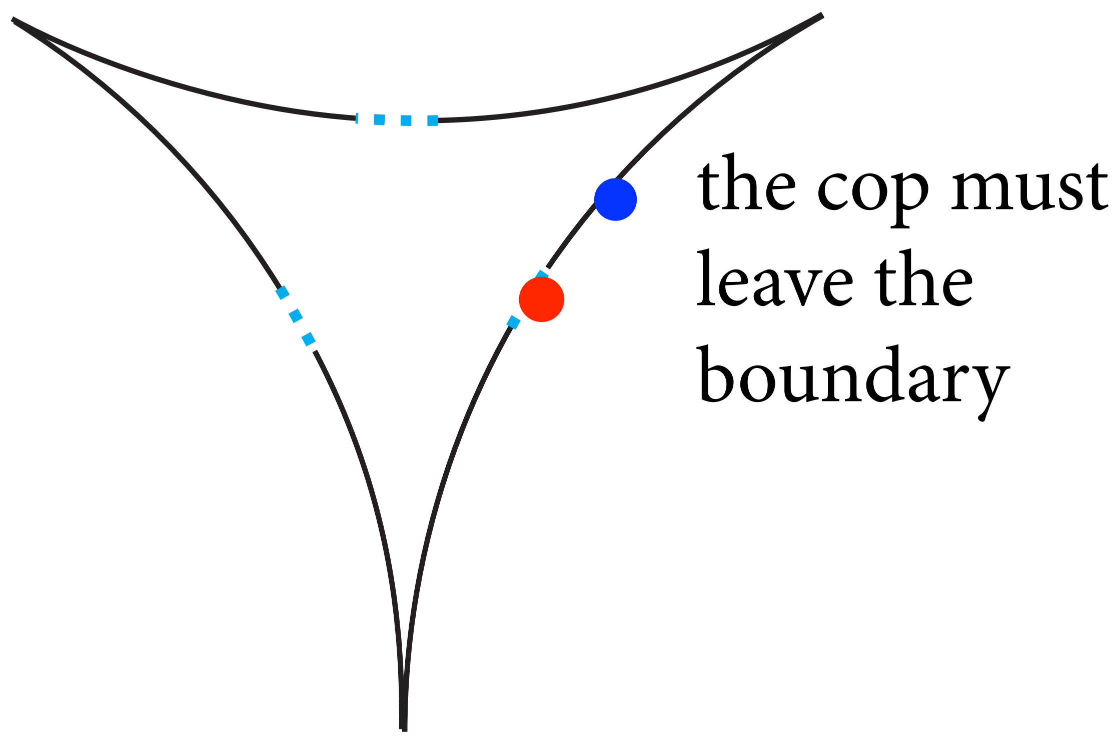

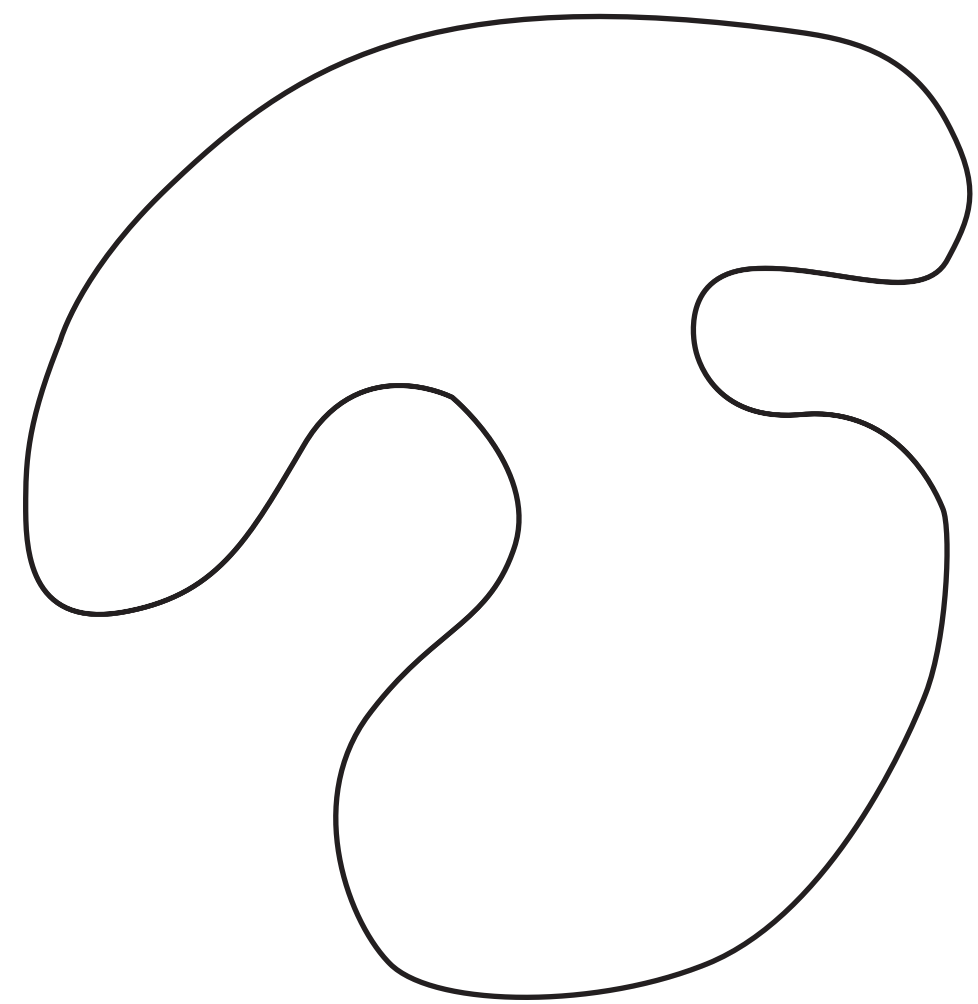

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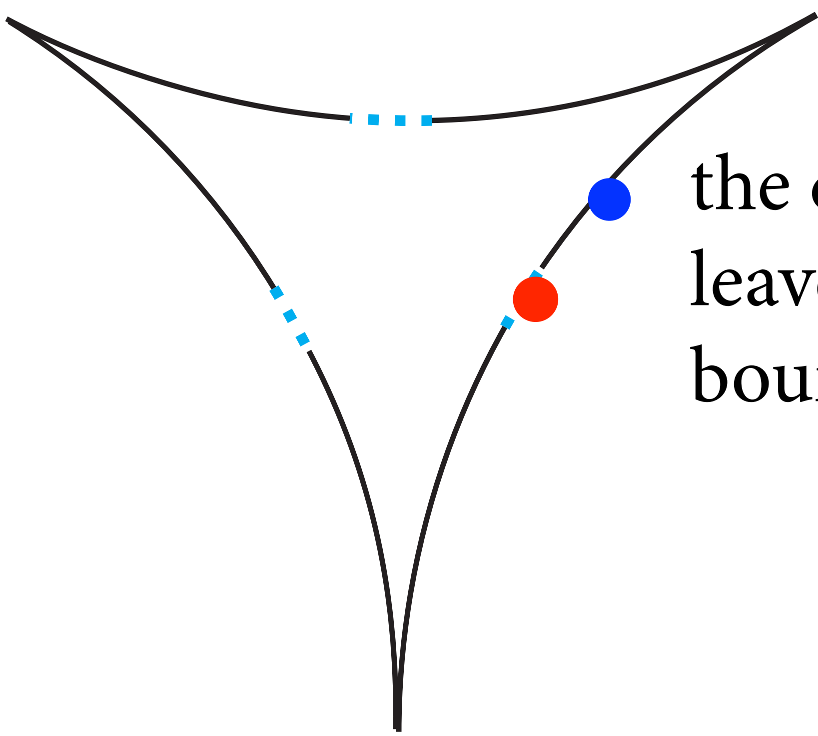

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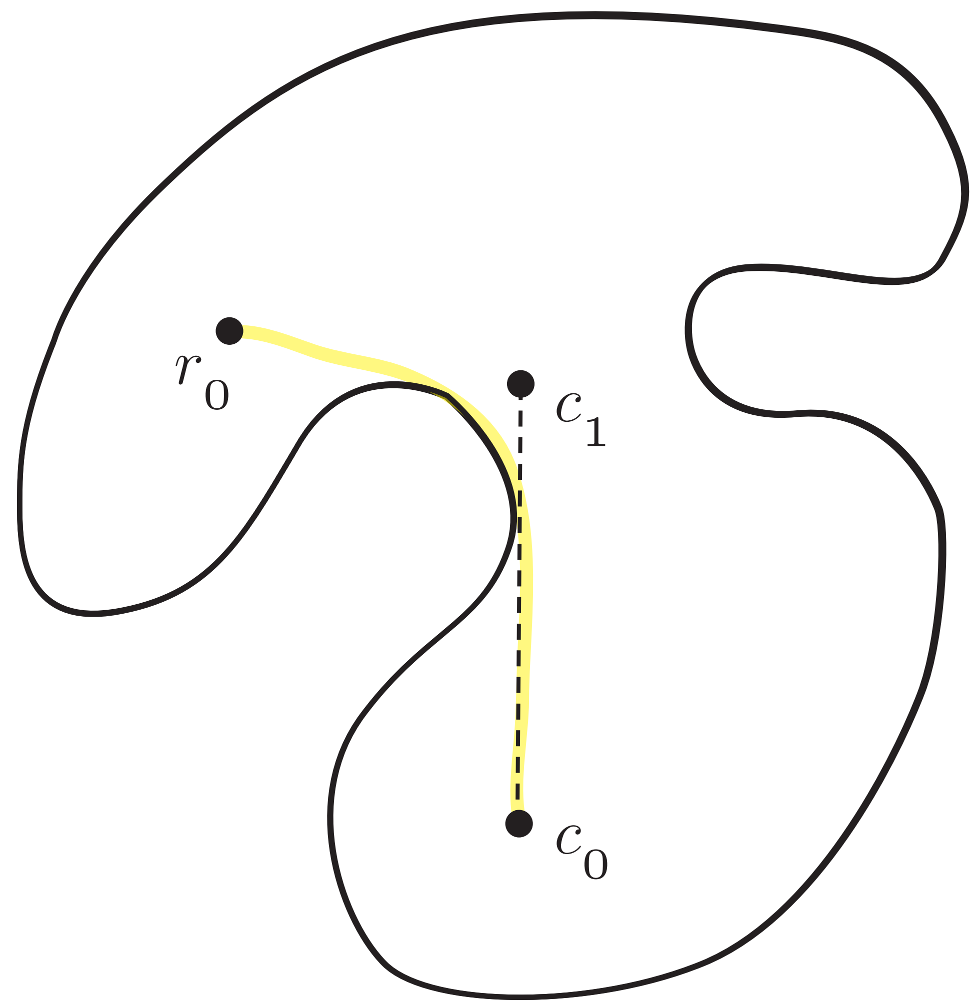

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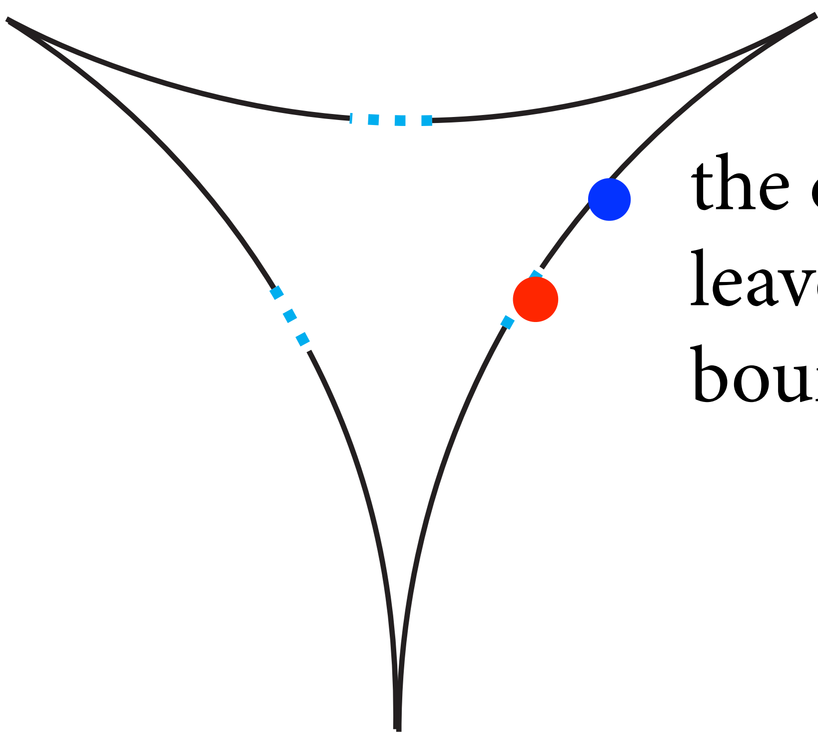

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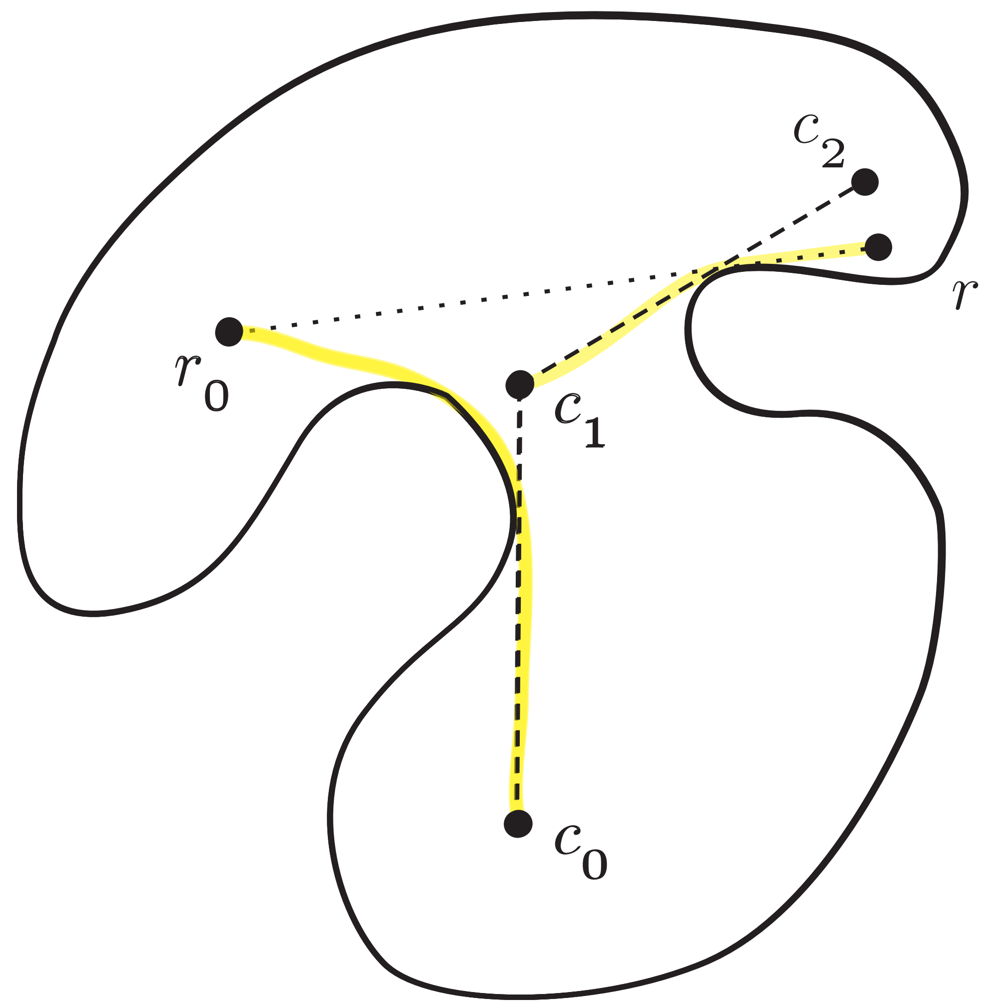

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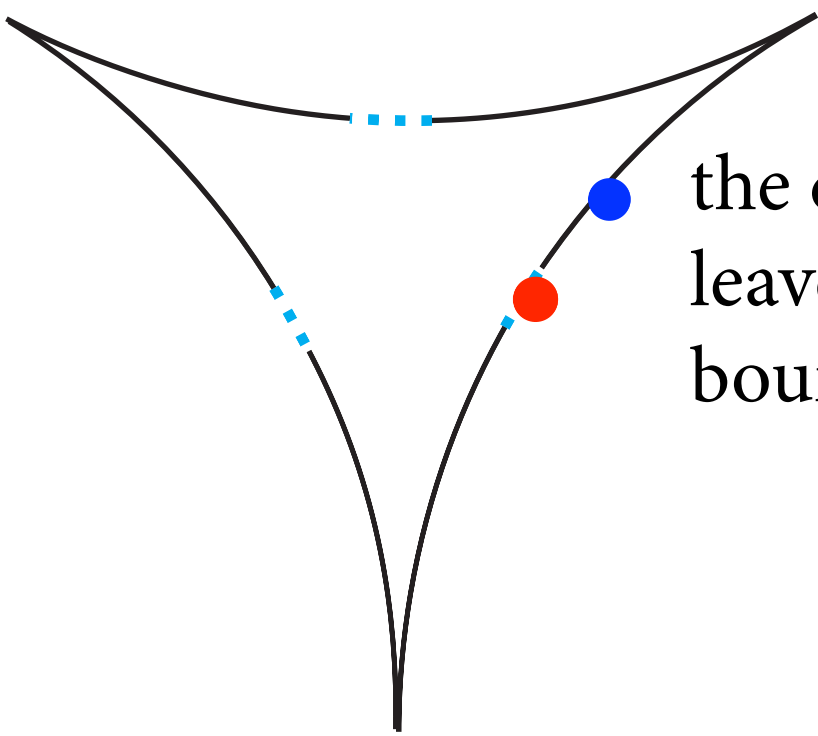

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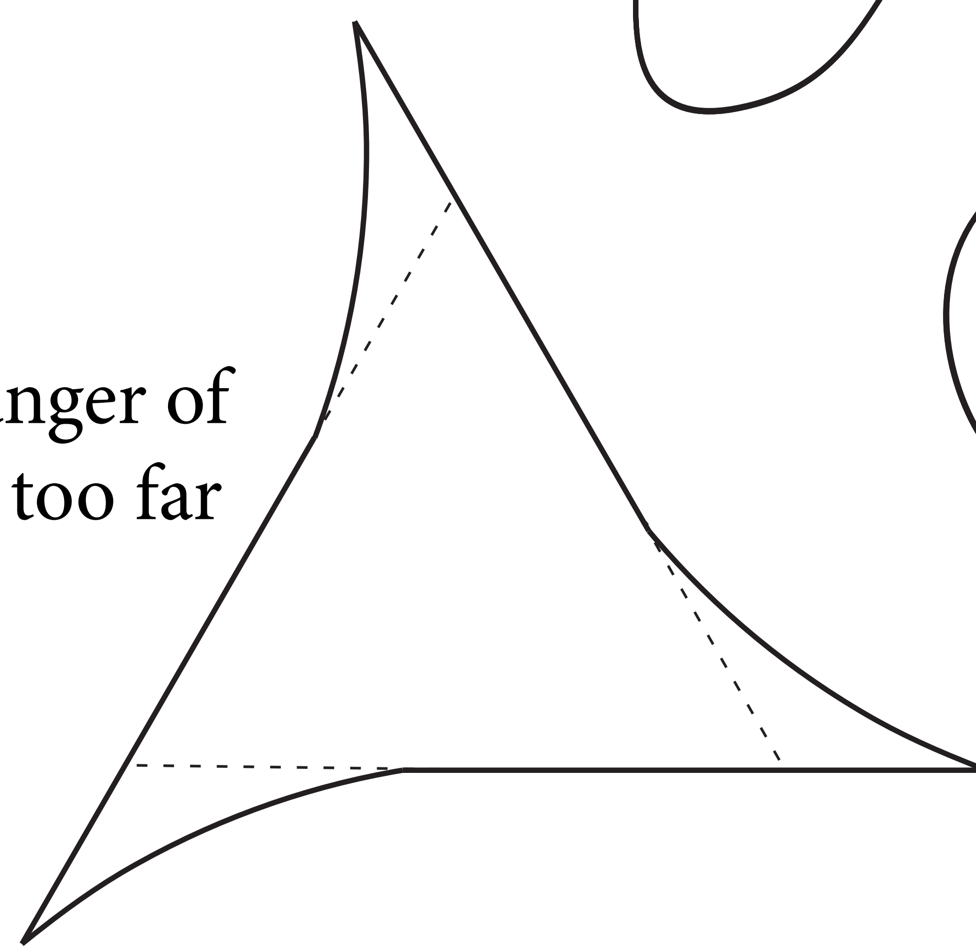
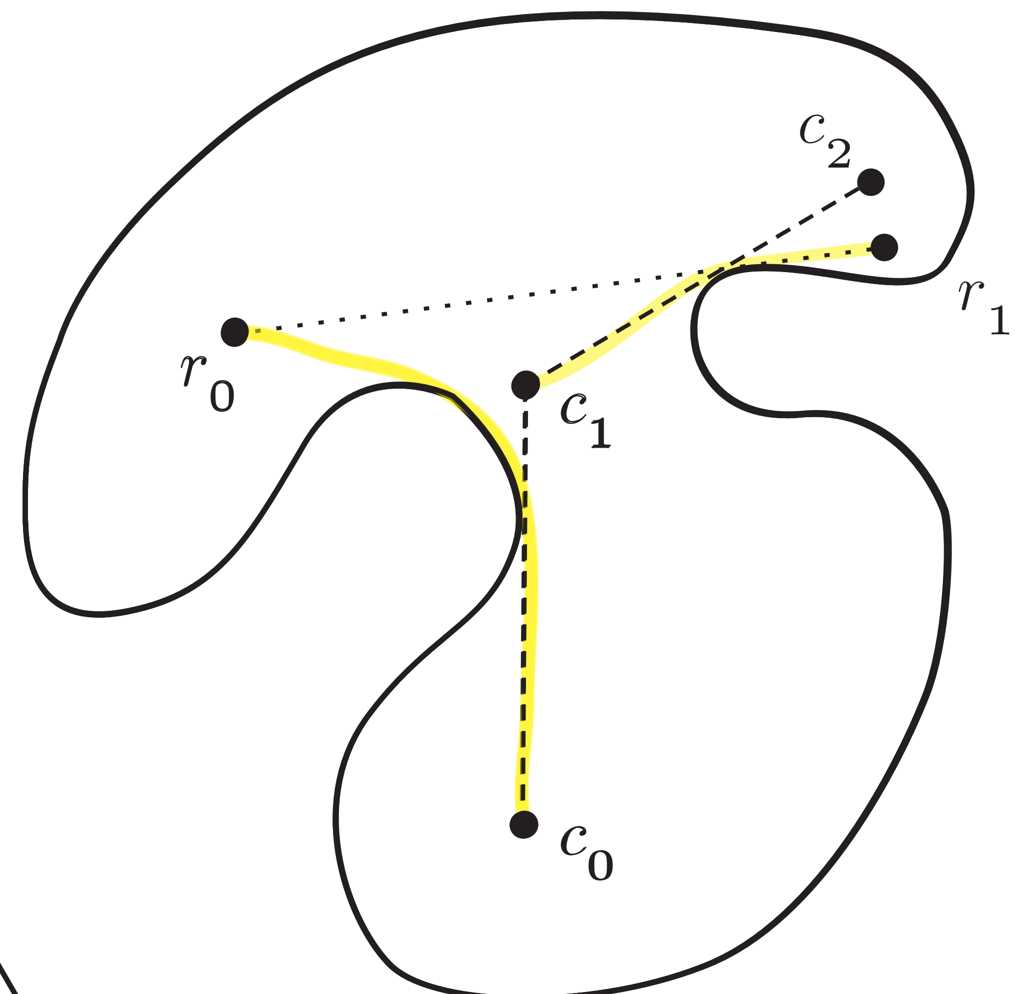


the danger of going too far



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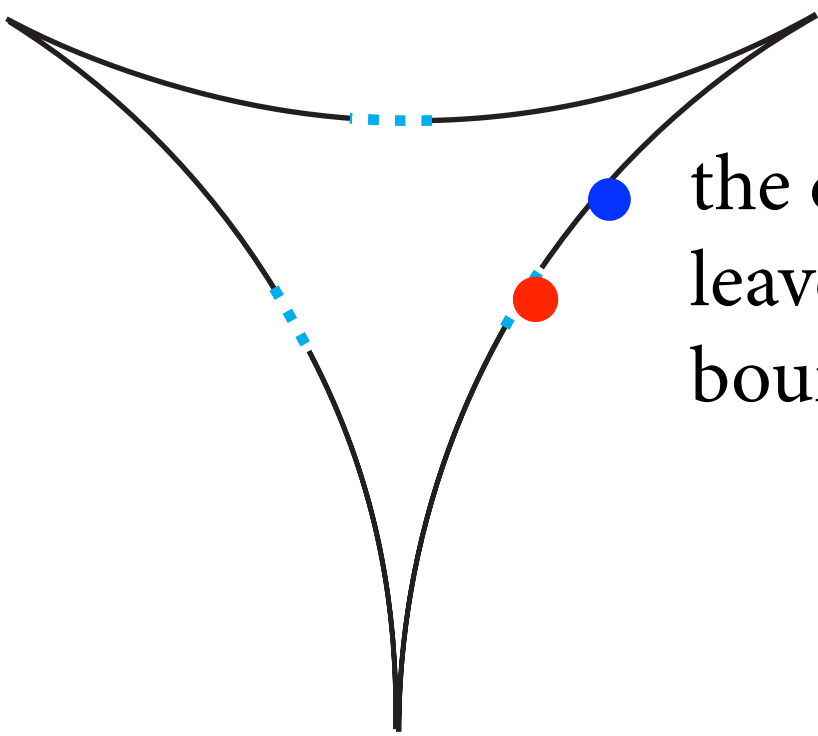

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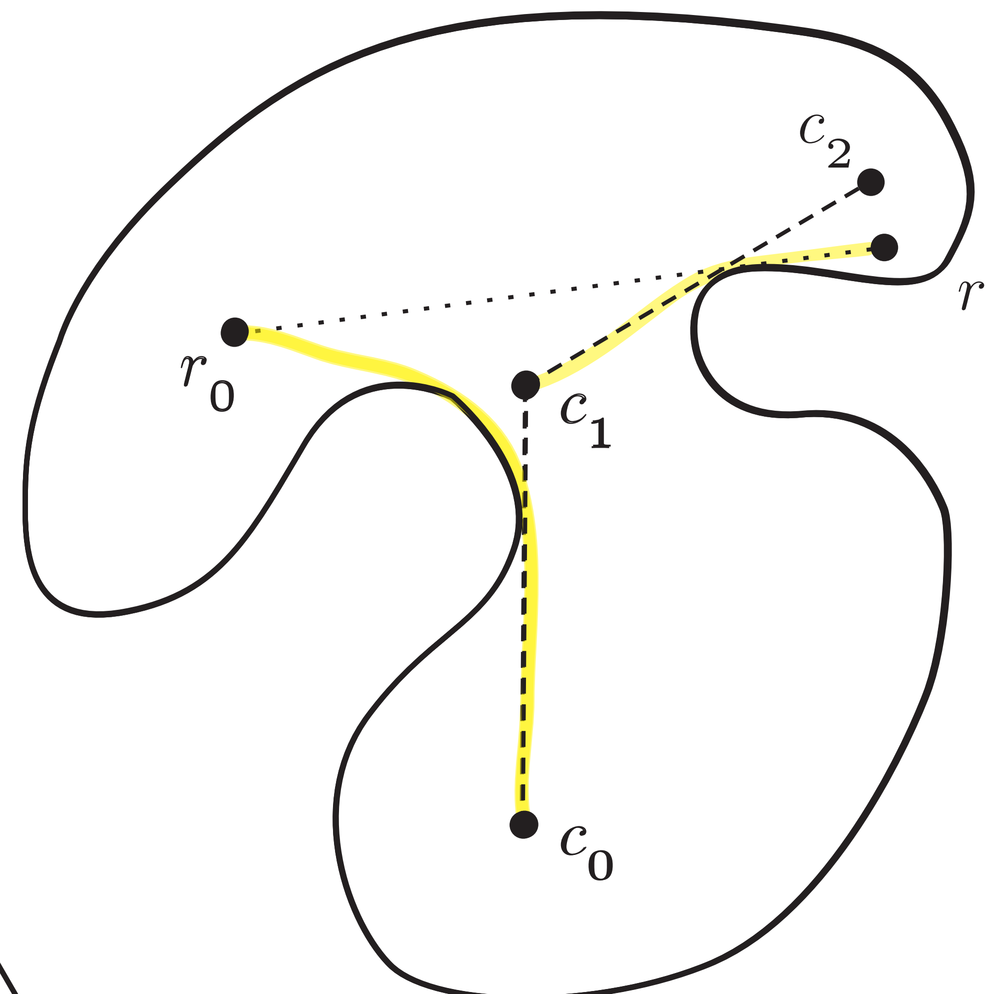

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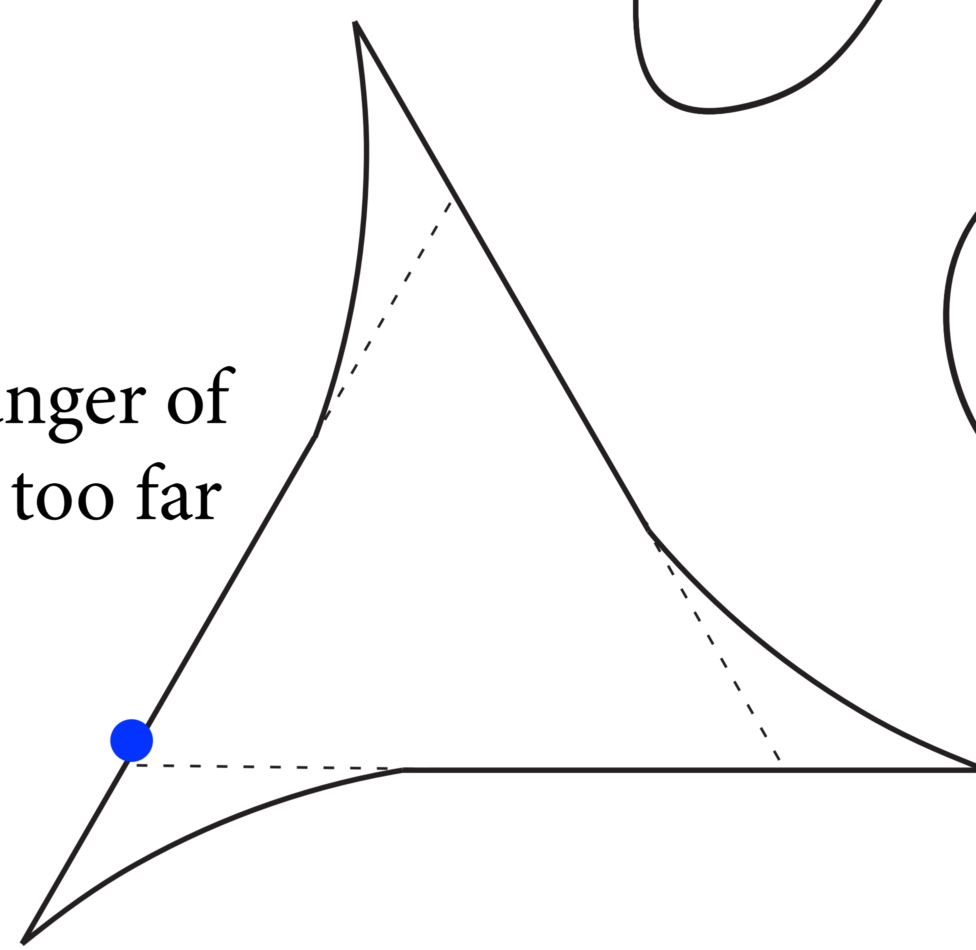

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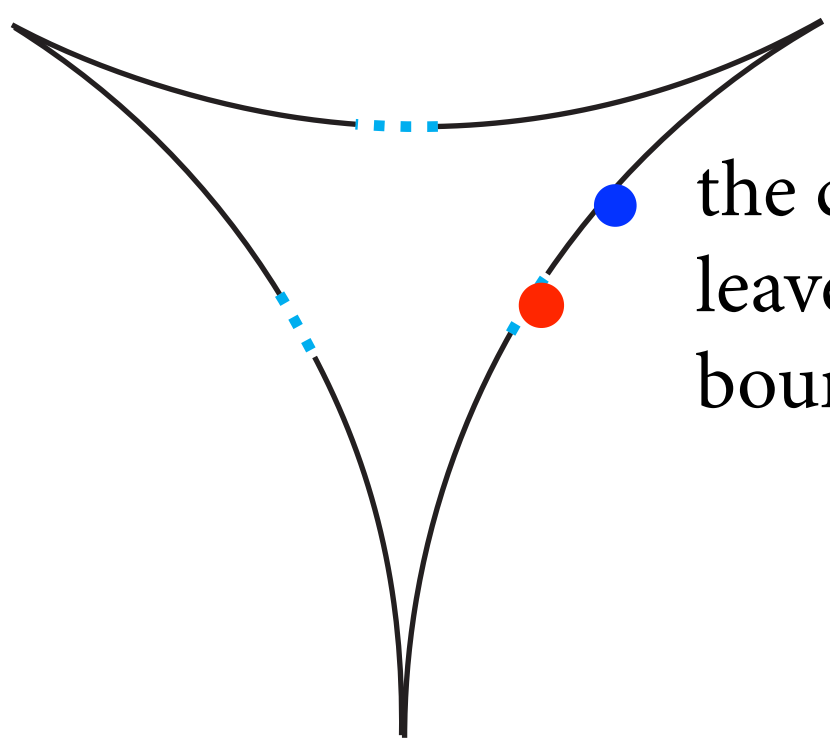

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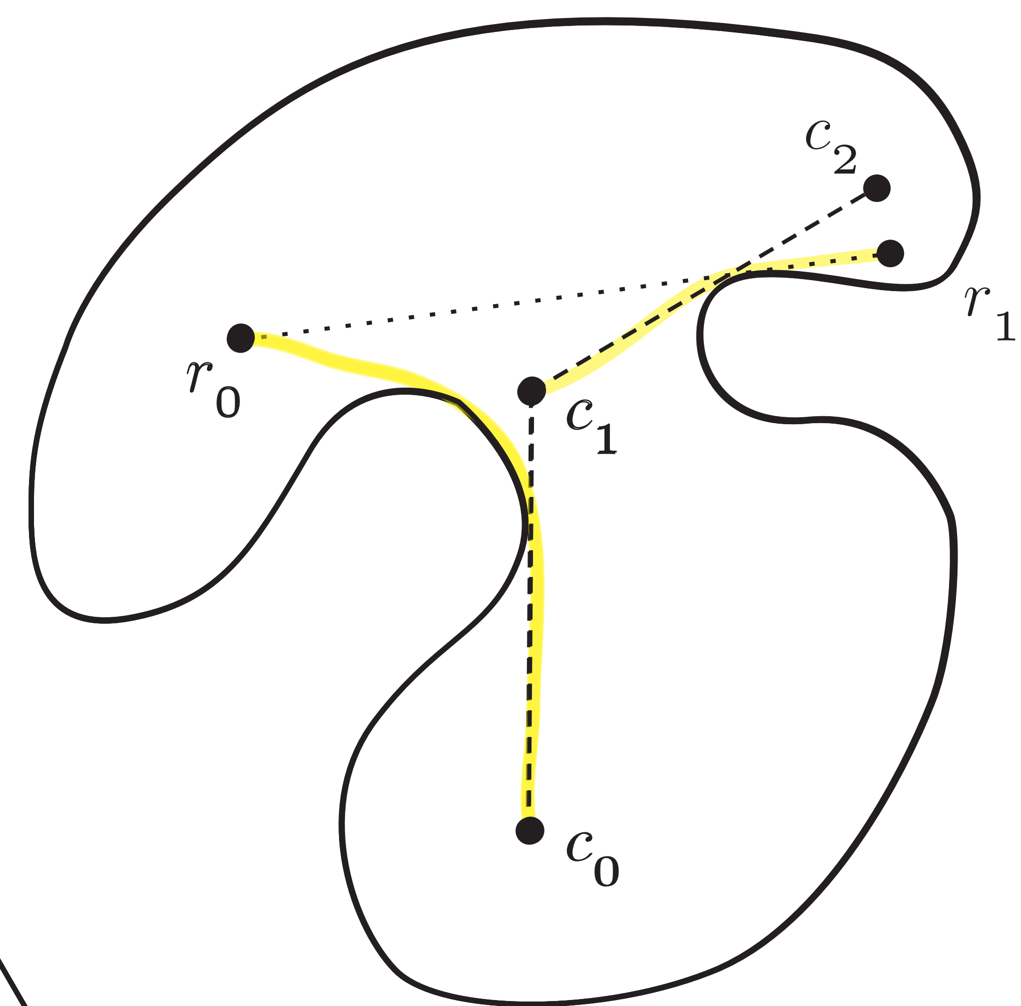

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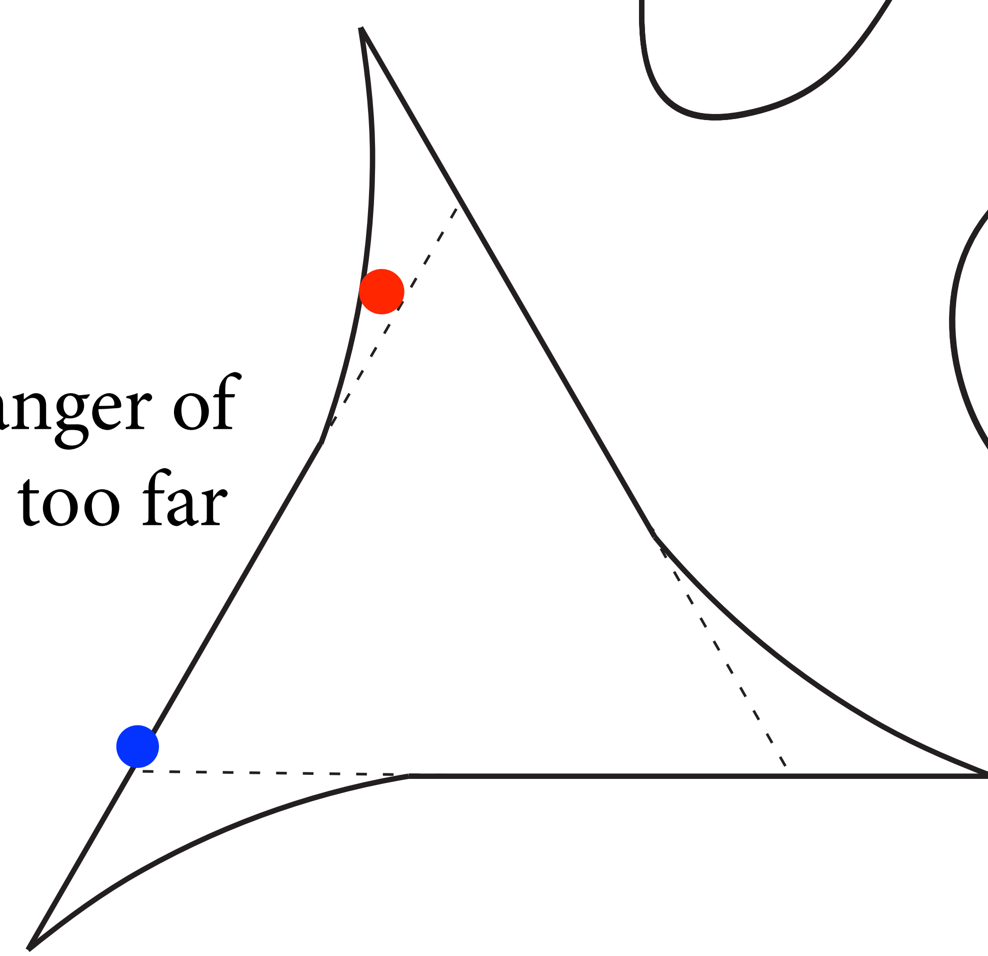

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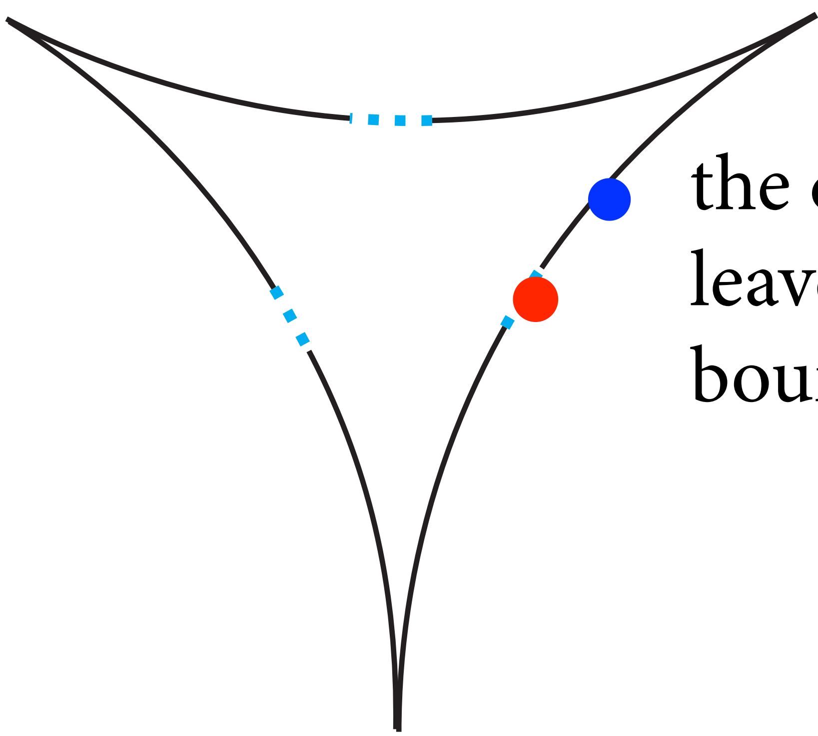

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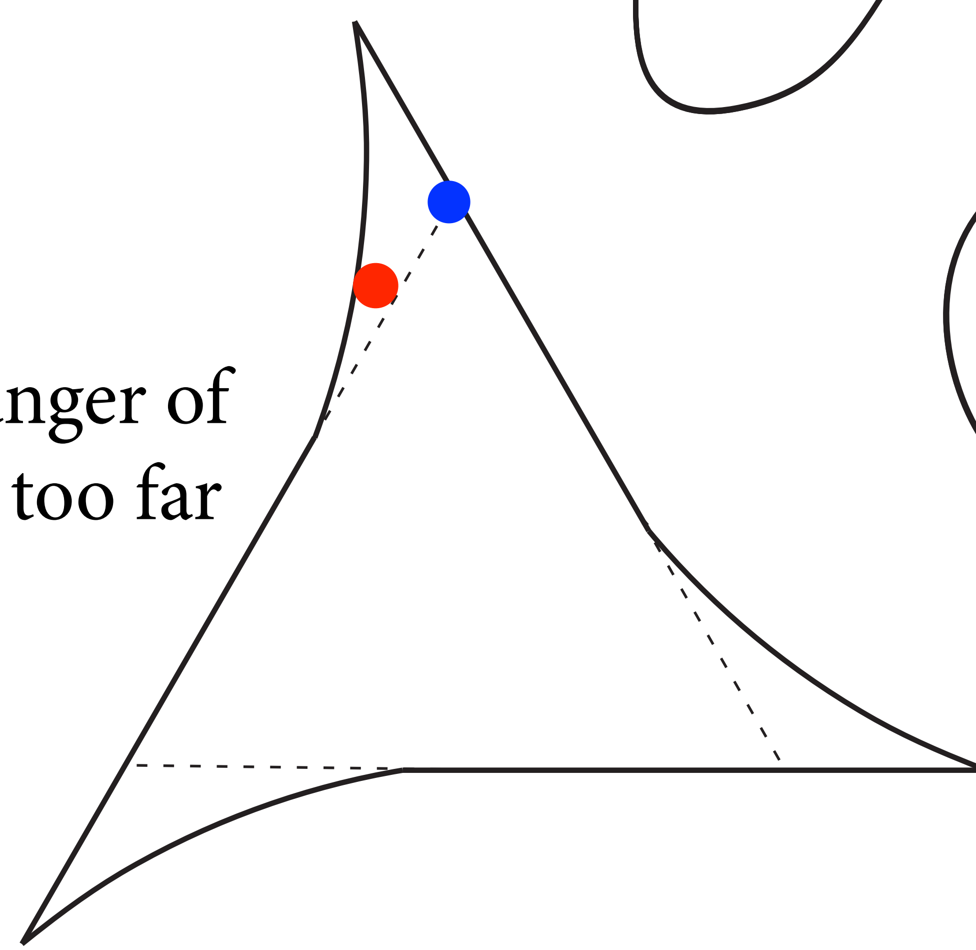

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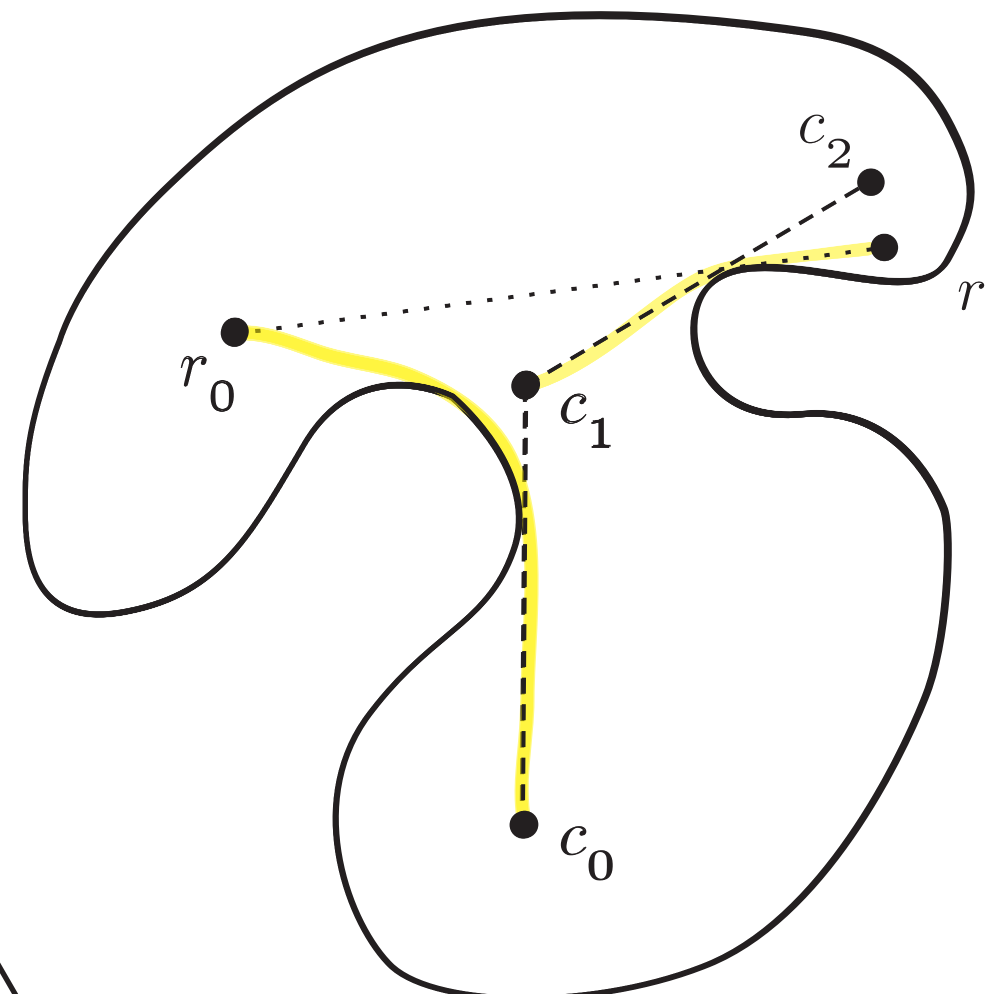

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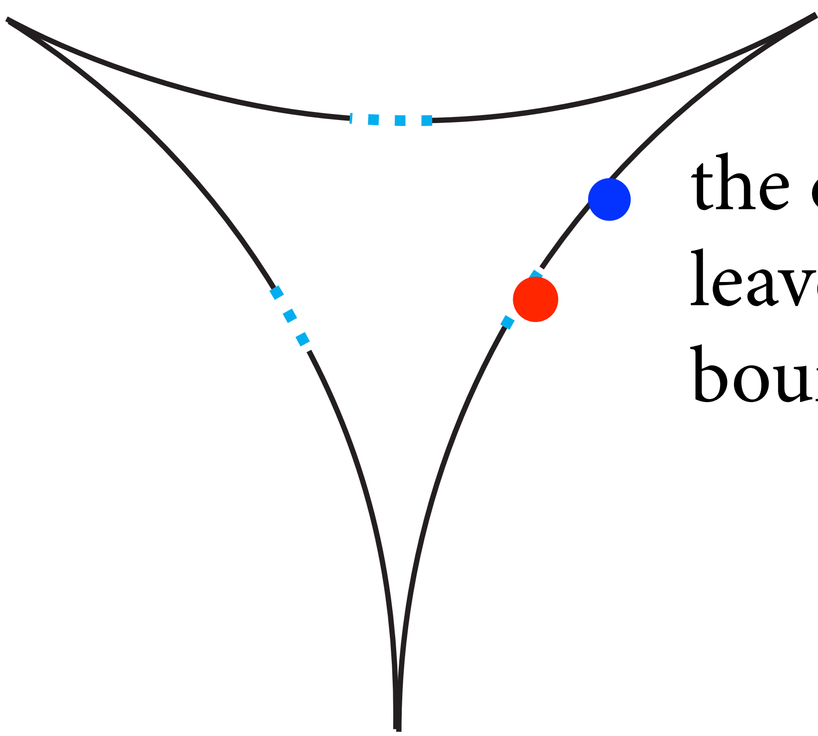

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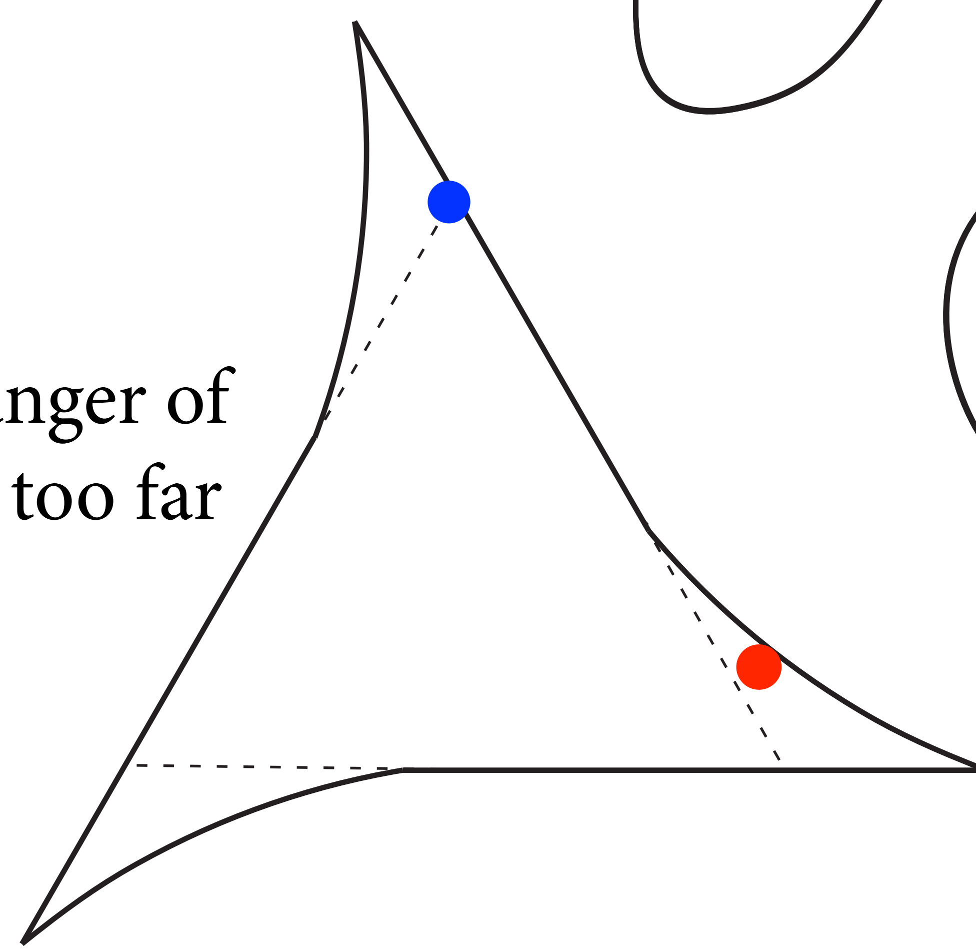

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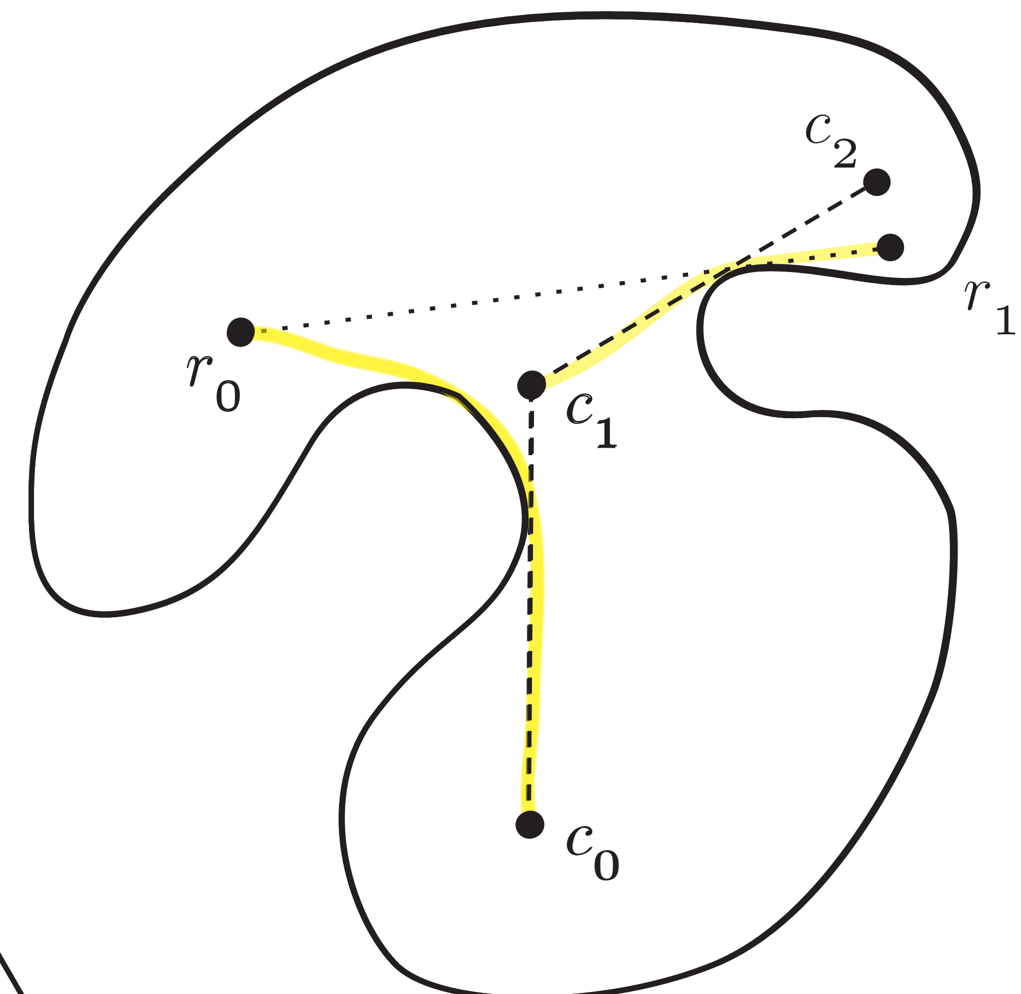
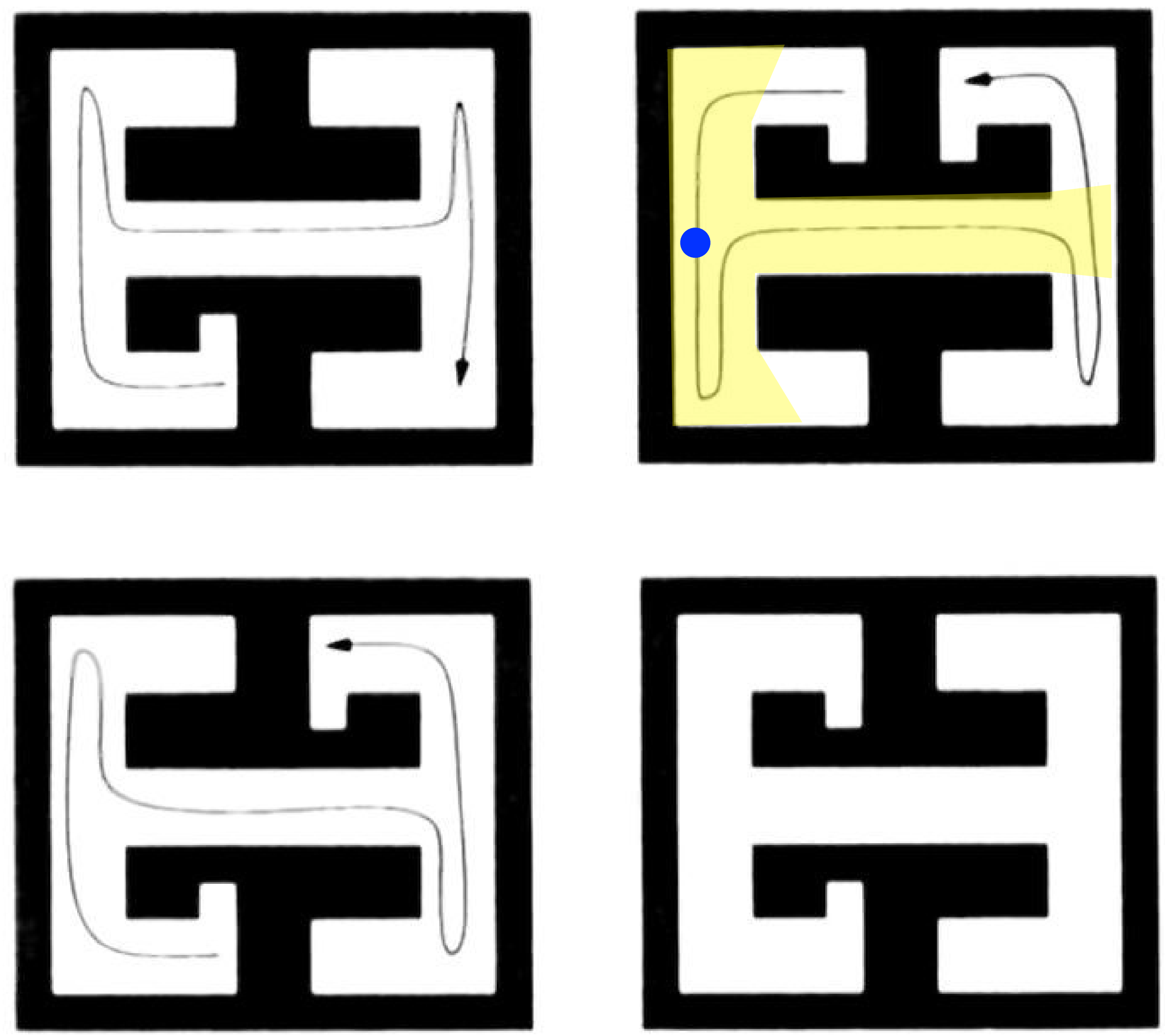


Visiblity-Based Pursuit Evasion

Guibas, Latombe, LaValle, Lin, Motwani. A visibility-based pursuit-evasion problem. 1999.

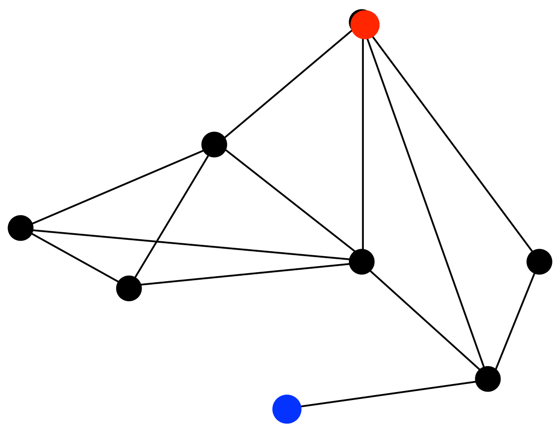
a fast evader is caught when seen by a pursuer

$O(\log n)$ pursuers suffice and are sometimes necessary.

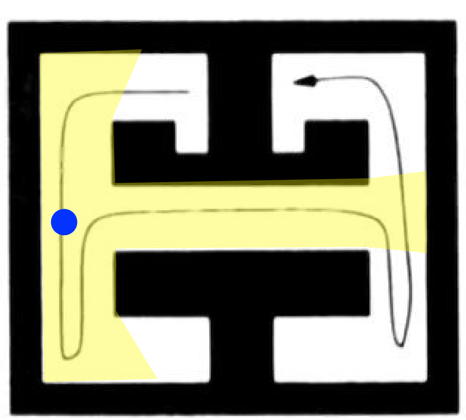


need 2 pursuers

Pursuit Evasion versus Cops and Robbers



	space is discrete/ continuous	capture by contact/ line-of-sight	full information/ no information	evader limited/ unlimited	players take turns/ move continuously
cops & robbers in a graph [Nowakowski & Winkler, 1983]				one edge	
visibility-based pursuit-evasion in a polygon [Guibas et al. 1999]					



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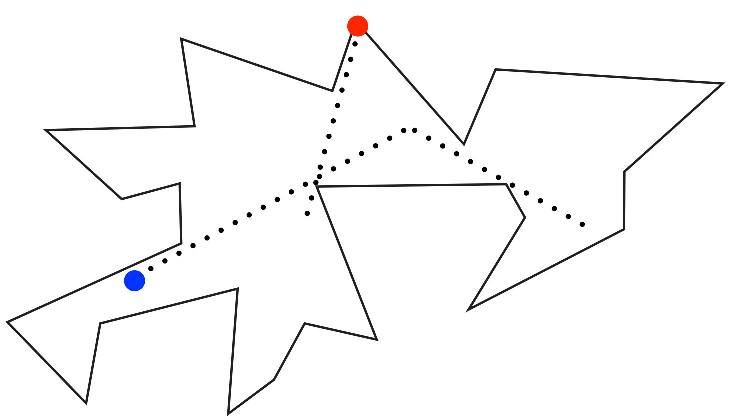

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graph searching (related to tree width) [Seymour & Thomas. 1993]					

Conclusions

- contributions to: visibility graphs, cops and robbers, pursuit evasion
  - visibility graphs $\subseteq$ dismantlable graphs
  - infinite visibility graphs are cop-win
  - the cop wins the cops and robbers game in a polygon (even played in all interior points, and even for curved regions)

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  - visibility graphs $\subseteq$ dismantlable graphs
  - infinite visibility graphs are cop-win
  - the cop wins the cops and robbers game in a polygon (even played in all interior points, and even for curved regions)
- OPEN. Do three cops suffice in polygonal regions with holes? (Three are sometimes necessary.)

Thank you